Analysis of 3-phase Induction Motor considering Current Regulator using DQ Transformation with Matrix Vector

Sun-Ki Hong* and Yoo-Chung Na**

Abstract – 3-phase Induction machines are being used in industry and dq transformation from 3 phase of a, b, c is commonly used to analyze these machines. The equivalent circuits of d and q axis are however generally cross coupled and not simple to analyze. In this study, an analysis method of 3ph induction motor considering current regulator using dq transformation and matrix vector is proposed and it can explain the 3ph induction motor physically. This model does not need the separating process of d and q components. With this technique, the model becomes simple, is easy to understand in physical, and can get the same results with those from the other dq models. These simulation results of the proposed model are compared with those of other models for the conformation of the proposed method.

Keywords: DQ transformation, Matrix vector, 3ph Induction motor, Current regulator

1. Introduction

3ph induction machines are very widely being used in industrial area. DQ transformation is commonly used to analyze the induction machines' characteristics [1]-[4]. The equivalent circuits of d and q axis are however generally cross coupled and that makes it difficult to calculate the characteristics of the machine. In this study, an analysis method of 3ph induction motor including current regulator using dq transformation with matrix vector is proposed. The model parameters can be matched with the 3ph induction machine physically and it does not need the separating process of d and q components. With this, the model becomes simple and is easy to understand in physical. These simulation results of the proposed model are compared with those of other models for the conformation of the proposed method.

2. Conventional Induction Motor Modeling

2.1 3ph induction motor model using $\psi = \omega_b \lambda$ at arbitrary reference frame

The dq voltage equations of stator and rotor from 3 phases become (1), (2) [3]. (3), (4) shows the magnetic flux linkage and torque equations.

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$$v_{qs} = P\lambda_{qs} + \omega\lambda_{ds} + r_s i_{qs}$$

$$v_{ds} = p\lambda_{ds} + \frac{\omega}{\omega_b}\lambda_{qs} + r_s i_{ds} \tag{1}$$

$$v'_{qr} = p\lambda'_{qr} + (\omega - \omega_r)\lambda'_{ds} + r'_r i'_{qr}$$

$$v'_{dr} = p\lambda'_{dr} + (\omega - \omega_r)\lambda'_{qs} + r'_r i'_{dr} \tag{2}$$

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_{qr} \\ \lambda'_{dr} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_m & 0 & L_m & 0 \\ 0 & L_{ls}+L_m & 0 & L_m \\ L_m & 0 & L_{ls}+L_m & 0 \\ 0 & L_m & 0 & L_{ls}+L_m \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} \tag{3}$$

$$T_{em} = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{ds} - \lambda_{qs} i_{ds}\right)$$

$$= \frac{3}{2}\frac{P}{2} L_m(i'_{dr} i_{qs} - i'_{qr} i_{ds}) \tag{4}$$

Where, p is differential operator, P is number of poles, ω is the speed of frame and ω_r is the speed of rotor, λ is flux linkage, L_{ls}, L_{lr} are leakage inductance, T_{em} is developed motor torque and ' means the transferred value from rotor to stator side. From these equations, Ong [3] adopted flux linkage per sec $\psi = \omega_b \lambda$ which dimension is voltage. With this, (1)~(4) become (5)~(8) [4] and the flux linkage equation (3) is changed to flux linkage per second equation (7)..

$$v_{qs} = \frac{p}{\omega_b}\psi_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + r_s i_{qs}$$

$$v_{ds} = \frac{P}{\omega_b}\psi_{ds} + \frac{\omega}{\omega_b}\psi_{qs} + r_s i_{ds} \tag{5}$$

$$v'_{qr} = \frac{p}{\omega_b}\psi'_{qr} + \left(\frac{\omega - \omega_r}{\omega_b}\right)\psi'_{ds} + r'_r i'_{qr}$$

$$v'_{dr} = \frac{p}{\omega_b}\psi'_{dr} + \left(\frac{\omega - \omega_r}{\omega_b}\right)\psi'_{qs} + r'_r i'_{dr} \tag{6}$$

$$\begin{bmatrix} \psi_{qs} \\ \psi_{ds} \\ \psi'_{qr} \\ \psi'_{dr} \end{bmatrix} = \begin{bmatrix} x_{ls}+x_m & 0 & x_m & 0 \\ 0 & x_{ls}+x_m & 0 & x_m \\ x_m & 0 & x_{ls}+x_m & 0 \\ 0 & x_m & 0 & x_{ls}+x_m \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} \tag{7}$$

$$T_{em} = \frac{3}{2}\frac{P}{2\omega_b}\left(\psi_{ds}\, i_{qs} - \psi_{qs}\, i_{ds}\right) \tag{8}$$

