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A NEW CLASS OF CYCLIC CODES USING ORDERED POWER PRODUCT OF POLYNOMIALS

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Abstract. The paper introduces a new product of polynomials defined over a field. It is a generalization of the ordinary product with inner polynomial getting non-overlapping segments obtained by multiplying with coefficients and variable with expanding powers. It has been called 'Ordered Power Product' (OPP). Considering two rings of polynomials $R_m[x] = F[x]$ modulo$x^m - 1$ and $R_n[x] = F[x]$modulo$x^n - 1$, over a field F, the paper then considers the newly introduced product of the two polynomial rings. Properties and algebraic structure of the product of two rings of polynomials are studied and it is shown to be a ring. Using the new type of product of polynomials, we define a new product of two cyclic codes and devise a method of getting a cyclic code from the 'ordered power product' of two cyclic codes. Conditions for the OPP of the generators polynomials of component codes, giving a cyclic code are examined. It is shown that OPP cyclic code so obtained is more efficient than the one that can be obtained by Kronecker type of product of the same component codes.

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1. Introduction

Linear algebra and in particular vector spaces are important mathematical structures and have numerous applications. There are areas, for example coding theory, finite geometries and statistical theory of designs, where vector spaces provide basic foundations. There it is felt that higher order and more efficient structures can be developed with advantage by suitably composing the lower order structures. There is thus interest in developing new mathematical composition laws on vectors, matrices and polynomials. Elias (1954) used Kronecker product of matrices to develop higher efficiency codes by combination of lower

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order codes. The method was powerful, but it could develop only a sparse class of product codes and the all important duality property was lost. Sharma (2008), introduced a new concept of matrix-multiplication called the partitioned product of matrices, and obtained a rather large class of rank-partitioned product codes in which the duality property was preserved. Cyclic codes, as is known, are ideally suited for implementation through shift registers. Most important codes like BCH, Goppa codes etc. are cyclic codes, which are best characterized through their generator polynomials. Composing higher order cyclic codes from lower order codes in terms of their generator polynomials has not been explored at any length. The reason for not been able to do that is that there does not exist a method of multiplying two polynomials leading to what may be used to consider product of two cyclic codes. In this paper, we start by introducing a new product of two polynomials defined over a field. It is a generalization of the ordinary product. For convenience we call the two polynomials as outer and inner polynomials. The new defined product then results in non-overlapping segments obtained by multiplying it with coefficients of outer polynomials and expanding powers of the variable. It is called Ordered Power Product and has elegant algebraic properties leading to new algebraic structures. Paper carries a section on applications of the above concepts in developing product of two cyclic codes, in terms of the new product defined by us of two polynomials. Paper is organized as follows: Section 2 gives basic definitions required for later study. In Section 3, the Ordered Power Product of two polynomials in introduced and its algebraic properties are reported. Section 4 carries Applications of Ordered Power Product of polynomials in Coding Theory and an example is given to illustrate the a cyclic code arising from the OPP of two cyclic codes. In Section 5, we refer to further work under study.

2. Main results

For our purpose, we begin with following simple well known algebraic ideas of polynomials over a field F. For details, one may refer to any standard text on Algebra as also Peterson and Weldon 1972 Chapters 2 and 6.

Definition 2.1. Ring of polynomials: Given a field F, we can consider a polynomial as,

$$f\left(x\right)=a_0+a_1x+a_2x^2+...+a_{n-1}x^{n-1}$$

where coefficients $a_{0,}a_1,a_2,...,a_{n-1}\in F$. The collection of all polynomials over field F, denoted by $F\left[x\right]$ under usual addition and product of polynomials, forms a ring, called 'ring of polynomials over F'.

Definition 2.2. Ring of polynomials : Algebra of Polynomial Residue Classes (Refer Peterson and Weldon (1972) The residue classes of the ring of polynomials $F\left[x\right]$ modulo $\left(x^n-1\right)$ a polynomial $f\left(x\right)$ of degree n form a commutative linear algebra of dimension n over the coefficient field F.

Definition 2.3. Cyclic code: An(n, k) block code C is said to be cyclic if it is linear and if for every code word $C = (c_0, c_1, c_2, ..., c_{n-1}) \in C$, its right cyclic shift $\bar{C} = (c_{n-1}, c_0, c_1, ..., c_{n-2}) \in \bar{C}$. An cyclic code is generated by a polynomial $g(x)$ of degree $n-k$ that is a factor of $(x^n - 1)$.

