Optimal Monetary Policy and Exchange Rate in a Small Open Economy with Unemployment[*]

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In this paper, we consider a small open economy under the New Keynesian model with unemployment of Galí (2011a, b) to discuss the design of the monetary policy. Our findings can be summarized in three parts. First, even with the existence of unemployment, the optimal policy is to minimize variance of domestic price inflation, wage inflation, and the output gap when both domestic price and wage are sticky. Second, stabilizing unemployment rate is important in reducing the welfare loss incurred by both technology and labor supply shocks. Therefore, introducing the unemployment rate as an another argument into the Taylor-rule type interest rate rule will be welfare-enhancing. Lastly, controlling CPI inflation is the best option when the policy is not allowed to respond to unemployment rate. Once the unemployment rate is controlled, however, stabilizing power of CPI inflation-based Taylor rule is diminished.

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I. Introduction

The New Keynesian model within dynamic stochastic general equilibrium (DSGE) framework has emerged in recent years. The model, however, is not perfect and has shortcomings although it has been popularized by policy practitioners and

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researchers. A major weakness of New Keynesian DSGE model is the lack of reflection on unemployment. Such a lack of consideration on unemployment and labor market frictions may be interpreted as suggesting that central banks need not consider unemployment and its fluctuations in designing monetary policy.

However, a recently growing body of studies has developed models by introducing labor market frictions and unemployment to the New Keynesian framework. (e.g., Blanchard and Galí, 2010; Christoffel et al., 2007; Faia, 2008, 2009; Thomas, 2008) Recent papers in the literature combine the nominal rigidities of New Keynesian model with labor market frictions by utilizing search- and-matching model. Walsh (2003, 2005) and Trigari (2009) examined the effects of monetary policy shocks within a model that combines labor market frictions with sticky prices and flexible wages. More recent contributions introduce various forms of nominal and real wage rigidities to study the effects of labor market frictions and unemployment on monetary policy design (e.g., Blanchard and Galí, 2010; Faia, 2008, 2009; Thomas, 2008).

Meanwhile, Galí (2011a, b) proposed different approach to introduce unemployment into the standard New Keynesian model. Galí's approach is the reinterpretation of the labor market of Erceg et al. (2000). The main advantage of Galí's approach is that the equilibrium levels of employment, the labor force, and unemployment rate can be easily determined within a standard representative household framework, not inducing labor market friction. In the model, the presence of market power in the labor markets results in unemployment whereas unemployment fluctuation results from nominal wage rigidities.

Despite substantial studies on labor market frictions and unemployment within the New Keynesian framework, few studies have extended the model to a small open economy. The recent works by Campolmi (2014), Campolmi and Faia (2014), and Rhee and Turdaliev (2013) are among notable exceptions. Campolmi (2014) and Rhee and Turdaliev (2013) studied optimal monetary policy for a small open economy in a model where both domestic prices and wages are sticky. Campolmi and Faia (2014) assess the design of the optimal exchange rate and currency regimes in presence of frictional labour markets using a two-country model. However, Campolmi (2014) and Rhee and Turdaliev (2013) do not consider unemployment fluctuations. Meanwhile, Rhee and Song (2013) extend Rhee and Turdaliev (2013) by incorporating real wage rigidities. Therefore, monetary policy design under the New Keynesian model with unemployment has been rarely explored in the context of open economy. More papers on the open economy issue include Hairault et al. (2004) and Rhee and Turdaliev (2013). This provides the motivation for our work.

In this paper, we develop a small open economy version of the New Keynesian model that allows for unemployment. For the structure of goods market, we follow Clarida et al. (2002), and Galí and Monacelli (2005). Monopolistically competitive domestic firms set their prices in staggered pricing fashion. For the labor market, individual households supply differentiated labor inputs to domestic firms that combine this labor services to produce domestic goods. An individual household with monopoly power in the labor market sets the nominal wages in staggered contract suggested by Calvo (1983). In this study, following Galí (2011b), we assume that a given portion of individuals either works a fixed number of hours or does not work at all.

Within the resulting framework, unemployment results from market power in the labor markets: the labor market power is tied by positive wage markup. Unemployment fluctuation is associated with variations in average wage markup resuslting from nominal wage rigidities. The wage markup also depends on the behavior of the terms of trade. Consequently, the terms of trade affects unemployment and its fluctuations through wage markup. Such a link between terms of trade and unemployment is a natural consequence of a small open economy framework.

