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Harmonic Winding Factors and MMF Analysis for Five-phase Fractional-slot Concentrated Winding PMSM

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Abstract – To enhance torque density by harmonic current injection, optimal slot/pole combinations for five-phase permanent magnet synchronous motors (PMSM) with fractional-slot concentrated windings (FSCW) are chosen. The synchronous and the third harmonic winding factors are calculated for a series of slot/pole combinations. Two five-phase PMSM, with general FSCW (GFSCW) and modular stator FSCW (MFSCW), are analyzed and compared in detail, including the stator structures, star of slots diagrams, and MMF harmonic analysis based on the winding function theory. The analytical results are verified by finite element method, the torque characteristics and phase back-EMF are also taken into considerations. Results show that the MFSCW PMSM can produce higher average torque, while characterized by more MMF harmonic contents and larger ripple torque.

Keywords: Winding factors, Winding function, MMF, GFSCW, MFSCW

1. Introduction

Multiphase permanent magnet synchronous motors offer many advantages such as high fault-tolerance ability, high efficiency and torque density with low torque ripple, and have been gaining more and more interest. A multiphase motor with full-pitch concentrated windings is capable of improving torque density by harmonic current injection[1]. Permanent magnet synchronous motors with fractional-slot concentrated windings (FSCW) feature short coil end, high slot fill factor and so on [2]. Besides, the mutual inductances between the phases can be effectively reduced, assuring good fault-tolerance ability [3]. Thus, a multiphase permanent magnet synchronous motor with FSCW is expected to incorporate the two advantages.

To realize the torque density enhancement by harmonic current injection, the winding factor of useful harmonics of multiphase windings should be high enough. Therefore, it is necessary to find the appropriate slot/pole combinations with relatively high synchronous and harmonic winding factors for the general FSCW (GFSCW) motor design. A kind of modular FSCW (MFSCW) motor, which is characterized with simplicity of assembly and lower difficulty of maintenance and replacement [4], is proposed recently. All the winding factors of MFSCW motor are unit because of the full-pitch concentrated winding. However, detailed MMF analysis is not presented in [4].

This paper compares the two motor types, GFSCW motor and MFSCW motor. Specifically, two five-phase motors with double-layer structures are exemplified. The stator structures and the star of slots diagrams are described. The stator air-gap MMF, phase back-EMF and torques produced in various running modes are quantitatively analyzed. The criteria and results for selecting the optimal slot/pole combinations are presented in Section II. In Section III, the stator air-gap MMF harmonic analysis is carried out based on winding function theory. Analysis results of the two motors by finite element method (FEM) are given in Section IV. The analytical results based on winding function theory are verified by FEM. The phase back-EMF and torques produced in three running modes are compared. And conclusions are drawn in Section V.

2. Selection of Slot/pole Combination

Slot/pole combination of FSCW motor determines largely the winding factors and MMF harmonic contents [5]. Besides, rotor loss, net radial force and ripple torque are influenced [2]. Therefore, choosing a optimized slot/pole combination is vital in the motor design process.

To improve the torque density through harmonic current injection for five-phase FSCW motor, proper slot/pole combination should be characterized by followings:

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Table 1. Synchronous Winding Factors of GFSCW Motor

s/p	2	4	6	8	10	12	14	16	18	20	22	24
5	0.5878	0.9511	0.9511	0.5878	-	0.5878	0.9511	0.9511	0.5878	-	0.5878	0.9511
10	-	0.5878	0.8090	0.9511	-	0.9511	0.8090	0.5878	0.3090	-	0.3090	0.5878
15	0.2049	0.4008	0.5878	0.7323	-	0.9511	0.9800	0.9800	0.9511	-	0.7323	0.5878
20	-	-	0.4484	0.5878	-	0.8090	0.8800	0.9511	0.9755	-	0.9755	0.9511
25	0.1234	0.2448	0.3623	0.4742	0.5878	0.6738	0.7584	0.8311	0.8906	0.9511	0.9668	0.9823
30	-	0.2049	-	0.4008	-	0.5878	0.6594	0.7323	0.8090	-	0.9002	0.9511
35	0.0882	0.1757	0.2618	0.3457	-	0.5047	0.5878	0.6474	0.7112	-	0.8212	0.8665
40	-	-	0.2299	-	-	0.4484	0.5145	0.5878	0.6395	-	0.7487	0.8090
45	0.0686	0.1369	0.2049	0.2712	-	0.4008	0.4619	0.5214	0.5878	-	0.6834	0.7323
50	-	0.1234	0.1844	0.2448	-	0.3623	0.4191	0.4742	0.5274	0.5878	0.6274	0.6738
55	0.0562	0.1121	0.1678	0.2228	-	0.3306	0.3830	0.4341	0.4837	-	0.5878	0.6227
60	-	-	-	0.2049	-	-	0.3527	0.4008	0.4484	-	0.5360	0.5878

