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DIFFERENCE CORDIALITY OF SOME SNAKE GRAPHS

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Abstract. Let G be a (p,q) graph. Let f be a map from $V(G)$ to $\{1,2,\ldots,p\}$. For each edge uv, assign the label $|f(u)-f(v)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0)-e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with admits a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordial labeling behavior of triangular snake, Quadrilateral snake, double triangular snake, double quadrilateral snake and alternate snakes.

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1. Introduction

Let G be a (p,q) graph. In this paper we have considered only simple and undirected graph. The number of vertices of G is called the order of G, denoted by $|V(G)|$ and the number of edges of G is called the size of G, denoted by $|E(G)|$. Labeled graphs are used in several areas such as astronomy, radar, circuit design and database management [1]. The notion of difference cordial labeling has been introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [3]. In [3, 4, 5, 6, 7], difference cordial labeling behaviour of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs have been investigated. In this paper we investigate the difference cordial labeling behaviour of Triangular snake, Quadrilateral snake, Alternate triangular snake, Alternate quadrilateral snake. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [2].

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2. Main results

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, p\}$ be a bijection. Foreach edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

Now we investigate the difference cordial labeling behavior of some snake graphs. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Theorem 2.2. *The Triangular snake T_n is difference cordial.*

Proof. Let P_n be the path $u_1 u_2 \ldots u_n$. Let $V(T_n) = V(P_n) \cup \{v_i : 1 \le i \le n-1\}$ and $E(T_n) = E(P_n) \cup \{u_i v_i, v_i u_{i+1} : 1 \le i \le n-1\}$. In this graph, $|V(T_n)| = 2n - 1$ and $|E(T_n)| = 3n - 3$. For $n > 4$, define $f : V(T_n) \to \{1, 2, \ldots, 2n-1\}$ by

$$
\begin{aligned}
f(u_i) &= 2i - 1 && 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\
f\left(u_{\lfloor \frac{n}{2} \rfloor + i}\right) &= 2\left\lfloor \frac{n}{2} \right\rfloor - 1 + i && 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\
f(v_{n-i}) &= \left\lfloor \frac{n}{2} \right\rfloor + n - 1 + i && 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\
f(v_i) &= 2i && 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1.
\end{aligned}
$$

The following table 1 shows that the labeling f defined above is a difference cordial labeing of T_n for $n > 4$.

Table 1

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{3n-4}{2}$	$\frac{3n-2}{2}$
$n \equiv 1 \pmod 2$	$\frac{3n-3}{2}$	$\frac{3n-3}{2}$

We now display a difference cordial labeling for T_2, T_3 and T_4 is given in figure 1.

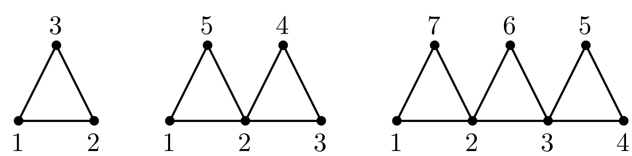

Figure 1.

□

The Quadrilateral snake Q_n is obtained from the path P_n by replacing each edge of the path by a cycle C_4.

Theorem 2.3. *All Quadrilateral snakes are difference cordial.*

Proof. Let P_n be the path $u_1u_2\dots u_n$. Let $V(Q_n) = \{v_i, w_i : 1 \le i \le n-1\} \cup V(P_n)$ and $E(Q_n) = E(P_n) \cup \{u_iv_i, v_iw_i, w_iu_{i+1} : 1 \le i \le n-1\}$. Note that $|V(Q_n)| = 3n-2$ and $|E(Q_n)| = 4n-4$. Define a map $f : V(Q_n) \to \{1, 2, 3, \dots, 3n-2\}$ by $f(v_1) = 3n-3$, $f(v_2) = 3n-2$, $f(w_1) = 3n-4$,

$$\begin{aligned} f(u_i) &= i && 1 \le i \le n \\ f(v_{n-i}) &= n+2i && 1 \le i \le n-3 \\ f(w_{n-i}) &= n+2i-1 && 1 \le i \le n-2. \end{aligned}$$

Here, $e_f(0) = e_f(1) = 2n-2$. It follows that f satisfies the edge condition of difference cordial graph. □