Next, we turn to the relation between unemployment and the design of optimal monetary policy in a small open economy. To find the optimal monetary policy, we derive a second-order approximation to the average welfare losses in the economy with both wage and price stickiness around a steady state with zero inflation. The resulting welfare function, even with the presence of unemployment in the labor market, can be expressed as the unconditional variances of the output gap, domestic price and wage inflation. In that sense, we suggest that the optimal policy seeks to minimize a weighted average of these variances. The welfare function and the associated optimal policy are similar to that derived in Campolmi (2014), and Rhee and Turdaliev (2013).[1]

In addition to the optimal policy, we consider three alternative simple policy rules in which the domestic nominal interest rate responds to both inflation and output gap. We refer to the first rule as a domestic inflation-based Taylor rule in which the domestic interest rate responds to domestic inflation. The second rule referred to as a CPI inflation-based Taylor rule assumes that the domestic interest rate responds to CPI inflation. In a similar way, we consider the third rule referred to as a wage inflation-based Taylor rule. We begin with analysis of unemployment and other variables under the optimal monetary policy and compare it to that under

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[1] Broadly related papers include Singh and Subramanian (2008), Birchwood (2011), Clarida et al. (1999), Casares (2007), and Boivin (2006).

alternative policy rules. Using the welfare function, we also evaluate the performance of alternative policy rules. From this exercise, we find that CPI inflation-based Taylor rule generates relatively small welfare losses incurred by both technology and labor supply shocks. The critical element that distinguishes CPI inflation-based Taylor rule relative to the Domestic inflation-based rule is the smoothness of the terms of trade and the nominal exchange rate. This in turn is reflected in muted response of the real wage under the CPI inflation-based rule. The welfare gains of CPI inflation-based Taylor rule comes from the fact that it is able to stabilize unemployment rate by reducing the fluctuations in real wage. This result suggests that introducing unemployment rate into the Taylor-rule type interest rate rule will improve the household welfare.

We also compute optimal simple interest rate rules among the class of alternative policy rules considered above. From this exercise, we find that, for all cases, the inflation coefficients are positive and above one, whereas the output coefficients are negative and small. The unemployment coefficients are negative and relatively larger than output coefficients in absolute value. We also find that the welfare losses are reduced significantly once the interest rate is allowed to react to the unemployment rate. This result points to the desirability of unemployment stabilization in the monetary policy. The optimized simple rule for the specification of CPI inflation-based Taylor yields to relatively small welfare lose when unemployment is not allowed in the policy. Once unemployment rate is controlled, however, stabilizing power of CPI inflation-based Taylor rule is diminished.

The plan of this paper is as follows. In section 2, we describe the unemployment-induced NK model in a small open economy version. Section 3 presents the equilibrium conditions and dynamic system of the model. Section 4 explores implications of unemployment for monetary policy design. In section 5, we concludes.

II. Model

The model in this paper is structured as follows. First, the goods market in the model follows Galí and Monacelli (2005). Secondly, we assume that two countries, home (H) and foreign (F), are common in their preferences, technology, and market structure except their size: the foreign country is a large economy while the home country is small. Following Galí (2011a, b), we modify the labor market and introduce unemployment into the small open economy NK model. We also treat labor as being indivisible in a sense that each period individual works a fixed number of hours or does not work at all. In that sense, all variations in labor input

corresponds to variations in employment. In this environment, we illustrate optimal behavior of households and firms.

1. Households

The country H consists of a large number of identical households. Each household is populated by a continuum of members represented by the unit square and indexed by a pair $(i,j) \in [0,1] \times [0,1]$. The index, $i \in [0,1]$ indicates the type of labor services and the index, $j \in [0,1]$ represents the disutility from work. We characterize labor disutilities by $X_t j^\varphi$ when people work and 0 otherwise. $\varphi \geq 0$ indicates the inverse of Frisch elasticity of labor supply while $X_t > 0$ represents an exogenous labor supply shock. The household's utility for each period can be expressed by the integral of its member's period utilities:

$$U(C_t, \{N_t(i)\}; X_t) \equiv \log C_t - X_t \int_0^1 \int_0^{N_{t(i)}} j^\varphi dj \, di$$

$$\equiv \log C_t - X_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} \, di,$$

where $N_t(i) \in [0,1]$ is the fraction of members who are employed in period t and provide type i labor. The composite consumption index, C_t, is defined by

$$C_t \equiv \left[(1-\delta)^{\frac{1}{\eta}} C_{H,t}^{(\frac{1-\eta}{\eta})} + \delta^{\frac{1}{\eta}} C_{F,t}^{(\frac{1-\eta}{\eta})} \right]^{\frac{\eta}{1-\eta}},$$