3. New Product - Ordered Power Product of polynomials

Linear algebraic structures, as is well known, are studied with advantage through polynomials. While direct polynomial addition and multiplications are commonly used as the operations on polynomial, wider algebraic structures and applications are done by considering polynomial algebra over a field. Here we shall consider polynomials over a field F, and consider the set of all polynomials $F[x]$. We will be considering structure of $F[x]$ modulo some polynomial, say, $g(x)$. It is proposed to define a new type of composition on polynomials, in which order of the polynomials multiplied is retained and segments of the product arise in terms of rising powers, we name it as 'Ordered Power product.' To motivate the new product, let us look at ordinary product of two polynomials $\lambda(x) = c_0 + c_1 x + c_2 x^2 + ... + c_{m-1} x^{m-1}$ and $V(x) = a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} x^{n-1}$,then

$$\lambda(x) * V(x) = c_0 V(x) + c_1 x V(x) + c_2 x^2 V(x) + ... + c_{m-1} x^{m-1} V(x).$$

Generalizing this by considering suitably indexed powers of x in the segments above, we arrive at the following what we call as the 'Ordered power Product' (OPP).

Definition 3.1. Ordered Power Product of Polynomials in non-overlapping sifting segments: Let$\lambda(x) = c_0 + c_1 x + c_2 x^2 + ... + c_{m-1} x^{m-1}$ and $V(x) = a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} x^{n-1}$ be two polynomials of degree $m-1$ and $n-1$ respectively over a field F. The ordered power product of $\lambda(x)$and $V(x)$ (in nth power of in different segments), denoted by $\lambda(x) * V(x)$, is defined as

$$\lambda(x) * V(x) = \lambda(x^{degree(V)+1}) V(x).$$

In short we shall call it 'Ordered Power Product' (OPP) or $*$product of $\lambda(x)$ and $V(x)$.

Remark 3.1. Clearly, the degree of the product polynomial $\lambda(x) * V(x)$ is $(m-1)n + (n-1) = mn - 1$.

Remark 3.2. We call it 'ordered' because as proved below, unlike ordinary product it is not commutative.

Remark 3.3. Just for convenience, in the product so defined, we will refer to $\lambda(x)$ as outer polynomial and $V(x)$ as inner polynomial.

Example 1. Let $\lambda(x) = 1 + 2x + 4x^3 + 5x^4$be a polynomial (outer) of degree 4, with $m = 5$ and $V(x) = 2 + x^2 + x^6$be the polynomial of degree 6 with $n = 7$ then

$\lambda(x) * V(x)$

$$\begin{aligned}&= 1\left(2+x^2+x^6\right)+2x^7\left(2+x^2+x^6\right)+0x^{14}\left(2+x^2+x^6\right)\\&\quad+4x^{21}\left(2+x^2+x^6\right)+5x^{28}\left(2+x^2+x^6\right)\\&= 2+x^2+5x^6+2x^9+10x^{13}+8x^{21}+4x^{23}+20x^{27}+10x^{28}+5x^{30}+5x^{34}\end{aligned}$$

this example also verifies the degree of outer product of two polynomials $\lambda(x) * V(x)$ mentioned above.

Definition 3.2. Vector Equivalence of OPP: Interestingly, if polynomials are replaced by their equivalent vectors

$$\lambda(x) = c_0 + c_1x + c_2x^2 + ... + c_{m-1}x^{m-1} \approx (c_0, c_1, c_2, ..., c_{m-1}) = \lambda \text{ and}$$

$$V(x) = a_0 + a_1x + a_2x^2 + ... + a_{n-1}x^{n-1} \approx (a_0, a_1, a_2, ..., a_{n-1}) = V$$

then it can be easily seen that $\lambda * V$ is the Kronecker product of vectorsλ and V.

Properties of Ordered Power of Product of Polynomials

(i) Non Commutativity: The * product of polynomials is in general non commutative, i.e.,

$$\lambda(x) * V(x) \neq V(x) * \lambda(x).$$

Proof: Choosing $\lambda(x)$ and$V(x)$ as above, we have

$$\begin{aligned}\lambda(x) * V(x) &= \lambda\left(x^{deg(V)+1}\right) * V(x)\\&= c_0V(x) + c_1x^nV(x) + c_2x^{2n}V(x) + ... + c_{m-1}x^{(m-1)n}V(x)\end{aligned} \tag{1}$$

Also,

$$\begin{aligned}V(x) * \lambda(x) &= V\left(x^{deg(\lambda)+1}\right) * \lambda(x)\\&= a_0\lambda(x) + a_1x^m\lambda(x) + a_2x^{2m}\lambda(x) + ... + a_{m-1}x^{(n-1)m}\lambda(x)\end{aligned} \tag{2}$$

In general (1) and (2) are different and this proves the result.