Next is the alternate triangular snake. An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Theorem 2.4. *Alternate triangular snakes are difference cordial.*

Proof. **Case 1.** Let the first triangle be starts from u_2 and the last triangle ends with u_{n-1}.
In this case, $|V(A(T_n))| = \frac{3n-2}{2}$ and $|E(A(T_n))| = 2n-3$. Define $f : V(A(T_n)) \to \left\{1, 2, \dots, \frac{3n-2}{2}\right\}$ by $f(u_1) = 1$,

$$\begin{aligned} f(u_{2i}) &= 3i+1 && 1 \le i \le \frac{n-2}{2} \\ f(u_{2i+1}) &= 3i && 1 \le i \le \frac{n-2}{2} \\ f(v_i) &= 3i-1 && 1 \le i \le \frac{n-2}{2}. \end{aligned}$$

and $f(u_n) = \frac{3n-2}{2}$. In this case $e_f(0) = n-1$ and $e_f(1) = n-2$ and hence f is difference cordial labeling.
Case 2. Let the first triangle be starts from u_1 and the last triangle ends with u_n. Here $|V(A(T_n))| = \frac{3n}{2}$ and $|E(A(T_n))| = 2n-1$. Define a map $f : V(A(T_n)) \to \left\{1, 2, \dots, \frac{3n}{2}\right\}$ by

$$\begin{aligned} f(u_{2i}) &= 3i-1 && 1 \le i \le \frac{n}{2} \\ f(u_{2i-1}) &= 3i-2 && 1 \le i \le \frac{n}{2} \\ f(v_i) &= 3i && 1 \le i \le \frac{n}{2}. \end{aligned}$$

Since $e_f(0) = n-1$ and $e_f(1) = n$, f is a required difference cordial labeling.
Case 3. Let the first triangle be starts from u_2 and the last triangle ends with u_n. Note that in this case, $|V(A(T_n))| = \frac{3n-1}{2}$ and $|E(A(T_n))| = 2n-2$. Define

an injective map $f : V(A(T_n)) \to \left\{1, 2, \ldots, \frac{3n-1}{2}\right\}$ by

$$f(u_1) = \frac{3n-1}{2}$$

$$\begin{array}{llll}
f(u_{4i-2}) = 6i-5 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n+1}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i+1}) = 6i-2 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n-3}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i-1}) = 3i & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n+1}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i}) = 6i & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n-3}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(v_i) = 3i-1 & 1 \le i \le \dfrac{n-1}{2}. & &
\end{array}$$

Here $e_f(0) = n-1$ and $e_f(1) = n-1$. Therefore, f is a difference cordial labeling.

Case 4. Let the first triangle be starts from u_1 and the last triangle ends with u_{n-1}. This case is equivalent to case 3. □

Now we look into alternate quadrilateral snake. An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $u_1 u_2 \ldots u_n$ by joining u_i, u_{i+1} (alternatively) to new vertices v_i, w_i respectively and then joining v_i and w_i. That is every alternate edge of a path is replaced by a cycle C_4.

Theorem 2.5. *All alternate quadrilateral snakes are difference cordial.*

Proof. **Case 1.** Let the first cycle C_4 be starts from u_2 and the last cycle be ends with u_{n-1}. Note that in this case, $|V(A(Q_n))| = 2n-2$ and $|E(A(Q_n))| = \frac{5n-8}{2}$. Define $f : V(A(Q_n)) \to \{1, 2, \ldots, 2n-2\}$ as follows:

$$f(u_1) = 1$$

$$\begin{array}{llll}
f(v_{2i-1}) = 8i-4 & 1 \le i \le \dfrac{n}{4} & \text{if} & n \equiv 0 \pmod 4 \\
 & 1 \le i \le \dfrac{n-2}{4} & \text{if} & n \equiv 2 \pmod 4 \\
f(v_{2i}) = 8i+1 & 1 \le i \le \dfrac{n-4}{4} & \text{if} & n \equiv 0 \pmod 4 \\
 & 1 \le i \le \dfrac{n-2}{4} & \text{if} & n \equiv 2 \pmod 4
\end{array}$$

$$
\begin{array}{llll}
f\left(w_{2i-1}\right)=8i-3 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n-2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(w_{2i}\right)=6i & 1\leq i\leq \dfrac{n-4}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n-2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(u_{2i}\right)=4i-2 & 1\leq i\leq \dfrac{n-2}{2} & & \\
f\left(u_{2i+1}\right)=4i-1 & 1\leq i\leq \dfrac{n-2}{2}. & &
\end{array}
$$

The table 2 given below shows that f is a difference cordial labeling.