where $C_{H,t}$ denotes an index of consumption of domestic goods given by

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(z)^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dz \right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}$$

and $C_{F,t}$ is an index of imported goods from the foreign country given by

$$C_{H,t} \equiv \left[\int_0^1 C_{F,t}(z*)^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dz* \right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}$$

where z and $z^* \in [0,1]$ are indices for the good varieties of monopolistically competitive firms at the country H and F, respectively. Notice that parameter ε_P is the elasticity of substitution between varieties of domestic and foreign goods, and parameter $\delta \in [0,1]$ is the share of imported goods in each country's domestic consumption. Parameter $\eta > 0$ represents the degree of substitutability between domestic and foreign goods. We also assume that $\xi_t \equiv \log X_t$ follows the AR(1) process:

$$\xi_t \equiv \rho_\xi \xi_{t-1} + \varepsilon_t^\xi,$$

where $\rho_\xi \in [0,1]$ and ε_t^ξ is a white noise process with zero mean and variance σ_ξ^2.

The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - X_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} \, di \right]$$

subject to the following budget constraints

$$P_t C_t + E_t \{ Q_{t,t+1} \ B_{t+1} \} \leq B_t + \int_0^1 W_t(i) N_t(i) \, di + T_t, \quad (1)$$

where the parameter β represents the subjective discount rate or time preference rate. B_t is the purchased amount of riskless and one-period discount bond paying one monetary unit, and Q_t is the price of that bond, W_t is the nominal wage for type i labor, and T_t is lump sum component of household income. The consumer price index (CPI) is defined by

$$P_t \equiv [(1-\delta) P_{H,t}^{(1-\eta)} + \delta P_{F,t}^{(1-\eta)}]^{\frac{1}{1-\eta}},$$

where the domestic price index ($P_{H,t}$) and the price index for goods imported from foreign country ($P_{F,t}$) are defined by the followings :

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(z)^{1-\varepsilon_p} dz \right]^{\frac{1}{1-\varepsilon p}}$$

$$P_{F,t} \equiv \left[\int_0^1 P_{F,t}(z*)^{1-\varepsilon_p} dz* \right]^{\frac{1}{1-\varepsilon p}}$$

We assume that the household can internationally trade a complete set of contingent claims. The riskless short-term nominal interest rate, R_t, is calculated as follows:

$$E_t \{Q_{t,t+1}\} = R_t^{-1}$$

1) Optimal Wage Setting

We now turn to the wage setting decisions of household. Following Calvo's formalism (Calvo 1983), we assume that workers specialized in a given type of labor (or the union representing them) reset their nominal wage with probability (1 - Θw) each period. That probability is independent of the time elapsed since those workers last reset their wage, and also independent across labor types. Thus, Θw is the probability that households keep their wage unchanged in any given period, representing index of nominal wage rigidities.

Consider a household re-optimizing its nominal wage in period t. Under the assumption of perfect consumption risk sharing across households, all households resetting their wage in any given period will choose the same wage in order to maximize

$$E_t \left\{ \sum_{k=0}^\infty (\beta\theta_w)^k \left[\log C_{t+k|t} - X_{t+k} \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right] \right\} \tag{2}$$

where $C_{t+k|t}$ and $N_{t+k|t}$ respectively denote the composite consumption and labor supply in period t+k of a household that last reset its wage in period t. $\overline{W_t}$ denotes the newly set wage. Maximization of (2) is subject to the labor demand schedules and budget constraints that are effective while $\overline{W_t}$ remains in place,

$$N_{t+k|t} = \left(\frac{\overline{W_t}}{W_{t+k}} \right)^{-\varepsilon w} \int_0^1 N_{t+k}(z) dz, \tag{3}$$

$$P_{t+k} C_{t+k|t} + E_t\{Q_{t,t+k+1} B_{t+k+1|t}\} \leq B_{t+k|t} + \overline{W_t} N_{t+k|t} + T_{t+k}$$

for k=0,1,2,... where $X_{t+k|t}$ denotes the value of X in period t+k of a household that last reset its wage in period t. The first-order condition is as follows:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \frac{N_{t+k|t}}{C_{t+k|t}} \left(\frac{\overline{W_t}}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_{w-1}} MRS_{t+k|t} \right) \right\} = 0, \qquad (4)$$

where $MRS_{t+k|t} \equiv X_{t+k} C_{t+k} N_{t+k|t}^{\varphi}$ denotes the marginal rate of substitution between consumption and labor supply in period t+k for the household resetting the wage in period t. Log-linearizing (4) around the zero inflation steady state yields

$$\overline{w_t} = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \}, \qquad (5)$$

