

# Fuzzy Controller of Three-Inertia Resonance System designed by Differential Evolution

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**Abstract** – In this paper, a new design method of vibration suppression controller for multi-inertia (especially, 3-inertia) resonance systems is proposed. The controller consists of a digital fuzzy controller for speed loop and a digital PI controller for current minor loop. The three scaling factor of the fuzzy controller and two PI controller gains are determined by Differential Evolution (DE). The DE is one of optimization techniques and a kind of evolutionary computation technique. In this paper, we have applied the *DE/rand/1/bin* strategy to design the optimal controller parameters. Comparing with the conventional design algorithm, the proposed method is able to shorten the time of the controller design to a large extent and to obtain accurate results. Finally, we confirmed the effectiveness of the proposal method by the computer simulations.

**Keywords:** 3-inertia resonance System, Vibration suppression control, Fuzzy controller, Differential Evolution

## 1. Introduction

In recent years, high precision and fast response motor drive systems are widely used in many industrial applications (e.g. Steel Rolling Mill, Blue-ray Disc Drive, Hard Disk Drive, Robot Manipulator, Electrical Vehicle and etc.). Advances of the control theory and the actuator technology have made it possible to widen the bandwidth of the control system for faster responses.

On the other hand, modern mechanical systems tend to lack stiffness due to miniaturization and weight reduction because constructions of those systems have become complicated. Therefore, the motor drive systems generally are multi-inertia systems with several inertia moments, gears and springs. It can be analyzed by an approximate 2-inertia system. More effective control methods to suppress vibrations of the 2-inertia resonance systems have been proposed: e.g. resonance ratio control, full state feedback control, coefficient diagram method (CDM),  $H^\infty$  control method, Pole Placement Method and Fractional Order PID<sub>k</sub> Control [1] - [3]. Ikeda, et al. have proposed a position control of the 2-inertia systems with the speed minor loop designed by the pole placement method [4].

To suppress the vibration and control precisely, however,

the system has to be treated as a multi-inertia system (system more than 3-mass model). For example, in electric vehicle, the system of the drive-train is 4-mass system. Also, in ball-screw driven stage, the system is 4-mass system. Furthermore, the turbines-generator system has 12 mass and includes 11-eigen frequencies. Here, some researcher presented the vibration control methods for the multi-inertia system [5], [6]. We proposed a vibration suppression control method for the 3-inertia system using a modified-IPD controller (Integral plus Proportional plus Derivative plus time lag of first order element) designed by Taguchi Method [7].

On the other hand, these control methods, however, may not be able to achieve the required results, in case that both the system equation and the real parameters of the system are not known. Fuzzy controller is one technique to solve such a problem.

Fuzzy controller is a nonmathematical control algorithm based on intuition and experience. This has been successfully applied in some applications such as a motor [8], [9].

In this paper, we propose the design of the control method for the 3-inertia resonance system in order to suppress the torsional vibration. The proposed controller consists of a digital fuzzy controller for speed loop and a digital PI controller for current minor loop. The controller uses motor side variables (a motor angular speed and a motor armature current) only.

The three scaling factors and two PI controller gains are determined by Differential Evolution algorithm (DE) [10]-[13]. The DE method proposed by Storn and Price proved

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to be a powerful global optimization technique. The DE makes it possible to give quick search of the required scaling factors and control gains and as a result to obtain the accurate control results.

Finally, the effectiveness of the proposal method is confirmed by the computer simulations.

## 2. 3-Inertia Vibration Suppression Control System

### 2.1 3-Inertia Model

The 3-inertia model, which consists of three rigid inertias with two torsional shafts, is shown in Fig. 1, where  $\omega_M$  is the angular speed of the motor,  $\omega_c$  is the angular speed of load 1,  $\omega_L$  is the angular speed of load 2,  $T_{in}$  is the input torque,  $T_L$  is the disturbance torque,  $J_M$  is the motor inertia,  $J_c$  is the inertia of load 1,  $J_L$  is the inertia of load 2,  $T_1$  is the torsional torque of shaft 1,  $T_2$  is the torsional torque of shaft 2,  $K_{s1}$  is the stiffness of shaft 1 and  $K_{s2}$  is the stiffness of shaft 2. In this research, we consider the current loop for the high speed torque control. And we have neglected the viscous frictions.