TABLE 2

Nature of n	$e_f\,(0)$	$e_f\,(1)$
$n\equiv 0\,(\bmod\ 4)$	$\frac{5n-8}{4}$	$\frac{5n-8}{4}$
$n\equiv 2\,(\bmod\ 4)$	$\frac{5n-10}{4}$	$\frac{5n-6}{4}$

Case 2. Let the first cycle C_4 be starts from u_1 and the last cycle be ends with u_n. Here, $|V\left(A\left(Q_n\right)\right)|=2n$ and $|E\left(A\left(Q_n\right)\right)|=\frac{5n-2}{2}$. Define $f:V\left(A\left(Q_n\right)\right)\rightarrow\{1,2,\ldots,2n\}$ by

$$
\begin{array}{llll}
f\left(u_{4i-3}\right)=8i-7 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n+2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(v_{4i-2}\right)=8i-6 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n-2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(u_{4i-1}\right)=8i-3 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n-2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(u_{4i}\right)=8i-2 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n-2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(v_{2i-1}\right)=8i-5 & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)\\
 & 1\leq i\leq \dfrac{n+2}{4} & \text{if} & n\equiv 2\,(\bmod\ 4)\\
f\left(v_{2i}\right)=8i & 1\leq i\leq \dfrac{n}{4} & \text{if} & n\equiv 0\,(\bmod\ 4)
\end{array}
$$

$$
\begin{array}{llll}
 & 1 \le i \le \dfrac{n-2}{4} & \text{if} & n \equiv 2 \pmod 4 \\
f(w_{2i-1}) = 8i-4 & 1 \le i \le \dfrac{n}{4} & \text{if} & n \equiv 0 \pmod 4 \\
 & 1 \le i \le \dfrac{n+2}{4} & \text{if} & n \equiv 2 \pmod 4 \\
f(w_{2i}) = 8i-1 & 1 \le i \le \dfrac{n}{4} & \text{if} & n \equiv 0 \pmod 4 \\
 & 1 \le i \le \dfrac{n-2}{4} & \text{if} & n \equiv 2 \pmod 4 .
\end{array}
$$

In this case the following table 3 shows that f is a difference cordial labeling.

TABLE 3

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{5n-4}{4}$	$\frac{5n}{4}$
$n \equiv 2 \pmod 4$	$\frac{5n-2}{4}$	$\frac{5n-2}{4}$

Case 3. Let the first cycle C_4 be starts from u_2 and the last cycle be ends with u_n. Note that $|V(A(Q_n))| = 2n-1$ and $|E(A(Q_n))| = \frac{5n-5}{2}$. Define $f : V(A(Q_n)) \to \{1, 2, \ldots, 2n-1\}$ by

$$
\begin{array}{llll}
f(u_1) = 2n-1 & & & \\
f(u_{4i-2}) = 8i-7 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n+1}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i-1}) = 8i-6 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n+1}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i}) = 8i-3 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n-3}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(u_{4i+1}) = 8i-7 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n-3}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(v_{2i-1}) = 8i-4 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4 \\
 & 1 \le i \le \dfrac{n+1}{4} & \text{if} & n \equiv 3 \pmod 4 \\
f(v_{2i}) = 8i-1 & 1 \le i \le \dfrac{n-1}{4} & \text{if} & n \equiv 1 \pmod 4
\end{array}
$$

$$
\begin{array}{lll}
& 1 \le i \le \dfrac{n-3}{4} & \text{if} \quad n \equiv 3 \pmod 4 \\
f(w_{2i-1}) = 8i-5 & 1 \le i \le \dfrac{n-1}{4} & \text{if} \quad n \equiv 1 \pmod 4 \\
& 1 \le i \le \dfrac{n+1}{4} & \text{if} \quad n \equiv 3 \pmod 4 \\
f(w_{2i}) = 8i & 1 \le i \le \dfrac{n-1}{4} & \text{if} \quad n \equiv 1 \pmod 4 \\
& 1 \le i \le \dfrac{n-3}{4} & \text{if} \quad n \equiv 3 \pmod 4 .
\end{array}
$$

The following table 4 shows that f is a difference cordial labeling.