where $\mu^w \equiv \log[\varepsilon_w/(\varepsilon_w - 1)]$ corresponds to the log of the optimal or desired wage markup, and parameter ε_w denotes the elasticity of substitution among labor varieties. Following $MRS_{t+k|t} \equiv X_{t+k} C_{t+k} N_{t+k|t}^{\varphi}$, we can express $mrs_{t+k|t}$, the log of $MRS_{t+k|t}$, as

$$mrs_{t+k|t} = mrs_{t+k} + \varphi(\eta_{t+k|t} - \eta_{t+k})$$
$$= mrs_{t+k} - \varepsilon_w \varphi(w_t - w_{t+k}),$$

where the last equality is due to (3). We therefore can re-express (5) as

$$\overline{w_t} = \frac{1 - \beta\theta_w}{1 + \varepsilon_w \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \{ \mu^w + mrs_{t+k} + \varepsilon_w \varphi w_{t+k} + p_{t+k} \}. \qquad (6)$$

2) Other Optimality Conditions

Not only the optimal wage setting condition, the solution to the household's problem also generates the optimal demand for each goods

$$C_{H,t}(z) = \left[\frac{P_{H,t}(z)^{-\varepsilon p}}{P_{H,t}} \right] C_{H,t} ; \quad C_{H,t}(z) = \left[\frac{P_{F,t}(z^*)^{-\varepsilon p}}{P_{F,t}} \right] C_{F,t}. \qquad (7)$$

At each time period, the optimal allocation of expenditures between domestic and imported goods is determined by

$$C_{H,t} = (1-\delta)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t; \quad C_{F,t} = \delta\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t.$$

Across two consecutive periods, the household allocate their expenditure in the following way:

$$\beta\left[\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right] = Q_{t,t+1}, \tag{8}$$

Equation (8) constitutes a standard Euler equation for intertemporal consumption decision and at the same time represents the expectational IS curve. By taking conditional expectations of both sides of (8) and rearranging with the riskless short-term nominal interest rate, we can express a standard stochastic Euler equation as

$$\beta R_t E_t\left[\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right] = 1, \tag{9}$$

For future reference, we take log-linearization to (9) and obtain

$$c_t = E_t\{c_{t+1}\} - [r_t - E_t\{\pi_{c,t+1}\} - \rho] \tag{10}$$

where lower case letters represent the log-deviations of the respective variables from their steady states, $\pi_{c,t+1} \equiv p_{t+1} - p_t$ denotes CPI inflation, and c_t is the log of total aggregate consumption. At last, $r_t \equiv -\log Q_{t+1}$ is the nominal yield on the one-period bond. In (10), we also assume $\rho = -\log \beta$ following notations in Galí (2011b).

2. Unemployment

In this section, we incorporate unemployment into the standard small open economy NK model and study its relation with the wage markup. The model mainly follows Galí (2011b). An individual that provides type i labor and experiences disutility of work $X_t J^\varphi$ will work in period t if and only if

$$\frac{W_t(i)}{P_t} \geq X_t C_t j^\varphi,$$

where the term on the right side represents the labor disutility expressed in marginal utility of consumption. If we let $L_t(i)$ denote type i labor supply or participation, marginal supplier of type i labor observes

$$\frac{W_t(i)}{P_t} = X_t C_t L_t(i)^{\varphi}. \tag{11}$$

Taking log of (11) and integrating over i, we can obtain the following approximation:

$$w_t - p_t = c_t + \varphi l_t + \xi_t, \tag{12}$$

where $w_t \cong \int_0^1 w_t(i)di$ and $l_t \cong \int_0^1 l_t(i)di$ are the first-order approximation of aggregate labor force or participation around its symmetric steady state. All the small letters denote log values of corresponding big letters while ξ_t represents the log of X_t. Equation (12) can be viewed as an aggregate labor supply or labor participation condition.

Following Galí (2011b), we consider the unemployment rate u_t as the log difference between the labor force and employment:

$$u_t \equiv l_t - n_t. \tag{13}$$

Also, the average wage markup M_t^W is assumed by

$$M_t^w = \frac{W_t/P_t}{MRS_t}.$$

Then, using the definition of marginal rate of substitution, we can express the average wage markup in log-linearized form as

$$\mu_t^{w} = (w_t - p_t) - (c_t + \varphi \eta_t + \xi_t), \tag{14}$$

where μ_t^w is a log of the average wage markup M_t^w. Combining (14) with (12) and (13), we can derive the following relation between wage markup and unemployment rate:

$$\mu_t^w = \varphi u_t. \tag{15}$$

Equation (15) shows that unemployment fluctuation is tighlty related to variations in the wage markup, which are the result of nominal wage rigidities. Now, we can define the natural rate of unemployment, u^n, that would prevail under fully flexible nominal wage to be

$$u^n = \frac{\mu^w}{\varphi}. \tag{16}$$

Equation (16) suggests that since the optimal wage markup is constant, a natural unemployment rate is also constant over time. It also shows that there exists unemployment even under fully flexible nominal wage as long as $\mu^w > 0$. Market power in the labor market, reflected in a positive optimal wage markup, accounts for the existence of unemployment.