Table 2. The Third Harmonic Winding Factors of GFSCW Motor

s/p	2	4	6	8	10	12	14	16	18	20	22	24
5	0.9511	0.5878	0.5878	0.9511	-	0.9511	0.5878	0.5878	0.9511	-	0.9511	0.5878
10	-	0.9511	0.3090	0.5878	-	0.5878	0.3090	0.9511	0.8090	-	0.8090	0.9511
15	0.5129	0.8300	0.9511	0.5129	-	0.5878	0.8300	0.8300	0.5878	-	0.5129	0.9511
20	-	-	0.8800	0.9511	-	0.3090	0.1394	0.5878	0.7939	-	0.7939	0.5878
25	0.3179	0.5911	0.7813	0.8618	0.9511	0.6653	0.4160	0.1082	0.2147	0.5878	0.7291	0.8482
30	-	0.5129	-	0.8300	-	0.9511	0.7060	0.5129	0.3090	-	0.2697	0.5878
35	0.2291	0.4416	0.6223	0.7582	-	0.8601	0.9511	0.7186	0.5665	-	0.1537	0.0772
40	-	-	0.5627	-	-	0.8800	0.8637	0.9511	0.7387	-	0.4527	0.3090
45	0.1788	0.3498	0.5129	0.6391	-	0.8300	0.8553	0.8553	0.9511	-	0.6391	0.5129
50	-	0.3179	0.4627	0.5911	-	0.7813	0.8364	0.8618	0.8567	0.9511	0.7567	0.6653
55	0.1466	0.2888	0.4226	0.5440	-	0.7360	0.8009	0.8423	0.8591	-	0.9511	0.7601
60	-	-	-	0.5129	-	-	0.7680	0.8300	0.8800	-	0.8513	0.9511

1) Relatively high synchronous and the third winding factors. In this case, more electromagnetic torque could be produced with the same phase currents.

2) High and even greatest common divisors of slot and pole numbers, K, and K should be as high as possible. An even K causes low net radial force on the motor. The higher K is, the lower the net radial force is.

3) High least common multiple of slot and pole numbers, L. The higher L is, the higher is the frequency of the cogging torque, resulting in smoother torque production.

4) Least loss on the rotor.

Besides, slot and pole numbers should satisfy the following equation [5-6]

$$s/K = mc_1,\ \ s/t = mc_2\,. \tag{1}$$

where s is the number of stator slots, c_1 and c_2 are both positive integers, t is the greatest common divisor of slots and pole pairs.

An optimal layout of five-phase FSCW motor can be set based on the method of synthesizing multiphase FSCW permanent magnet synchronous motors proposed in [6]. Then, the winding factors can be calculated. There are several methods for winding factor calculation [7-9]. The star of slots theory is adopted in this paper.

The harmonic winding factor is defined as

$$k_{wv} = k_{pv} k_{dv} \tag{2}$$

k_{wv} is the v^{th} harmonic winding factor, k_{pv} is the v^{th} harmonic pitch factor. k_{dv} is the v^{th} harmonic distribution factor. In FSCW motor, the pitch in slot number, y_p, equals to 1.

The harmonic pitch factors and distribution factors can be calculated respectively [9].

$$k_{pv} = \sin\left(2\pi v / s\right) \tag{3}$$

$$k_{dv} = \begin{cases} \dfrac{2\sin\left(q_{ph}\alpha_{phv}/4\right)}{q_{ph}\sin\left(\alpha_{phv}/2\right)} & \dfrac{s}{mt}\ \textit{is even} \\[2ex] \dfrac{\sin\left(q_{ph}\alpha_{phv}/4\right)}{q_{ph}\sin\left(\alpha_{phv}/4\right)} & \dfrac{s}{mt}\ \textit{is odd} \end{cases} \tag{4}$$

where q_{ph} is the number of spokes per phase, α_{phv} is the angle between two v^{th} spokes.