The system parameters used in this paper are listed in Table 1. Here, these parameters are expressed by nominal values which are normalized. Fig. 2 shows the frequency response of the nominal 3-mass model. Two peaks of gain characteristic are observed. In this figure, these frequencies of the peaks are the resonance frequencies. The purpose of the vibration suppression control is to reduce these peaks. The continuous state equation of the 3-mass model is given by equation (1).

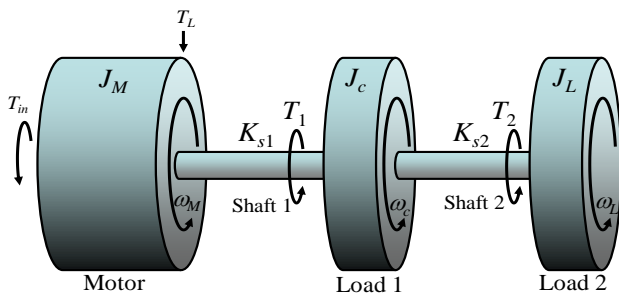


Fig. 1. 3-Inertia Model

Table 1. Nominal Parameters of 3-Inertia Model

Characteristics	Specification
Inertia Moment of Motor $J_M$	$3.645 \times 10^{-5}$ [kgm <sup>2</sup> ]
Inertia Moment of Load 1 $J_c$	$6.920 \times 10^{-5}$ [kgm <sup>2</sup> ]
Inertia Moment of Load 2 $J_L$	$3.18 \times 10^{-5}$ [kgm <sup>2</sup> ]
Stiffness of Shaft 1 $K_{s1}$	64.3 [Nm/rad]
Stiffness of Shaft 1 $K_{s2}$	80.4 [Nm/rad]

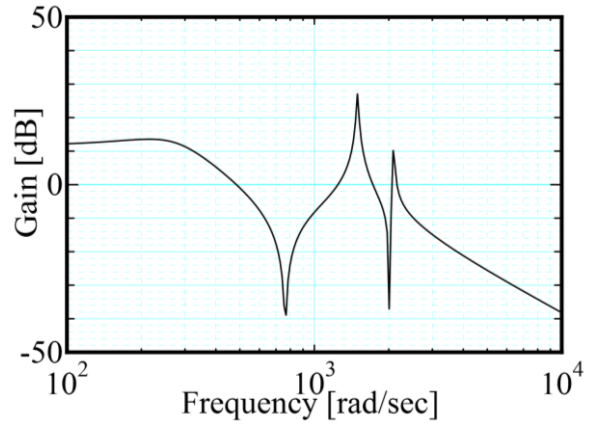


Fig. 2. Frequency Response of 3-Inertia Model

$$\begin{pmatrix} J_M s & 1 & 0 & 0 & 0 \\ -1 & \frac{1}{K_{s1}} s & 1 & 0 & 0 \\ 0 & -1 & J_c s & 1 & 0 \\ 0 & 0 & -1 & \frac{1}{K_{s2}} s & 1 \\ 0 & 0 & 0 & -1 & J_L s \end{pmatrix} \begin{pmatrix} \omega_M \\ T_{d1} \\ \omega_c \\ T_{d2} \\ \omega_L \end{pmatrix} = \begin{pmatrix} T_{in} \\ 0 \\ 0 \\ 0 \\ T_L \end{pmatrix} \quad (1)$$

From the above equation of the 3-inertia model, the transfer function of the  $T_{in}$  to  $\omega_M$  is obtained below.

$$\frac{\omega_M}{T_{in}} = \frac{(s^2 + \omega_{a1}^2)(s^2 + \omega_{a2}^2)}{J_M s (s^2 + \omega_{r1}^2)(s^2 + \omega_{r2}^2)} \quad (2)$$

Where,  $\omega_{r1}$  and  $\omega_{r2}$  are resonance frequencies and  $\omega_{a1}$  and  $\omega_{a2}$  are anti-resonance frequencies. The resonance frequencies of the nominal model in this paper are

$$\begin{aligned} \omega_{r1} &= 2070.0 \text{ [rad/sec]}, & \omega_{r2} &= 1419.2 \text{ [rad/sec]} \\ \omega_{a1} &= 1990.7 \text{ [rad/sec]}, & \omega_{a2} &= 757.3 \text{ [rad/sec]} \end{aligned} \quad (3)$$

Here, we assume that the driving motor is dc motor. Table 2 show the nominal parameters of the armature circuit of the dc motor. Fig. 3 shows the block diagram of the 3-inertia system including the armature current loop.