2.2. 3ph Induction motor model using flux linkage λ

As mentioned in 2.1, ψ model needs somewhat complex mathematical equations and has physical ambiguities in the model block. In this section, flux linkage λ itself is not modified but used directly to derive the integration form equations as (9) - (11) and (3) for physical meaning.

$$i_{qs} = \frac{1}{L_{ls}+L_m}\int \left(v_{qs} - r_s i_{qs} - \omega\lambda_{ds} - L_m\frac{d}{dt}i'_{qr}\right)dt$$

$$i_{ds} = \frac{1}{L_{ls}+L_m}\int \left(v_{ds} - r_s i_{ds} - \omega\lambda_{qs} - L_m\frac{d}{dt}i'_{dr}\right)dt \tag{9}$$

$$i'_{qr} = \frac{1}{L'_{lr}+L_m}\int \left(-r'_r i'_{qr} - (\omega-\omega_r)\lambda'_{dr} - L_m\frac{d}{dt}i_{qs}\right)dt$$

$$i'_{dr} = \frac{1}{L'_{lr}+L_m}\int \left(-r'_r i'_{dr} - (\omega-\omega_r)\lambda'_{qr} - L_m\frac{d}{dt}i_{ds}\right)dt \tag{10}$$

$$T_{em} = \frac{3}{2}\frac{P}{2}L_m\left(i^s_{qs}\, i^s_{dr} - i^s_{ds}\, i^s_{qr}\right) \tag{11}$$

This model is derived using λ which has physical meaning although it looks a little bit more complex. Actually it doesn't have sub-blocks, so it becomes simpler than the previous model using $\psi = \omega_b\lambda$.

2.3. 3ph induction motor model using complex vector

Complex vector has real and imaginary axis. In 3ph induction motor modeling, q-axis is real and d-axis is imaginary axis. This kind of modeling becomes simpler than the common dq transformation method because it does not need to separate the d and q components With Simulink tool, it is easy to realize to complex vector model and the complex calculation is possible in the model block. For example, the dq voltage equation can be expressed as $v_{qd} = v_q + jv_d$, where the bold character means complex vector. The following equations of (12) and (13) show the

voltage and flux linkage equations of complex vector form in stator and rotor [5].

$$\boldsymbol{v}_{qd} = r_s\boldsymbol{i}_{qd} + \frac{d}{dt}\boldsymbol{\lambda}_{qd} + j\omega\boldsymbol{\lambda}_{qd}$$

$$\boldsymbol{v}_{qdr} = r_s\boldsymbol{i}_{ds} + \frac{d}{dt}\boldsymbol{\lambda}_{qd} + j\omega\boldsymbol{\lambda}_{qd} \tag{12}$$
$$\boldsymbol{\lambda}_{qd} = L_s\boldsymbol{i}_{qd} + L_m\boldsymbol{i}_{qdr}$$

$$\boldsymbol{\lambda}_{qdr} = L_r\boldsymbol{i}_{qdr} + L_m\boldsymbol{i}_{qd} \tag{13}$$

In this approach, current complex vector $\boldsymbol{i}_{qd}$ and magnetic flux linkage complex vector $\boldsymbol{\lambda}_{qdr}$ become the variables to solve therefore $\boldsymbol{i}_{qdr}$ and $\boldsymbol{\lambda}_{qd}$ need to be erased. With (13), the following equation of (14) is derived.

$$\boldsymbol{\lambda}_{qd} = \sigma L_s\boldsymbol{i}_{qd} + \frac{L_m}{L_r}\boldsymbol{\lambda}_{qdr} \tag{14}$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \tag{15}$$