TABLE 4

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 4$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$
$n \equiv 3 \pmod 4$	$\frac{5n-7}{4}$	$\frac{5n-3}{4}$

Case 4. Let the first cycle C_4 be starts from u_1 and the last cycle be ends with u_{n-1}. This case is equivalent to case 3. □

Next investigation is about the irregular triangular snakes. The irregular triangular snake IT_n is obtained from the path $P_n : u_1u_2 \dots u_n$ with vertex set $V(IT_n) = V(P_n) \to \{v_i : 1 \le i \le n-2\}$ and the edge set $E(IT_n) = E(P_n) \cup \{u_iv_i, v_iu_{i+2} : 1 \le i \le n-2\}$.

Theorem 2.6. *The irregular triangular snake is difference cordial.*

Proof. Clearly $|V(IT_n)| = 2n-2$ and $|E(IT_n)| = 3n-5$. Define $f : V(IT_n) \to \{1, 2, \dots, 2n-2\}$ as follows:

Case 1. n is odd.

$$
\begin{array}{ll}
f(u_{2i-1}) = 2i-1 & 1 \le i \le \left\lceil \dfrac{n}{2} \right\rceil \\
f(u_{2i}) = 2\left\lceil \dfrac{n}{2} \right\rceil + 2i - 1 & 1 \le i \le \left\lfloor \dfrac{n-2}{2} \right\rfloor \\
f(v_{2i-1}) = 2i & 1 \le i \le \left\lceil \dfrac{n-2}{2} \right\rceil \\
f(v_{2i}) = 2\left\lceil \dfrac{n-2}{2} \right\rceil + 2i & 1 \le i \le \left\lfloor \dfrac{n-2}{2} \right\rfloor .
\end{array}
$$

Case 2. n is even. Label the vertices u_i $(1 \le i \le n-1)$ and v_i $(1 \le i \le n)$ as in case 1 and assign the label $2n-2$ to the vertex u_n. The following table 5 gives the nature of the edge condition of the above labeling f. It follows that f is a difference cordial labeling. □

The difference cordial labeling of irregular triangular snake IT_{12} is given in figure 2.

TABLE 5

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \,(\mathrm{mod}\ 2)$	$\frac{3n-4}{2}$	$\frac{3n-6}{2}$
$n \equiv 3 \,(\mathrm{mod}\ 2)$	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$

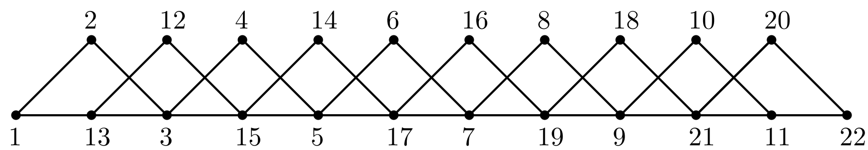

Figure 2.

The irregular quadrilateral snake IQ_n is obtained from the path $P_n : u_1u_2 \dots, u_n$ with vertex set $V(IQ_n) = V(P_n) \cup \{v_i, w_i : 1 \le i \le n-2\}$ and edge set $E(IQ_n) = E(P_n) \cup \{u_iv_i, w_iu_{i+2}, v_iw_i : 1 \le i \le n-2\}$.

Theorem 2.7. *The irregular quadrilateral snake is difference cordial.*

Proof. Clearly, $|V(IQ_n)| = 3n-4$ and $|E(IQ_n)| = 4n-7$ respectively. Define $f : V(IQ_n) \to \{1, 2, 3, \dots, 3n-4\}$ by

$$\begin{aligned} f(u_i) &= i && 1 \le i \le n \\ f(v_i) &= n + 2i - 1 && 1 \le i \le n-2 \\ f(w_i) &= n + 2i && 1 \le i \le n-2. \end{aligned}$$

Since $e_f(0) = \frac{4n-8}{2}$ and $e_f(1) = \frac{4n-6}{2}$, it follows that f is a difference cordial labeling. □

A double triangular snake DT_n consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path $u_1, u_2 \dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i $(1 \le i \le n-1)$ and to a new vertex w_i $(1 \le i \le n-1)$.