Table 2. Specification of DC Motor

Characteristics	Specification
Armature Resistance $R_a$	1.6 [ $\Omega$ ]
Armature Inductance $L_a$	6.0 [mH]
Back EMF Constant $K_e$	0.264 [Vsec/rad]
Torque Constant $K_t$	0.264 [Nm/A]
Converter Gain $K_0$	25 [V/pu]

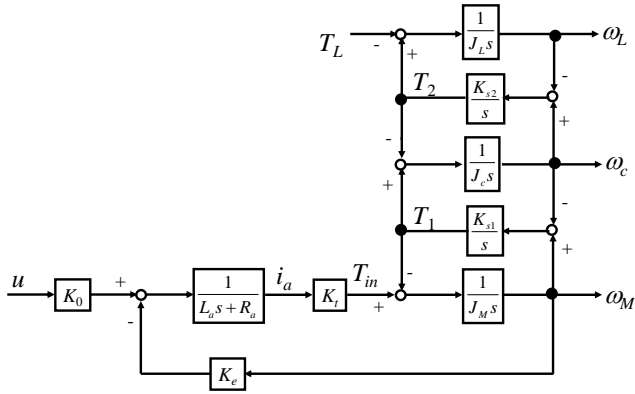


Fig. 3. Block Diagram of 3-Inertia Model

## 2.2 Fuzzy Controller

The fuzzy control is able to successfully apply to control nonlinear complex systems using an operator experiences or control engineering knowledge without any mathematical model of the plant. The fuzzy controller is designed for the model shown in Fig. 4. The control system consists of the fuzzy speed controller and the current PI controller, where  $S_1$ ,  $S_2$  and  $S_3$  are the scaling factors of the fuzzy controller. And the current PI controller has two controller gains ( $K_{pc}$  and  $K_{ic}$ ). Furthermore, considering the application to the experimental apparatus, we construct the digital control system which has discrete controller.

If all parameters and state variables are known, it is possible to design the desired closed-loop system poles to any position on the S plane. But the load side variables are hard to be measured in general, because of bad environment and sometimes narrow space.

We propose the vibration suppression control method, which needs no information about the load side variables, namely load 1 angular speed  $\omega_c$ , load 2 angular speed  $\omega_L$ , torsional torque 1  $T_1$  and torsional torque 2  $T_2$ . In this paper, only the motor angular speed  $\omega_M$  and the armature current  $i_a$  are observable.

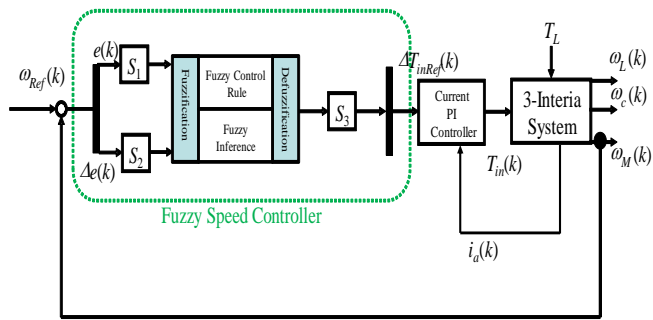


Fig. 4. Block diagram of the proposed controller

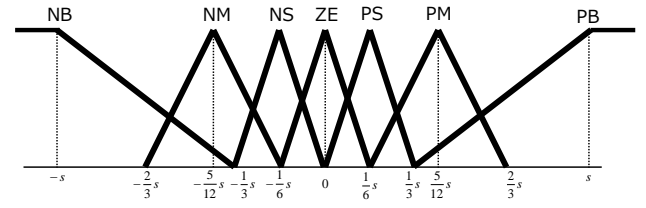


Fig. 5. Membership functions of the antecedence

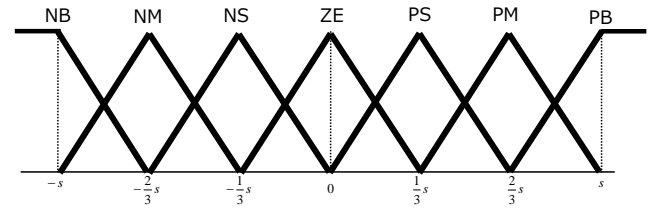


Fig. 6. Membership functions of the consequence

Here, we apply the triangular membership function for the antecedence and the consequence variables. The membership function of the antecedence and the consequence are illustrated in Fig. 5 and 6, respectively. Where, the  $s$  is the scaling factor, PB: Positive Big, NB: Negative Big, PM: Positive Medium, NM: Negative Medium, PS: Positive Small, NS: Negative Small and ZE: Zero. The antecedence variables  $e_{\omega_M}(k)$  and  $\Delta e_{\omega_M}(k)$  are defined as,