(ii) Associativity : The * product of two polynomials is in general associative, i.e.,

$$\lambda(x) * (U(x) * V(x)) = (\lambda(x) * U(x)) * V(x).$$

Proof: Associativity: in the above notation associativity is easy to prove by calculations like

$$\begin{aligned}(L(x) * U(x)) * V(x) &= (\lambda(x^{deg(U)+1})U(x)) * V(x)\\&= \lambda\left(\left(x^{deg(V)+1}\right)^{deg(U)+1} U\left(x^{deg(V)+1}\right)\right)V(x)\\&= \lambda(x^{(deg(V)+1)(deg(U)+1)})U(x^{deg(V)+1))V(x)}\\&= \lambda(x^{(deg(V)deg(U)+deg(V)+deg(U+1)})U(x^{deg(V)+1))V(x)}\\&= \lambda(x) * \left(U\left(x^{deg(V)+1}V(x)\right)\right)\\&= \lambda(x) * (U(x) * V(x))\end{aligned}$$

(iii) Distributive over addition: The * product of two polynomials is such that:
(a) The outer polynomial distributes over the sum of inner polynomials,

$$\lambda(x) * (U(x) + V(x)) = (\lambda(x) * U(x)) + \lambda(x) * V(x).$$

(b) The inner polynomial distributing over the sum of outer polynomials,

$$(C(x) + D(x)) * V(x) = (C(x) * V(x)) + D(x) * V(x).$$

Where $U(x)$ and $V(x)$ are of same degree $n-1$ and $C(x)$ and $D(x)$ are of same degree $m-1$.
Proof: We prove both the forms

$$\begin{aligned}\lambda(x) * (U(x) + V(x)) &= \lambda(x^{deg(U or V)+1})(U(x) + V(x)) \\ &= \lambda(x^{deg(U)+1})U(x) + \lambda(x^{deg(V)+1})V(x) \\ &= (\lambda(x) * U(x)) + \lambda(x) * V(x).\end{aligned}$$

$$\begin{aligned}(C(x) + D(x) * V(x)) &= C\left(x^{V(x)+1}\right) V(x) + D\left(x^{V(x)+1}\right) V(x) \\ &= (C(x) * V(x)) + D(x) * V(x).\end{aligned}$$

4. Ordered Power Product of Rings of Polynomials

In this section we extend the idea considered above to product of two sets of polynomials defined over the same field F. In particular, let us consider two rings of polynomials namely, $R_m[x]\, modulo\, (x^m - 1)$ and $R_n[x]\, modulo\, (x^n - 1)$, where m and n are any positive integers. Obviously these rings contain respectively all polynomials of degree $m-1$, $n-1$ and less.

Definition 4.1. Let $\lambda_j(x) = c_{0j} + c_{1j}x + c_{2j}x^2 + ... + c_{(m-1)j}x^{m-1} \in R_m(x)$for different values of j be polynomials of degree $m-1$ or less and $V_i(x) = a_{0i} + a_{1i}x + a_{2i}x^2 + ... + a_{(n-1)i}x^{n-1} \in R_n(x)$, for different values of$i$, be polynomials of degree $n-1$ or less. We define the set of ordered power product of $R_m[x]$ and $R_n[x]$ as
$\Gamma(x) = R_m(x) * R_n(x) = \{\lambda_j(x) * V_i(x) \mid \lambda_j(x) \in R_m(x), V_i(x) \in R_n(x)\}$.

Theorem 4.2. *If $R_m[x] = F(x)\, modulo\, (x^m - 1)$, $R_n[x] = F(x)\, modulo\, (x^n - 1)$, m and n being any positive integers, are two rings of polynomial over a field F, then*

$$\Gamma(x) = R_m(x) * R_n(x) = \{\lambda_j(x) * v_i(x) \mid \lambda_j(x) \in R_m(x), v_i(x) \in R_n(x)\}$$

is a ring of polynomials of degree mn – 1, under ordinary addition and ordered product of elements of $\Gamma(x)\, modulo\, (x^m - 1, x^n - 1)$ over F.