The synchronous and the third harmonic winding factors are calculated and presented in Table 1 and Table 2, respectively.

It should be noted that the calculations are specific to the double-layer FSCW motor.

Table 1 and Table 2 show that

1) Combinations with slot per pole per phase(SPP) equal to 1, which highlighted in yellow, have the highest third harmonic winding factors. However, the synchronous winding factors are not high enough. Combinations in this group are not preferred.

2) Combinations with SPP >1 have low synchronous and the third harmonic winding factors. Thus, the combinations in this group are not suggested to utilize.

3) Combinations with SPP <1 have relatively higher synchronous and the third harmonic winding factors. There are five combinations that the synchronous winding factors are higher than 0.955 and the third harmonic winding factors are higher than 0.79. These are 20/18, 20/22, 15/14, 15/16, 25/24. In Table 1 and Table 2, the former two combinations are highlighted in dark green and the others in light green. These combinations are recommended.

4) Among the recommended combinations, only 20/18 and 20/22 have an even K value, and the L value is 180 and 220, respectively.

Finally, the 20/18 and 20/22 are regarded as the most appropriate combinations. In MFSCW motors, there is a relation [4]

$$2p = mk_1\left(k_2 + k_3/m\right). \qquad (5)$$

where k1 is the number of sectors in one sector group, k2 is the number of large teeth in one sector, and k3 is the equivalent number of small teeth that attaches two sectors, 1 or 2. It is obvious that the 20/18 motor can not satisfy the MFSCW structure. For consistency, the following two analyzed motors are both 20/22 type.

3. MMF Analysis Based on Winding Function

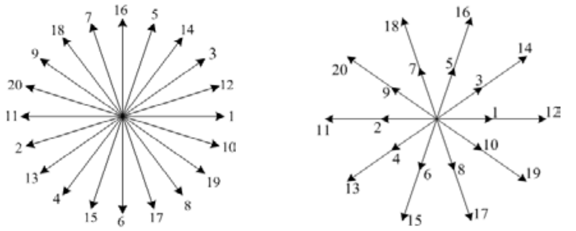
3.1. Winding synthesis

The stator layouts of the two five-phase 20/22 motor are shown in Fig. 1. The distribution sequences are the same in the two motors. In MFSCW motor, there are two sector groups, each group has five sectors, A, B, C, D, E, and each sector has two large teeth and one small tooth. The large tooth width is equal to polar pitch and the small tooth width is 1/5 of the pole pitch. Every large tooth is wound with windings, whereas none of small teeth are wound. The difference between the two motors can also be noticed from their respective star of slots, as shown in Fig. 2.

From the diagrams of star of slots, it is can be seen that the synchronous winding factor of the MFSCW motor is unit. Actually, all harmonic winding factors of the MFSCW motor are units because of the concentrated full-pitch winding. For the GFSCW motor, winding factors are all lower, though it is an optimum layout. However, it should be noted that high winding factors do not necessarily represent high MMF and EMF values.

(a) GFSCW motor (b) MFSCW motor

Fig. 1. Stator layouts of 20/22 five-phase motors



(a) GFSCW motor (b) MFSCW motor

Fig. 2. Star of slots for the two 20/22 five-phase motors

3.2. Winding function

Winding function method is a mathematical description of winding distribution of a motor, and it is useful to calculate the stator air-gap MMF, winding factor and inductance[8]. Once the structure is fixed, the winding function is then specified. For simplicity, the slotting effect and skewing are not considered when specifying the winding functions [10].

The winding functions of GFSCW motor and MFSCW motor are shown in Fig. 3. In the figure, the horizontal axis shows the mechanical angle of the air-gap space, the vertical axis shows the phase MMF when the number of turns per coil is 1 and the instantaneous phase current is 1A.