$$e_{\omega_M}(k) = \omega_{ref} - \omega_M(k) \quad (4)$$

$$\Delta e_{\omega_M}(k) = \omega_M - \omega_M(k-1) \quad (5)$$

Then, the consequence variable is the variation width of the torque input  $\Delta T_{in}(k)$ . The control rule table is shown in Fig. 7. The rule is included the rising correction of the angular speed response.

$e \backslash \Delta e$	NB	NM	NS	ZE	PS	PM	PB
NB				NB	NM		
NM				NM			
NS				NS	ZE		PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM		ZE	PS			
PM				PM			
PB			PM	PB			

Fig. 7. Control rule table

### 3. Determination of Controller Parameters by Differential Evolution

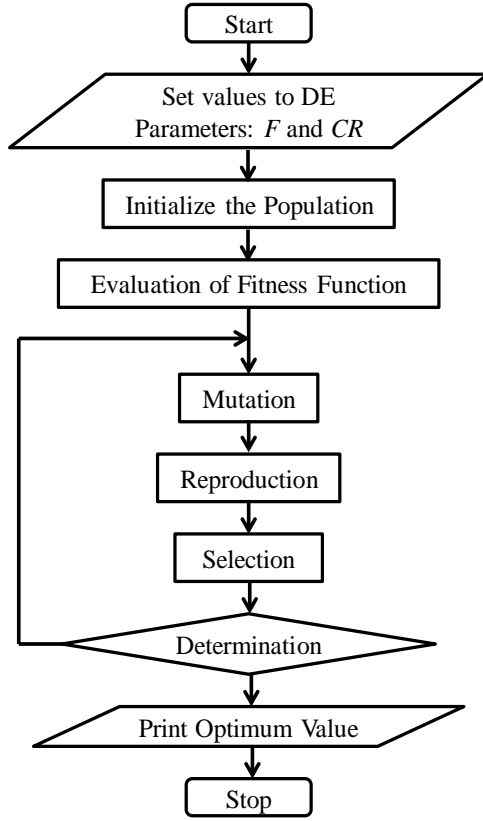


Fig. 8. Flow of DE algorithm

In general, since determinations of the scaling factors and the PI controller gains are a rule of thumb, it takes a long time to determine these factors by trial and error or some other methods. In this paper, we apply the DE for the determination of five design parameters ( $S_1$ ,  $S_2$ ,  $S_3$ ,  $K_{pc}$  and  $K_{ic}$ ).

DE algorithm is one of the evolutionary algorithms [10]-[13]. Unlike simple GA that uses binary coding for representing problem parameters, DE uses real-valued vectors. The crucial idea behind DE is a scheme for generating trial parameter vectors.

The design procedure using the DE consists of 4 steps (Initial Population, Mutation, Crossover and Selection). Fig. 8 shows the flow of DE algorithm.

There are several variants of DE design. In this paper, we utilize the DE combination *DE/rand/1/bin* strategy.

A set of  $D$  optimization parameters is called an individual. It is represented by  $D$ -dimensional parameter vector. A population consists of  $NP$  parameter vector  $\mathbf{x}_{i,G}$ .

Where,  $i=1, 2, \dots, NP$ ,  $NP$  denotes the number of members in one population and  $G$  indicates one generation. We have one population for each generation. Here, the initial population vector is determined randomly.