Proof. Proof is simply followed by the definition. □

5. Applications of OPP of Polynomials in Coding Theory

In this section, we consider cyclic binary codes represented by their generator polynomials and study the codes obtained by their OPP-composition. It may be pointed out that in the binary case, $(x^n - 1)$ may be taken as $(x^n + 1)$, and that 1+1 = 0. Since cyclic codes of length n are generated by factors of $(x^n - 1)$, we next give some useful results for getting its factors.

Lemma 5.1. *For p and q distinct prime numbers and r and s are natural numbers, we have*

(i) $x^p + 1 = (1 + x)\left(1 + x + x^2 + ... + x^{p-1}\right)$

(ii) $$x^{p^r} + 1 = (1 + x)\left(1 + x + x^2 + \ldots + x^{p-1}\right)\left(1 + x^p + x^{2p} + \ldots + x^{(p-1)p}\right)\ldots$$
$$\left(1 + x^{p^{r-1}} + x^{2p^{r-1}} + \ldots + x^{(p-1)p^{r-1}}\right)$$
$$\left(1 + x^{p^r} + x^{2p^{r-1}} + \ldots + x^{(q-1)p^{r-1}}\right)\ldots$$
$$\left(1 + x^{qp^r} + x^{2qp^{r-1}} + \ldots + x^{(q-1)qp^{r-1}}\right)$$
$$\left(1 + x^{q^{s-1}p^r} + x^{2q^{s-1}p^{r-1}} + ... + x^{(q-1)q^{s-1}p^{r-1}}\right)$$
$$= (1 + x)\prod_{\alpha=1}^{\alpha=r}\left(1 + x^{p^{\alpha-1}} + x^{2p^{\alpha-1}} + ... + x^{(p-1)p^{\alpha-1}}\right)$$

(iii) $$x^{p^r p^s} + 1 = (1 + x)\left(1 + x + x^2 + \cdots + x^{p-1}\right)\left(1 + x^p + x^{2p} + \cdots + x^{(p-1)p}\right)$$
$$\ldots\left(1 + x^{p^{r-1}} + x^{2p^{r-1}} + \cdots + x^{(p-1)p^{r-1}}\right).$$

Proof. The results can be easily verified, and extended for , for any n when n is expressed as product of powers of primes. □

Theorem 5.2. *If (n_1, k_1) and (n_2, k_2) are two cyclic codes with $g_1(x)$ and $g_2(x)$ as their generator polynomials then the * product of their generator polynomials given by $g(x) = g_1(x) * g_2(x)$ of degree $(n_1 - k_1)\, n_2 + (n_2 - k_2)$code, will generate a cyclic(n, k) code, where $n = n_1 n_2$ and $k = (k_1 - 1)\, n_2 + k_2$, whenever $g_2(x)$ divides $x^n - 1$.*

Proof. Let $g(x) = g_1(x) * g_2(x)$. It can be written as a simple product: $g(x) = g_1(x^{n_2})\, g_2(x)$ where $g_1(x)$ and $g_2(x)$ are divisors of $x^{n_1} - 1$ and $x^{n_2} - 1$ respectively. For $g(x)$to represent a cyclic code of length$n_1 n_2 - 1$, $g(x)$ must a factor of $x^{n_1 n_2 - 1}$. So we examine the conditions under which $g(x) = g_1(x) * g_2(x)$ is a divisor of $x^{n_1 n_2} - 1$. First we show that $g_1(x^{n_2})$ is a divisor of $x^{n_1 n_2} - 1$. This follows from the fact that $g_1(x)$ is a divisor of $x^{n_1 - 1}$, and by replacing x by x^{n_2}. Thus we get that $g_1(x^{n_2})$ is a divisor of $(x^{n_2})^{n_1 - 1} = x^{n_1 n_2} - 1$. It remains to investigate the conditions for$g_2(x)$ is a factor of$x^{n_1 n_2} - 1$. This will not happen for all choices of $n_1 n_2$, as is evident by the Example 3 below. We therefore can use the result of the Lemma above for determining if$g_2(x)$ divides $x^{n_1 n_2} - 1$. In fact any one or products of any numbers of these factors can be taken for$g_2(x)$ to get the *-product cyclic code from generator polynomials of component codes. □

Note: It may have been noted that while choice of , the inner-polynomial, is limited, there is no limitation on choice of outer polynomial in forming OPP codes.