3. Firm

1) Technology

Next, we consider the supply side of the economy. Domestic goods market in the country H is populated by a continuum of domestic firms acting as monopolistic competitors indexed by $z \in [0,1]$. Production function for differentiated goods for individual firm z takes the following form:

$$Y_t(z) = A_t N_t(z)^{1-\alpha}, \tag{17}$$

where $a_t \equiv \log A_t$ follows the AR(1) process, $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, and $N_t(z)$ is an index of labor input used by firm i and defined by

$$N_t(z) \equiv \left[\int_0^1 N_t(z,i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \tag{18}$$

where $N_t(z,i)$ denotes the quantity of i type labor employed by firm z in period t. We also assume a continuum of labor types indexed by $i \in [0,1]$.

Let $W_t(i)$ denote the nominal wage for i type labor in period t. As mentioned above, wages are set by workers of each labor type and taken as given by firms. Given the wages at any point in time and firm's total employment $N_t(z)$, cost minimization yields the following demand schedules for each firm z and labor type i :

$$N_t(z,i) = \left(\frac{W_t(i))}{W_t} \right)^{-\varepsilon_w} N_t(z) \tag{19}$$

for all z and $i \in [0,1]$, where

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\varepsilon_w} di \right]^{\frac{1}{1-\varepsilon_w}} \tag{20}$$

is the aggregate nominal wage index.

Furthermore, it is assumed that each firm receives a subsidy of τ percent of its wage bill. As a result, all the firms face the same real marginal cost:

$$mc_t = \nu + w_t - p_{H,t} - a_t + an_t, \tag{21}$$

where $v = \log(1-\tau) - \log(1-\alpha)$. Next, we present an approximate aggregate production function in relation to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(z)dz = \frac{Y_t D_t}{A_t},$$

where $D_t \equiv \int_0^1 \frac{Y_t(z)}{Y_t} dz$. Around the perfect foresight steady state, equilibrium variation in $d_t \equiv \log D_t$ are of second order (See Galí and Monacelli, 2005). Thus, up to a first order approximation, we have an aggregate production relation

$$y_t = a_t + (1-\alpha)n_t.$$

2) Price-setting

Following Calvo (1983), we assume that a fraction $(1-\theta_p)$ of (randomly selected) domestic firms reset their prices each periods. We also assume that the

fraction is independent of the time elapsed since its last price-reset. As shown in Galí and Monacelli (2005), the newly reset prices in period t can be approximated by the (log-linear) rule

$$\overline{p_{H,t}} = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{ \frac{1-a}{1-a+a\varepsilon_p} mc_{t+k} + p_{H,t} \right\}, \qquad (22)$$

where $\overline{p_{H,t}}$ denotes the log of newly set domestic prices, and $\mu^p \equiv \log M^p = \log\{\varepsilon_{p/}(\varepsilon_{p-1})\}$ corresponds to the log of price markup in a flexible price equilibrium or in the steady state.

3) Foreign country

We assume that the foreign country F is large so that

$$P_{F,t}^* = P_t^*$$
$$C_t^* = Y_t^*.$$

Similarly to the country H, the following first-order conditions hold for the country F:

$$\beta\left\{ \frac{C_t^*}{C_{t+1}^*} \quad \frac{P_t^*}{P_{t+1}^*} \right\} = Q_{t,t+1}^*, \qquad (23)$$

where $1/E_t[Q_{(t,t+1)}^*] = R_t^*$ is the riskless short-term foreign nominal interest rate. Goods produced in the country H are sold to people in the country F. The country F's demand for the country H's output z is given by

$$C_{H,t}^*(z) = \delta \left[\frac{P_{H,t}^*(z)^{-\varepsilon_p}}{P_{H,t}^*} \right] C_{H,t}^*,$$

and the optimal allocation of expenditures for domestic goods is given by

$$C_{H,t}^* = \delta \left[\frac{P_{H,t}^*}{P_t^*} \right]^{-\eta} C_t^*.$$

III. Equilibrium

1. Aggregate Demand

Goods market clearing in the country H requires

$$Y_t(z) = C_{H,t}(z) + C^*_{H,t}(z)$$

$$= \left[P_{H,t} \frac{(z)^{-\epsilon_p}}{P_{(H,t)}(z)} \right] \left[(1-\delta) \left(\frac{P_t}{P_{H,t}} \right)^{-\eta} C_t + \delta \left(\frac{P_t}{P^*_{H,t}} \right) C^*_t \right] . \tag{24}$$