It can be observed that the winding functions for the two 20/22 motor are basically similar, and the slight difference is that the non-zero part of the winding function for the MFSCW motor is wider than that of the GFSCW motor. Since the both winding functions repeat conversely in every half period (mechanical space in radical unit), the harmonic phase MMF exist only in odd orders.

3.3 MMF Analysis

Once the winding function is determined, the stator air-gap MMF can be got with the phase winding injected with specific currents. Theoretically, the phase currents may be any values. However, in a view of designing rotating electric machine, the currents should produce a rotating magnetic field. In other words, the phase currents for five-phase motor should be

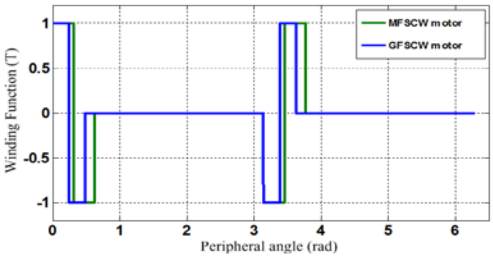


Fig. 3. Winding functions for the two 20/22 motors

$$
\begin{aligned}
i_A &= \sum_{k=1,3,5\ldots}^{\infty} I_k \cos k(\omega t) \\
i_B &= \sum_{k=1,3,5\ldots}^{\infty} I_k \cos k(\omega t - 2\pi/5) \\
i_C &= \sum_{k=1,3,5\ldots}^{\infty} I_k \cos k(\omega t - 4\pi/5) \\
i_D &= \sum_{k=1,3,5\ldots}^{\infty} I_k \cos k(\omega t - 6\pi/5) \\
i_E &= \sum_{k=1,3,5\ldots}^{\infty} I_k \cos k(\omega t - 8\pi/5)
\end{aligned}
\quad . \qquad (6)
$$

where I_k is the amplitude of the k_{th} current harmonic, ω is the fundamental frequency of the current. In this case, the stator air-gap MMF varies periodically.

If k is large enough, there will be various rotating magnetic fields in the air-gap. Some of them rotate in the same direction as the fundamental field with same or different speed. Some rotate in the opposite direction. There will also be pulsating magnetic fields.

For research purpose, the fundamental sinusoidal current (k=1), fundamental sinusoidal plus the third harmonic current (k=1, 3) and square current (k=1, 3, 5...) are chosen. The corresponding motor operation modes are termed as permanent magnet synchronous motor (PMSM), harmonic current injection (HCI) and brushless DC motor (BLDC), respectively.

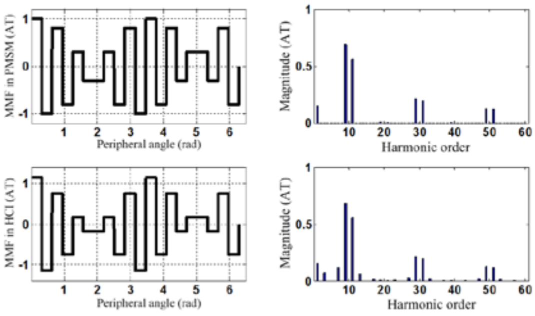
To analyze the stator air-gap MMF produced by different currents for the two motors, two steps should be taken

1) Calculate the phase MMF with the winding function and phase currents,

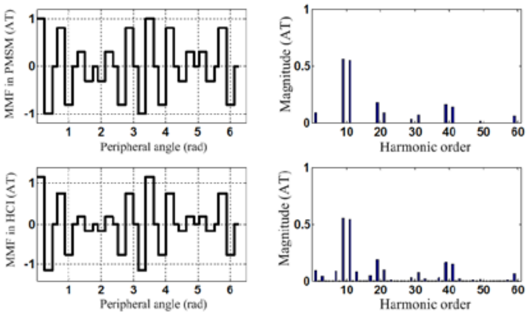
2) Synthesize the phase MMF.

Fig.4 shows the stator air-gap MMF produced in PMSM and HCI modes, which locate in upper row and lower row of the figure for each motor, respectively. It is very necessary to mention that phase currents in both modes have the same rms value and they are both 1A.

It can be noticed from Fig. 4 that, the 9th MMF harmonic of GFSCW motor is larger than that of the MFSCW motor. While there exist more harmonic content in the MMF of the MFSCW motor. In PMSM mode, there are only the (5k±1)th odd MMF harmonics. While in HCI mode, the (5k±2)th odd MMF harmonics are added.