Theorem 2.8. *Double triangular snake DT_n is difference cordial iff $n \le 6$.*

Proof. For $n \le 6$, the difference cordial labeling is given in figure 3. Conversely, suppose $n > 6$ and f is a difference cordial labeling of the double triangular snake. Here, $|V(DT_n)| = 3n-2$ and $|E(DT_n)| = 5n-5$. We observe that the maximum value of $e_f(1)$ does not exceed $1 + 2(n-1) + 1 = 2n$. Hence $e_f(0) \ge q - 2n \ge 3n-5$. Therefore, $e_f(0) - e_f(1) \ge n-5$, a contradiction. □

A double quadrilateral snake DQ_n consists of two triangular snakes that have a common path.

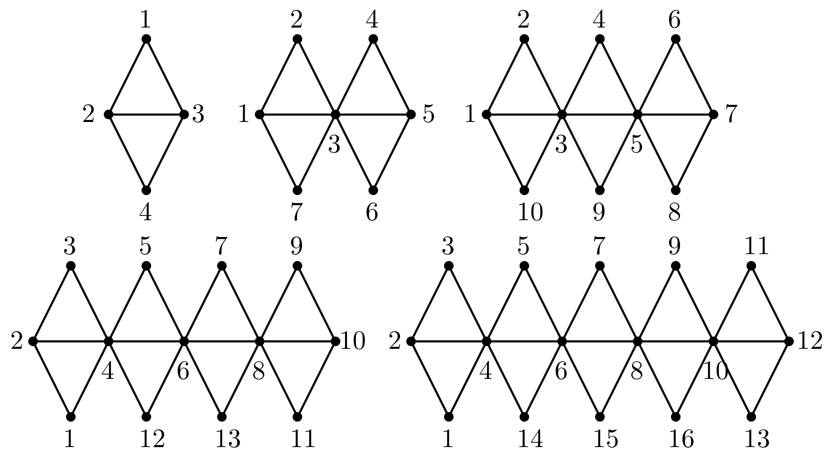

Figure 3.

Theorem 2.9. *The double quadrilateral snake is difference cordial.*

Proof. Let $V(DQ_n) = \{u_i : 1 \le i \le n\} \cup \{v_i, w_i, x_i, y_i : 1 \le i \le n-1\}$ and $E(DQ_n) = \{u_iu_{i+1}, u_iv_i, v_iw_i, u_ix_i, x_iy_i, w_iu_{i+1}, y_iu_{i+1} : 1 \le i \le n-1\}$. Clearly, $|V(DQ_n)| = 5n-4$ and $|E(DQ_n)| = 7n-7$. Define a map $f : V(DQ_n) \to \{1, 2, \ldots, 5n-4\}$ as follows:

$$
\begin{array}{ll}
f(u_i) = 3i-2 & 1 \le i \le \left\lfloor \frac{n+2}{2} \right\rfloor \\
f(v_i) = 3i-1 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\
f(w_i) = 3i & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\
f(x_i) = 3n+2i-3 & 1 \le i \le n-1 \\
f(y_i) = 3n+2i-2 & 1 \le i \le n-1 \\
f\left(u_{\lfloor \frac{n+2}{2} \rfloor + i}\right) = 3\left\lfloor \frac{n+2}{2} \right\rfloor + i - 2 & 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil \\
f\left(v_{\lfloor \frac{n}{2} \rfloor + i}\right) = 3\left\lfloor \frac{n+2}{2} \right\rfloor + \left\lceil \frac{n-2}{2} \right\rceil + 2i - 3 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\
f\left(w_{\lfloor \frac{n}{2} \rfloor + i}\right) = 3\left\lfloor \frac{n+2}{2} \right\rfloor + \left\lceil \frac{n-2}{2} \right\rceil + 2i - 2 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1.
\end{array}
$$

The following table 6 shows that $f$is a difference cordial labeling of DQ_n. □

TABLE 6

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{7n-8}{2}$	$\frac{7n-6}{2}$
$n \equiv 1 \pmod 2$	$\frac{7n-7}{2}$	$\frac{7n-7}{2}$

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i.