We also define aggregate output as

$$Y_t \equiv \left[\int_0^1 Y_t(z)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dz \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} .$$

Combining (24) with the definition of aggregate domestic output index and the international risk-sharing condition, $C_t = Q_t C^*_t$, yields

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t [(1-\delta) + \delta Q_t^{\eta-1}], \tag{25}$$

where $Q_t = \varepsilon_t P^*_t / P_t$ is the real exchange rate.[2]

Taking the first order log-linear approximation to (25) around the steady state, we have

$$\begin{aligned} y_t &= c_t - \eta(p_{H,t} - p_t) + \delta(\eta-1)q_t \\ &= c_t + \delta\sigma s_t, \end{aligned} \tag{26}$$

where the last equality follows from the definition of the terms of trade, $s_t = p_{F,t} - p_{H,t}$ and the real exchange rate $q_t = (1-\delta)s_t$, and $\sigma \equiv \eta + (1-\delta)(\eta-1)$. For the country F, $y^*_t = c^*_t$. The analogous to (26) for the country F is

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[2] ε is a nominal exchange rate, which is defined as the price of foreign currency in terms of domestic currency.

$$c_t^* = y_t^* + (1-\delta)s_{t.} \tag{27}$$

Combining (26) with (10), we obtain

$$y_t = y_t^* + \phi s_{t,} \tag{28}$$

where $\phi \equiv (1-\delta) + \delta\sigma > 0$. Finally, combining (26) with Euler equation (10), we get

$$y_t = E_t\{y_{t+1}\} - \psi^{-1}[r_t - E_t\{\pi_{H,t+1}\} + \delta\phi^{-1}(\sigma-1)E_t\{\triangle y_{t+1}^*\} - \rho], \tag{29}$$

where $\psi \equiv 1 + \delta\phi^{-1}$.

2. Supply side

Following Galí and Monacelli (2005), the (log-linearized) optimal price-setting condition (22) can be combined with the (log-linearized) domestic inflation equation, which is a function of deviations of marginal cost from its steady state value:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} - \lambda_{pH}\,\hat{\mu}_t^{pH}, \tag{30}$$

where $\hat{\mu}_t^{\;pH} \equiv \mu_t^{\;pH} - \mu^{pH} = -\widehat{mc_t}$ and $\lambda_{pH} \equiv \dfrac{(1-\theta_{pH})(1-\beta\theta_{pH})}{\theta_{pH}(1-a+a\varepsilon_p)}(1-a)$. Let $\hat{\mu}_t^{\;w} \equiv \mu_t^{\;w} - \mu^{w}$ denote the deviation of the economy's (log) average wage markup μ_t^{w} from its steady state level μ^{w}. Then, (6) can be rewritten as

$$w_t = \beta\theta_w E_t\{\overline{w_{t+1}}\} + (1-\beta\theta_w)\big[w_t - (1-\epsilon_w\varphi)^{-1}\hat{\mu}_t^{\;w}\big]. \tag{31}$$

Since we assume that all households newly setting their wage choose the same wage, the aggregate wage index appears to be

$$w_t = \Big[\theta_w w_{t-1}^{1-\varepsilon_w} + (1-\theta_w)\overline{w}_{t-1}^{1-\varepsilon_w}\Big]^{\frac{1}{1-\varepsilon_w}}. \tag{32}$$

A first-order Taylor expansion of (32) around the zero inflation steady state yields

$$W_t = \theta_W w_{t-1} + (1-\theta_W)\overline{W}_t. \tag{33}$$

Combining (31) with (33) leads to

$$\pi_{W,t} = \beta E_t \{\pi_{W,t+1}\} - \lambda_W \widehat{\mu}_t^{\,W}, \tag{34}$$

where $\pi_{W,t} = w_t - w_{t-1}$, and $\lambda_w = \dfrac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$. Note that this wage inflation equation is similar to (30) that describes the dynamics of domestic price inflation.

Equations (34), (15), and (16) can be combined to derive a relation between wage inflation and unemployment:

$$\pi_{W,t} = \beta E_t \{\pi_{W,t+1}\} - \lambda_w \varphi (u_t - u)^n.$$

Galí (2011b) refers to this equation as the New Keynesian wage Phillips curve.