(a) GFSCW motor



(b) MFSCW motor

Fig. 4. Stator air-gap MMF and their spectrums

4. FEM Verification

FEM is used to verify the results obtained by winding function method. Design parameters of the two motors are shown in Table 3.

4.1 Phase Back-EMF

In HCI mode, the current amplitude ratio of the third harmonic to the fundamental should be the same as that of the corresponding harmonics of the phase back-EMF[11]. Therefore, the phase back-EMF should be calculated at first. The no-load phase back-EMF waveforms of the two motors are shown in Fig. 5.

The two EMF waveforms are both nearly trapezoidal, which is favored to improve torque density for multiphase motors. It is also can be found that the flat-top part of the back-EMF waveform of MFSCW is slightly wider than GFSCW, meaning more torque resulting from the same stator currents.

Table 3 Design Parameters of The Two 20/22 Motors

Symbol	Parameter	GFSCW motor	MFSCW motor
s	Number of stator slots	20	10(large)/20(small)
Wm	PM width	17.6mm	17.6mm
Lm	PM length	6.5 mm	6.5 mm
Ws	Slot width	16 mm	16/14 mm
Hs	Slot height	22.6 mm	22.6 mm
Wt	Tooth width	12 mm	10 mm
Nt	Series turns per phase	33	33
Dso	Stator outer diameter	200 mm	200 mm
Dro	Rotor outer diameter	139 mm	139 mm
Dri	Rotor inner diameter	114 mm	114 mm
Lg	Air-gap length	0.5 mm	0.5 mm

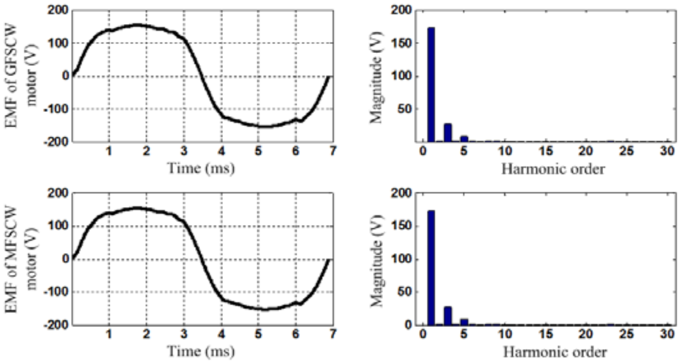


Fig. 5. Phase back-EMF waveforms of the two motors

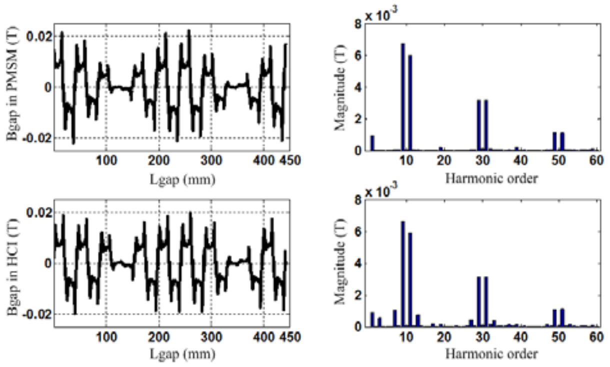
FFT results of the two waveforms show that the amplitude ratio of the third harmonic to the fundamental are nearly same, and they are both 0.166. In this case, if the rms phase current in PMSM mode is 1 A, the fundamental current and the third harmonic current should be 0.98 A and 0.16 A in HCI mode.

4.2. Flux Density

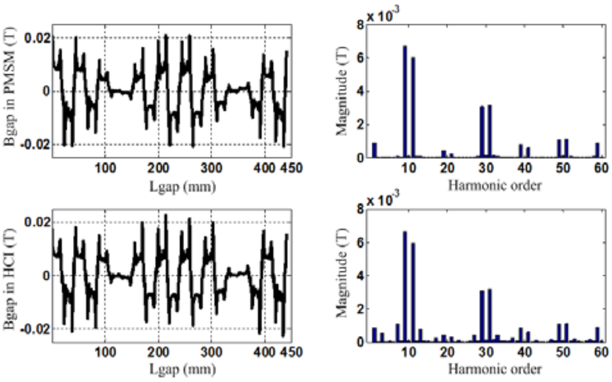
With the calculated currents, waveforms of air-gap flux density together with their FFT analysis results can be obtained and they are presented in Fig. 6.