In this DE optimization,  $F$  is the scaling factor and  $CR$  is the crossover rate. The scaling factor  $F$  works for creating a mutation vector  $\mathbf{v}_{i,G}$ . For each target vector  $\mathbf{x}_{i,G}$ , the mutation vector  $\mathbf{v}_i$  are generated according to

$$\mathbf{v}_{i,G+1} = \mathbf{x}_{r1,G} + F(\mathbf{x}_{r2,G} - \mathbf{x}_{r3,G}) \quad r_1 \neq r_2 \neq r_3 \neq i \quad (6)$$

Where,  $r_1$ ,  $r_2$  and  $r_3$  are distinct. In the crossover, the target vector  $\mathbf{x}_{i,G}$  mixed with the mutation vector  $\mathbf{v}_{i,G+1}$ , using following scheme for  $j = 1, 2, \dots, D$ ,

$$\mathbf{u}_{ji,G+1} = \begin{cases} \mathbf{v}_{ji,G+1}, & \text{if } \text{rand}() \leq CR \text{ or } j = \text{start point} \\ \mathbf{x}_{ji,G}, & \text{if } \text{rand}() > CR \text{ or } j \neq \text{start point} \end{cases} \quad (7)$$

$\mathbf{u}_{i,G+1}$  is the trial vector.  $\text{rand}()$  is the  $j$ th evaluation of uniform random generator number. The start point (1, 2, ...  $D$ ) is a randomly chosen index which ensures that  $\mathbf{u}_{i,G+1}$  gets at least one element from  $\mathbf{v}_{i,G+1}$ .

A selection algorithm is utilized

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{u}_{i,G+1}, & \text{if } y(\mathbf{u}_{i,G+1}) > y(\mathbf{x}_{i,G}) \\ \mathbf{x}_{i,G}, & \text{for maximization problems} \\ \mathbf{x}_{i,G}, & \text{otherwise} \end{cases} \quad (8)$$

for  $j = 1, 2, \dots, D$ .

In this paper, the number of problem dimension is five ( $S_1$ ,  $S_2$ ,  $S_3$ ,  $K_{pc}$  and  $K_{ic}$ ), the population size is 200, the scaling factor  $F$  is 0.5 and the crossover rate  $CR$  is 0.9. Equation (9) is the index function, where  $\omega_M$  is the motor angular speed and  $\omega_{ref}$  is the angular speed reference command which is the shape of the step response of the second lag element.

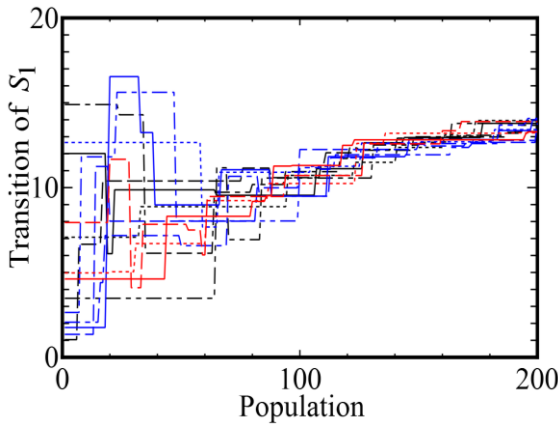
$$y = \int_0^\infty t |\omega_{ref} - \omega_M(k)| dt \quad (9)$$

### 4. Simulation Results

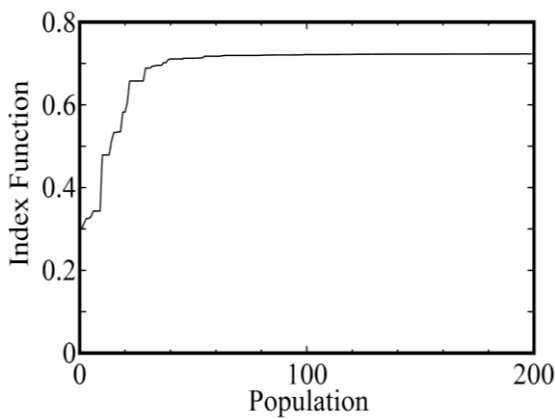
In this section, we verify the validity of the proposed method. And all simulation is executed by MATLAB / Simulink. Fig. 9 shows the transitions of the  $S_1$  vector, where each parameter has 13 individuals. Fig. 10 shows the convergence of the maximum index function  $y$ .

Fig. 11 shows the responses using the conventional PI controller. Here, the disturbance torque input  $T_L$  is changed from 0 to 10 [%] at  $t = 0.3$  [sec]. From this figure, all the angular speed (the motor angular speed  $\omega_M$ , the load 1 angular speed  $\omega_L$  and the load 2 angular speed  $\omega_L$ ) are observed the transient errors. Then, the speed is oscillating after the torque input. And the controller performance of the speed control has been slow.