3. Equilibrium dynamics

First, we derive linearized equilibrium dynamics for the domestic price and wage inflation in term of output gap $\widetilde{y_t} = y_t - y_t^n$ following the recent standard New Keynesian small open economy model. Secondly, we introduce a new variable, the real wage gap $\widetilde{\omega}_t^{\,R}$:

$$\widetilde{\omega}_t^{\,R} \equiv \omega_t^R - (\omega_t^R)^n,$$

where $(w_t^R)^n$ is the natural real wage, i.e., the real wage that would prevail in the flexible prices and wages equilibrium. The real marginal cost is

$$\begin{aligned} mc_t &= v + w_t - p_{H,t} - a_t + \alpha n_t \\ &= v + w_t^R + \delta s_t - y_t + n_t, \end{aligned} \tag{35}$$

where last equality follows $p_t = p_{H,t} + \delta s_t$ and aggregate production relation. Then the natural real wage can be expressed as

$$(\omega_t^R)^n = -v - \mu^p - \delta s_t^n + y_t^n - n_t^n, \tag{36}$$

where s_t^n, y_t^n, and n_t^n are the terms of trade, output, and employment at the natural level. Next, we relate the average price markup $\widehat{\mu}_t^{\,pH}$ to the output and real wage

gaps. Using the fact that $\widehat{\mu}_t^{pH} = -(mc_t - mc_t^n)$,

$$
\begin{aligned}
\widehat{\mu}_t^{pH} &= [(y_t - y_t^n) - (n_t - n_t^n) - (\omega_t^R - (\omega_t^R)^n) - \delta(s_t - s_t^n)] \\
&= -\tilde{\omega}_t^R - \delta s_t \\
&= -\tilde{\omega}_t^R - \left(\frac{\delta}{\phi}\right) y_t.
\end{aligned}
\tag{37}
$$

Hence, combining (30) and (37) yields

$$
\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda_{p_H}\tilde{\omega}_t^R + \kappa_{p_H}\tilde{y}_t,
\tag{38}
$$

where $\kappa_{p_H} = \left(\dfrac{\alpha + \delta\varnothing^{-1}}{1-\alpha}\right)\lambda_{p_H}$. Equation (38) represents equation for domestic price inflation, which is similar to the equation for price inflation of Erceg, Henderson, and Levin (2000). We also relate the average wage markup $\widehat{\mu}_t^w$ to the output and real wage gaps as

$$
\begin{aligned}
\widehat{\mu}_t^w &= \omega_t^R - [c_{t+}\varphi n_{t+}\xi_t] - \mu^w \\
&= \tilde{\omega}_t^R - (1-\delta)\varnothing^{-1}y_t - \varphi\tilde{n} \\
&= \tilde{\omega}_t^R - \left[(1-\delta)\sigma^{-1} + \frac{\varphi}{1-\alpha}\right] y_t,
\end{aligned}
\tag{39}
$$

where last equality makes use of (28) and aggregate production relation. Therefore, we can derive the following wage inflation equation

$$
\pi_{W,t} = \beta E_t\{\pi_{W,t+1}\} + \kappa_w\tilde{y}_{t-}\lambda_w\tilde{\omega}_t^R,
\tag{40}
$$

where $\kappa_w = \lambda_w\left[(1-\delta)\sigma^{-1} + \dfrac{\varphi}{1-\alpha}\right]$. Combining (15) and (39), we can obtain the following equation describing relation between the unemployment rate and the output and wage gaps as:

$$
\tilde{u}_t = \varphi^{-1}\left\{\tilde{\omega}_t^R - \left[(1-\delta)\varnothing^{-1} + \frac{\varphi}{1-\alpha}\right]\tilde{y}_t\right\}.
\tag{41}
$$

Then, we observe the following identity relating the change in the real wage gap to domestic price inflation, wage inflation, and the natural wage

$$\tilde{\omega}_t^R = \tilde{\omega}_{t-1}^R + \pi_{W,t} - \pi_{H,t} - \delta \triangle s_t - \triangle (\omega_t^R)^n.$$

Meanwhile, the dynamic IS equation for the small open economy can be obtained by rewriting (26) in terms of output gap as

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \psi^{-1}[r_t - E_t\{\pi_{H,t+1}\} - \bar{rr}_t], \qquad (42)$$

where $\bar{rr}_t \equiv \rho + \psi E_t\{\triangle y_{t+1}\} - \delta(\sigma-1)\o^{-1}E_t\{\triangle y_{t+1}^*\}$ is the small open economy's natural rate of interest. In order to close the model, we assume a Taylor-type interest rule of the form

$$r_t = \rho + \o_\pi \pi_{C,t} + \o_y \tilde{y}_t + v_t, \qquad (43)$$

where $\pi_{C,t} = \pi_{H,t} + \delta \triangle s_t$ is CPI inflation, and v_t is an exogenous monetary policy component, which is assumed to follow an AR(1) process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v,$$

where $\rho_{v \in [0,1]}$ and ϵ_t^v is a white noise process with zero mean and variance σ_v^2.