σ is defined as (15) for simplicity. (14) is substituted for (12) and the following differential equations are derived.

$$\sigma\frac{d}{dt}\boldsymbol{i}_{qd} = \boldsymbol{v}_{qd} - (r_s + j\omega\sigma L_s)\boldsymbol{i}_{qd}$$

$$-\frac{L_m}{L_r}\left(\frac{d}{dt}\boldsymbol{\lambda}_{qdr} + j\omega\boldsymbol{\lambda}_{qdr}\right) \tag{16}$$

$$\frac{d}{dt}\boldsymbol{\lambda}_{qdr} = -\frac{r_r}{L_r} - j(\omega-\omega_r)\boldsymbol{\lambda}_{qdr} + r_r\frac{L_m}{L_r}\boldsymbol{i}_{qd} \tag{17}$$

Fig. 1. IM model using complex vector for arbitrary ref. frame.

Fig. 1 shows the 3ph induction motor model derived from (16) and (17). This model does not need to extra parameter like ψ and does not look complex as the flux linkage model [3]. Each icon has physical meaning and is

easily realized as the model in Fig. 1. In the figure, the integral symbol is a sub block that has two integral processes of each *q* and *d* axis equation. After the integrations, they are combined again to make complex vector.

With these results, the IM modeling with complex vector and *dq* transformation has simple structure, is easy to debug and to analyze IM characteristics.

2.4. 3ph induction motor model using matrix vector

The complex vector model of IM is very useful to analyze induction machine, however the model is not able to apply to permanent magnet synchronous machine because the applied machine with complex vector method needs to be symmetric.

Eq. (17)-(20) shows the 3ph induction motor's stator, rotor, and field equations where bold character means not complex but matrix form and (3) is the flux linkage equation.

$$V_{qd} = r_s i_{qd} + \frac{d}{dt}\lambda_{qd} + j\omega\lambda_{qd} \tag{17}$$

$$0 = r_r i_{qdr} + \frac{d}{dt}\lambda_{qdr} + j(\omega - \omega_r)\lambda_{qdr} \tag{18}$$

$$V_{fr} = r_f i_{fr} + \frac{d}{dt}\lambda_{qdr} \tag{19}$$

$$\lambda_{qdr} = L_r i_{qdr} + L_m(i_{qd} + i_{fr}) + \lambda_{pm} \tag{20}$$

In those equations, bold *j* is not just a imaginary symbol but is matrix vector form as (21). L_s is also a matrix as follows.

$$j = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{21}$$

$$L_s = \begin{bmatrix} L_{qq} & L_{qd} \\ L_{dq} & L_{dd} \end{bmatrix} \tag{22}$$

With these equations, 3ph induction motor model for arbitrary reference frame with matrix vector can be induced. (23) and (24) shows the differential equations from (17)-(20). This model can also be applied to PM synchronous motor because it considers the field current PM flux linkage.

$$\frac{d}{dt}L_s\sigma i_{qd} = V_{qd} - r_s i_{qd} - j\omega L_s\sigma i_{qd}$$

$$-\left(\frac{d}{dt}L_m L_r^{-1}\lambda_{qdr} + j\omega L_m L_r^{-1}\lambda_{qdr}\right) - (I - L_m L_r^{-1})$$

$$\{\frac{d}{dt}(L_m i_{fr} + \lambda_{pm}) + j\omega(L_m i_{fr} + \lambda_{pm})\} \tag{23}$$

$$\frac{d}{dt}\lambda_{qdr} = r_r L_r^{-1}[L_m i_{qd} + (L_m i_{fr} + \lambda_{pm})$$