Theorem 2.10. *Double alternate triangular snake $DA(T_n)$ is difference cordial.*

Proof. **Case 1.** The triangles starts from u_1 and end with u_n. In this case, $|V(DA(T_n))| = 2n$ and $|E(DA(T_n))| = 3n - 1$. Define an injective map $f : V(DA(T_n)) \to \{1, 2, \ldots, 2n\}$ by

$$f(u_i) = 2i - 1 \qquad 1 \le i \le n$$
$$f(v_i) = 4i - 2 \qquad 1 \le i \le \frac{n}{2}$$
$$f(w_i) = 4i \qquad 1 \le i \le \frac{n}{2}.$$

Since $e_f(1) = \frac{3n}{2}$ and $e_f(0) = \frac{3n-2}{2}$, f is a difference cordial labeling of $DA(T_n)$.
Case 2. The triangles starts from u_2 and end with u_{n-1}. Note that In this case, $|V(DA(T_n))| = 2n - 2$ and $|E(DA(T_n))| = 3n - 5$. Label the vertices v_i and w_i $\left(1 \le i \le \frac{n-2}{2}\right)$ as in case 1 and define $f(u_i) = 2i - 3,\ 2 \le i \le n - 1$, $f(u_1) = 2n - 3$ and $f(u_n) = 2n - 2$. Since $e_f(1) = \frac{3n-6}{2}$ and $e_f(0) = \frac{3n-4}{2}$, f is a difference cordial labeling of $DA(T_n)$.
Case 3. The triangles starts from u_2 and end with u_n. It is clear that in this case, $|V(DA(T_n))| = 2n - 1$ and $|E(DA(T_n))| = 3n - 3$. Label the vertices v_i and w_i $\left(1 \le i \le \frac{n-1}{2}\right)$ as in case 1 and label u_i $(2 \le i \le n - 1)$ as in case 2 and define $f(u_1) = 2n - 1$. Since $e_f(1) = e_f(0) = \frac{3n-3}{2}$, f is a difference cordial labeling of $DA(T_n)$.
Case 4. The triangles starts from u_1 and end with u_{n-1}. This case is equivalent to case 3. □

Finally, we look into the graph double alternate quadrilateral snake. A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, it is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i, x_i and w_i, y_i respectively and adding the edges v_iw_i and x_iy_i.

Theorem 2.11. *All double alternate quadrilateral snakes are difference cordial.*

Proof. **Case 1** The squares starts from u_1 and end with u_n. In this case, $|V(DA(Q_n))| = 3n$ and $|E(DA(Q_n))| = 4n-1$. Define a map $f : V(DA(Q_n)) \to \{1, 2, \ldots, 3n\}$ by

$$f(u_i) = 3i - 1 \qquad 1 \le i \le n$$
$$f(v_i) = 6i - 5 \qquad 1 \le i \le i \le \frac{n}{2}$$
$$f(w_i) = 6i - 2 \qquad 1 \le i \le i \le \frac{n}{2}$$
$$f(x_i) = 6i - 3 \qquad 1 \le i \le i \le \frac{n}{2}$$
$$f(y_i) = 6i \qquad 1 \le i \le i \le \frac{n}{2}.$$

Since $e_f(1) = 2n$ and $e_f(0) = 2n-1$, f is a difference cordial labeling of $DA(Q_n)$.

Case 2. The squares starts from u_2 and end with u_{n-1}. In this case, $|V(DA(Q_n))| = 3n-4$ and $|E(DA(Q_n))| = 4n-7$. Label the vertices v_i, w_i, x_i, y_i $\left(1 \le i \le \frac{n-2}{2}\right)$ as in case 1 and define $f(u_i) = 3i-4$, $2 \le i \le n$, and $f(u_1) = 3n-5$. Since $e_f(1) = 2n-3$ and $e_f(0) = 2n-4$, f is a difference cordial labeling of $DA(Q_n)$.

Case 3. The square starts from u_2 and end with u_n. In this case, $|V(DA(Q_n))| = 3n-2$ and $|E(DA(Q_n))| = 4n-4$. Label the vertices v_i, w_i, x_i, y_i $\left(1 \le i \le \frac{n-1}{2}\right)$ as in case 1 and label u_i $(2 \le i \le n)$ as in case 2 and define $f(u_1) = 3n-2$. Since $e_f(1) = e_f(0) = 2n-2$, f is a difference cordial labeling of $DA(Q_n)$.

Case 4. The square starts from u_1 and end with u_{n-1}. This case is equivalent to case 3. □

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