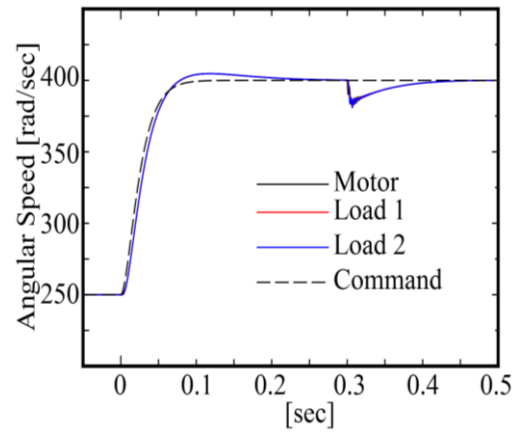
Fig. 12 shows the responses using the proposed method. It is observed that the speed errors reduced very well and the vibration suppressed well. Moreover, the effect of disturbance torque input was found to be insignificant. Fig. 13 shows the load 2 angular speed  $\omega_L$  including the error of the nominal inertia value. And Fig. 14 shows the  $\omega_L$  including the error of the nominal stiffness of the shaft. From these figure, the control algorithm work well up to a certain extent, regardless of the variation in the load inertia and the stiffness of the shaft. Thus the results show that the proposed method has the effectiveness for the vibration suppression and the high robustness.



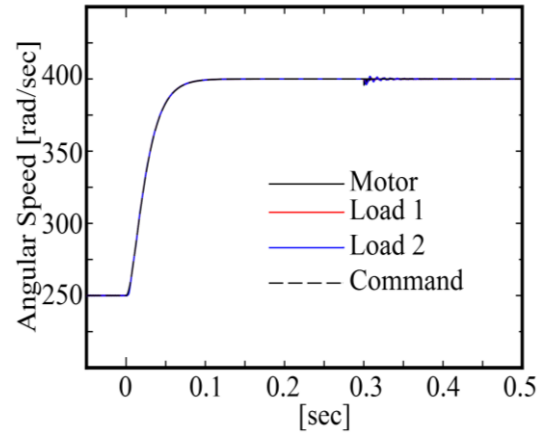
**Fig. 9.** Transition of Scaling Factor  $S_1$



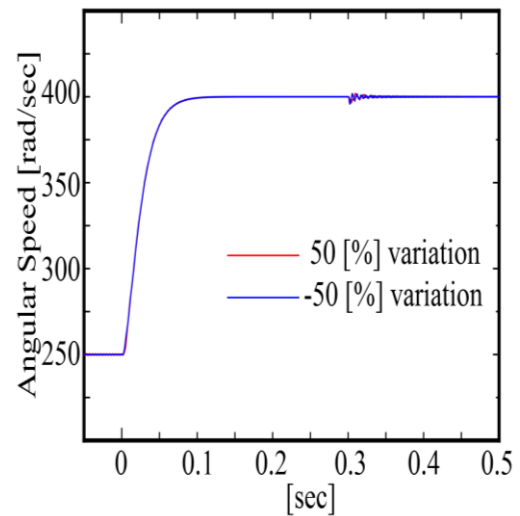
**Fig. 10.** Convergence of Index Function  $y$



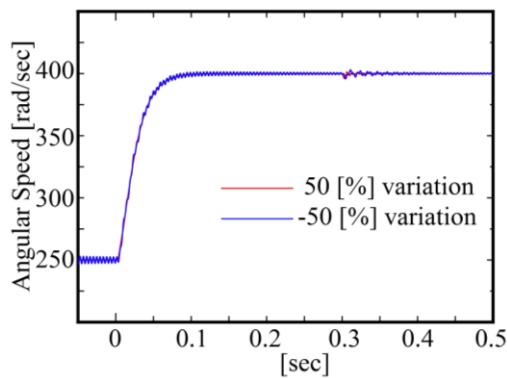
**Fig. 11.** Simulation Results Using Conventional PI Controller



**Fig. 12.** Simulation Results Using Proposed Method



**Fig. 13.** Load 2 Angular Speed Including Load Inertia Errors



**Fig. 14.** Load 2 Angular Speed Including Stiffness Errors

### 5. Conclusion

In this paper, we proposed the vibration suppression controller for multi-inertia system using the fuzzy speed controller and the current PI controller. Then, the five controller design parameters were determined by the DE to design quickly. We confirmed the effectiveness of the controller structure by the computer simulations. The topic of our future researches is to investigate the robustness of the fuzzy controller and the influence of gear backlash of the proposed method.

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