Theorem 5.3. *If $(n_1, k_1)$and(n_2, k_2) are two cyclic codes with $g_1(x)$and$g_2(x)$ as their generator polynomials and $g_2(x)$divides$x^{n_1 n_2} - 1$ then the * product $g(x) = g_1(x) * g_2(x)$ generates $(n_1 n_2, k)$code, where $k = (k_1 - 1) n_2 + k_2$. Also then $k - 1$ code polynomials $g(x), xg(x), x^2 g(x), ..., x^{k-1} g(x)$ span cyclic code C.*

Proof. The proof follows directly from the definition of cyclic code generated by g(x). □

Example 2. Let us consider two cyclic codes C_1andC_2 where C_1 is (7, 4) code and C_2 is (3, 1) cyclic code with generator polynomials $g_1 = \left(1 + x + x^3\right)$ and $g_2 = \left(1 + x + x^2\right)$ respectively. Then

$$\begin{aligned}
g(x) &= 1 + x + x^2 + x^3 + x^4 + x^5 + x^9 + x^{10} + x^{11} \\
xg(x) &= x + x^2 + x^3 + x^4 + x^5 + x^6 + x^{10} + x^{11} + x^{12} \\
x^2 g(x) &= x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^{11} + x^{12} + x^{13} \\
x^3 g(x) &= x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^{12} + x^{13} + x^{14} \\
x^4 g(x) &= x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{13} + x^{14} + x^{15} \\
x^5 g(x) &= x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{14} + x^{15} + x^{16} \\
x^6 g(x) &= x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{15} + x^{16} + x^{17} \\
x^7 g(x) &= x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{16} + x^{17} + x^{18} \\
x^8 g(x) &= x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{17} + x^{18} + x^{19} \\
x^9 g(x) &= x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{18} + x^{19} + x^{20}
\end{aligned}$$

Example 3. Let us consider two cyclic codes C_1andC_2 where C_1 is (3, 2) code and C_2 is (7, 4) cyclic code with generator polynomials $g_1 = (1 + x)$ and $g_2 = \left(1 + x + x^3\right)$ respectively. Then

$$g_1(x) * g_2(x) = 1 + x + x^3 + x^7 + x^8 + x^{10}.$$

Here $g_2(x) = \left(1 + x + x^3\right)$ is not a divisor of $x^{21} - 1$.

Theorem 5.4. *If two linear binary cyclic codes (n_1, k_1) and (n_2, k_2) have rates R_1 and R_2 then the rate R of their cyclic Code is an increasing function of the rate of either component code.*

Proof. Let R be the rate of the product cyclic code. Then, we have

$$R = \frac{(k_{1-1}) n_2 + k_2}{n_1 n_2} = \frac{(k_{1-1}) n_2}{n_1 n_2} + \frac{k_2}{n_1 n_2} = R_1 + \frac{(R_{2-1}) R_1}{k_1} = \frac{1}{k_1}\left(R_1 (k_1 - 1) + R_1 R_2\right)$$

$$R - R_1R_2 = \frac{1}{k_1}\left(R_1\left(k_1-1\right)+R_1R_2\right) - R_1R_2 = \frac{k_1-1}{k_1}R_1\left(1+R_2\right)$$

and trivially, $(R = R_1R_2)$, when $k = 1$. In general the difference is directly proportional to R_1, and increases with R_2. This shows that the newly introduced OPP cyclic codes are a new class of codes that have rates better than those that be obtained by their Kronecker product codes. □

6. Concluding Remarks

We have considered ordered power product of two polynomials in such powers of x that the various segments of the inner polynomial are laterally advanced without having overlaps amongst them, with coefficients multiplied by those of the outer polynomial. For this choice the motivation was to consider a new composition suitable for developing new product type of codes, which are more efficient. However, an ordered power product may be defined more generally, when this is not the case, that is when $\lambda\left(x\right) = c_0 + c_1x + c_2x^2 + ... + c_{m-1}x^{m-1}$ and $V\left(x\right) = a_0 + a_1x + a_2x^2 + ... + a_{n-1}x^{n-1}$; but$\lambda\left(x\right) * V\left(x\right) = c_0V\left(x\right) + c_1x^kV\left(x\right) + c_2x^{2k}V\left(x\right) + ... + c_{m-1}x^{(m-1)k}V\left(x\right)$. where $k < n$, or even when $k > n$. In such cases we can perhaps use the notation for OPP using an index *like* $*^k$ *−opp*. These interesting cases are being studied separately.

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