It can be noted that the air-gap flux density is different from MMF in nature, but their distributions in air-gap are basically identical. The MMF waveforms and FFT results by FEM are consistent with those gotten by winding function method, showing the effectiveness of the winding function method.

There are more spikes in the flux density waveforms since the slotting effect of the models are considered in FEM analysis. As a result, harmonic contents in flux density increase with limited magnitudes.



(a) GFSCW motor



(b) MFSCW motor

Fig. 6. Stator air-gap flux densities and their FFT spectrums

4.3. Torque Analysis

To analyze the electromagnetic torques of the two motors by FEM, various operating modes should be included. The mentioned three modes, PMSM, HCI and BLDC, are all taken into consideration. In the five-phase case, the corresponding phase currents in BLDC mode are square currents with magnitude for 144 electric degrees both in positive and negative half-cycles. In the HCI mode, like in the MMF calculation, the current amplitude ratio of the 3rd harmonic to the fundamental is same as that of the corresponding back-EMF, and the ratio is 0.166. All the phase currents have the same rms value, 17A.

Electromagnetic torque waveforms of the two motors in various operating modes are calculated and presented in Fig.7. The average values of torque are enumerated in Table 4. It can be seen that both of motors produce higher average torque values in HCI mode than in PMSM or BLDC modes, revealing the effectiveness of torque density improvement by harmonic current injection. For a fixed flattop width of back-EMF, the torque density in BLDC mode may be lower than that in PMSM mode[11].

The torque ripple of the two motors have the same frequency in all operating modes. In BLDC mode, commutation torque ripple also exists. The average torques of MFSCW motor are slightly higher than those in GFSCW motor, while torque ripples of MFSCW motor are much higher.

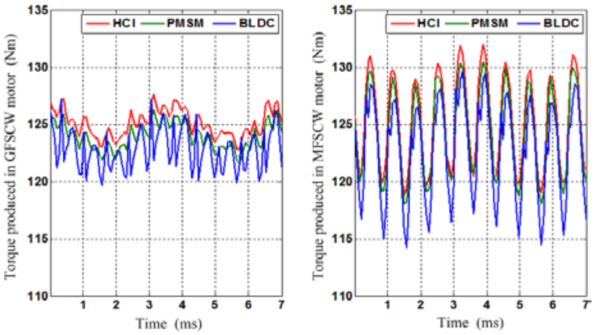


Fig.7. Torque waveforms for the two motors

Table 4 Average Torque Comparison

Average torque (Nm)	GFSCW motor	MFSCW motor
HCI	125.11	125.43
PMSM	123.95	124.25
BLDC	122.87	122.95

5. Conclusions

The suitable slot/pole combines are selected for five-phase FSCW PMSM, which is intended to be fed by HCI method to enhance torque density. Two types of five-phase FSCW PMSM with 20/22 combination, MFSCW and GFSCW motor, are analyzed and compared in detail, including the stator structures, star of slots diagrams, MMF harmonics, back-EMF and electromagnetic torques produced in various operating modes. It can be found that

1) In GFSCW motor, most of the synchronous winding factors are not high enough and so are the third harmonic winding factors. Thus, attention should be paid to pick out the slot/pole combination that synchronous and the third harmonic winding factors are both high enough. Task of this kind is not necessary for MFSCW motor because all its winding factors are unit.

2) MMF harmonic contents in MFSCW motor are richer and higher, which may cause higher rotor loss.

3) Driven with same rms phase currents, both of the GFSCW and MFSCW motors running at HCI mode produce the highest torque, proving the effectiveness of harmonic currents injection method to improve the torque density .The MFSCW motor can produce slightly higher average torque than the GFSCW motor in three running modes. However, larger ripple torque can also be found.

Acknowledgements

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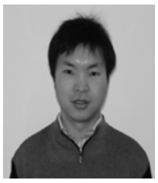

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