$$- \{I + L_r r_r^{-1}j(\omega - \omega_r)\}\lambda_{qdr}] \tag{24}$$

Fig. 2 shows the AC machine model made from (23)-(24). It doesn't have cross coupling and has simple structure and is physically well explained.
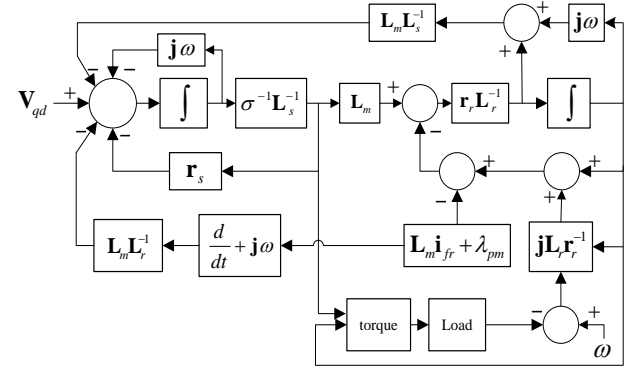
Fig. 2. Generalized AC machine model.

3. Current Regulator Model with Matrix Vector

Due to the previous simple 3ph induction motor model, the simulation of motor characteristics becomes easy and the machine model can be combined with current controller without any modification. Figure 3 shows the 3ph induction motor model with current controller [3].
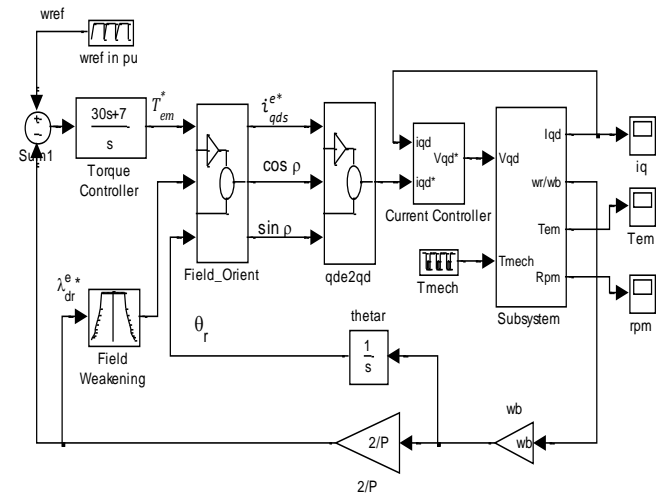

Fig. 3. Generalized AC machine model with current controller.

The current controller is the proportional-gain controller which is generally used in current control. At first, speed command wref is applied and this is compared with the

motor speed to generated torque command T_{em}^*. In the field oriented block, q and d axis currents for synchronous ref. frame are transformed to the stationary ref. frame currents $\boldsymbol{i}_{qds}^*$, which can be seen in Fig. 4.

This current command vector is compared with the feedback current vector of i_{qds} to generate the voltage command vector v_{qd}^*. As mentioned before, the current controller adopts the proportional-gain control. The last block is same with Fig. 2.
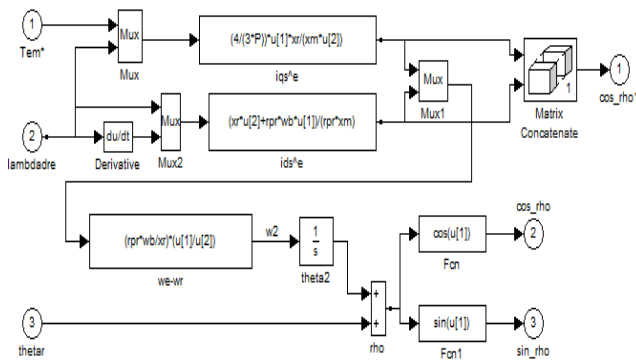

Fig. 4. Field orient block in Fig. 3.

3. Simulation and Comparison

The simulation is executed for a 3ph induction motor to compare the results with each other. The specification of sample IM is that 220 V, 60 Hz, 4 poles, stator and rotor resistance are 0.435 Ω and 0.816 Ω, leakage and magnetization inductance are 2 mH and 69.3 mH and moment of inertia is 0.089. Fig. 5 shows the simulation block when 3-phase voltage is applied to the induction motor of Fig. 2. The 3-phase voltage is transformed to qd voltage and the qd voltage is applied to the IM. The motor block in Fig.2 becomes 3ph induction motor when the permanent flux linkage is 0 and field current is 0.
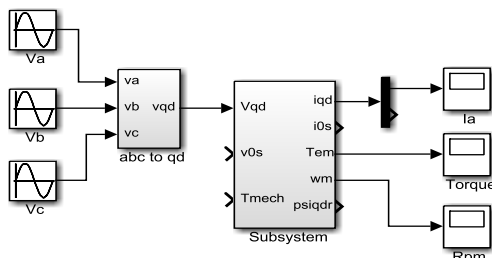
Fig. 5. Induction motor simulation block for 3-phase voltage.

Fig. 6(a) shows the speed response when the speed command is 1,800 rpm. Fig 6(b) shows the torque response waveform of an induction motor and the results are well

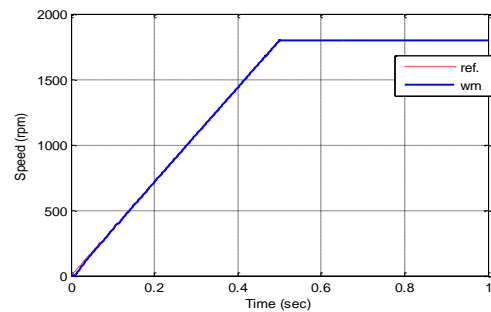matched with the results of other models [3, 4] because the sample specification is same.
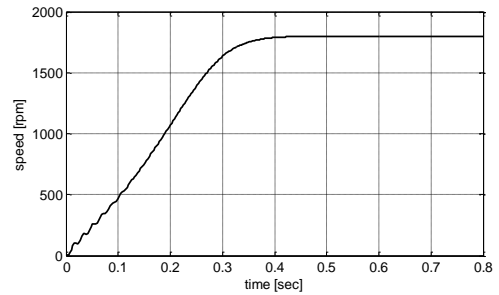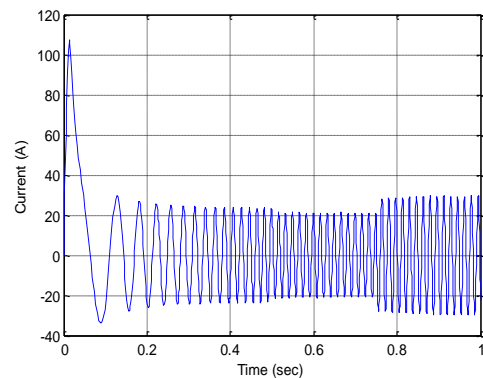

(a) speed response
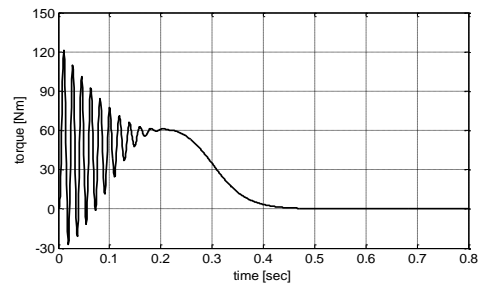
(b) torque response

Fig. 6. IM waveforms using matrix vector for stationary ref. frame.

(a) speed response

(b) current waveform of phase *a*

Fig. 7. IM waveforms under speed control with current controller.

The block diagram shown in Fig. 2 has simple structure and can be applied to not only 3ph induction motor but also permanent magnet type synchronous motor.

Fig. 7 shows the simulation results for the system of Fig. 3. Fig. 7(a) shows the comparison between speed reference and the response. The speed command is ramp input from 0 to 0.5 (sec) and after that the speed command is constant as 1800 (rpm). They are well matched almost without overshoot because the input command is not step but ramp. Fig. 7(b) shows the *a*-phase current wave form. At 0.75 (sec), the load torque is applied and the phase current is increased to match with the load variance.

4. Conclusions

Analyzing method for 3ph induction motor including current regulator using *dq* transformation and matrix vector are surveyed. That 3ph induction model is combined with current controller model and simulated for speed control. The proposed method looks very useful for 3ph induction motor analysis and simulation because of its simple structure and even no problem to apply to asymmetric machine of SM.

Acknowledgements

## References

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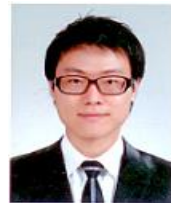
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