As a result, we observe that $(\omega_t^R)^n = mrs_t^n$ for all t in the labor market equilibrium under flexible prices. Thus, the natural level of employment in the open economy is given by

$$n_t^n = -(v - \mu^{p_H})\Gamma^{-1} - \Gamma^{-1}\xi_t + (\Omega - \alpha\o^{-1})\Gamma^{-1}(a_t - y_t^*), \qquad (44)$$

where $\Gamma \equiv \dfrac{1}{\alpha + \varphi + (1-\alpha)(1-\alpha) + (1-\alpha)\delta\o^{-1}}$ and $\Omega \equiv \dfrac{\sigma\delta}{1-\delta+\delta\sigma}$. After some algebra, we derive the following expression for the natural values of the output, and wages:

$$(\omega_t^R)^n = -(v - \mu^{p_H}) + \frac{1}{1-\alpha}a_t + \delta\o^{-1}y_t^* - \frac{\alpha + (1-\alpha)\delta\o^{-1}}{1-\alpha}y_t^n,$$
$$y_t^n = a_t + (1-\text{a})n_t^n.$$

IV. Monetary Policy Design

This section draws implication of the existence of unemployment in a small open economy for the conduct of monetary policy. First, we consider the efficient allocation, i.e., the equilibrium allocation under fully flexible price and wage. In order to keep the analysis as simple as possible, we assume the special case of $\sigma = \eta = 1$. The efficient allocation is equivalent to the solution of a sequence of static social planner's problem as follows:

$$\max U(C_t, \{N_t(i)\}; \chi_t),$$

subject to (i) the technological constraint $Y_t(z) = A_t N_t(z)^{1-\alpha}$, (ii) the index of labor input (18) used by firm i, (iii) the market clearing condition (25), and (iv) a consumption/output possibilities set, namely, the international risk-sharing condition, $C_t = Q_t C_t^*$.[3] Then, the efficient allocation are conditioned upon

$$W_t(i) = W_t, \quad \forall i,$$
$$N_t(z,i) = N_t, \quad \forall i \ and \ z$$
$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = MPN_t,$$

where $MPN_t = (1-\alpha)(1-\delta)\dfrac{C_t}{N_t}$. The efficient allocation also can be illustrated by $C_t N_t^{\varphi} = (1-\alpha)(1-\delta)\dfrac{C_t}{N_t}$. Notice that the flexible price and wage equilibrium satisfies

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = MPN_t \frac{C_t}{N_t} \frac{1}{(1-\tau)M^W M^p}.$$

Then, at the flexible prices equilibrium, we have

$$C_t N_t^{\varphi} = (1-\alpha)(1-\delta)\frac{C_t}{N_t} \frac{1}{(1-\tau)M^w M^p}.$$

---

[3] This optimization is to seek for optimal monetary policy. Therefore, in the current section, we do not impose any monetary policy including Taylor rule in advance.

In this economy, there are two types of distortions in the market. The first distortion comes from the presence of market power in goods markets, represented by the optimal price markup (M^P). The second distortion also results from the presence of market power in labor markets, represented by the optimal wage markup (M^W). The presence of markup distortions leads to an inefficiently low level of employment and output. The above inefficiency resulting from the presence of market power can be eliminated through the suitable choice of an employment subsidy (τ) which are financed by means of lump-sum taxes. By setting $(1-\tau) = 1/(1-\delta)M^w M^p$, the condition for the efficient allocation also holds, thus guaranteeing the efficiency of the flexible price equilibrium allocation. We assume that the efficiency allocation under the flexible price equilibrium holds through this study.

1. Optimal monetary policy

In order to derive the optimal policy in this context, we begin by providing a second order approximation to the representative household's welfare losses due to domestic price and wage rigidities. In the Appendix, we derive the second-order approximation to the average utility losses around the zero-inflation steady state, under the assumption of $\sigma = \eta = 1$. The resulting welfare loss is

$$W = \frac{1-\delta}{2} E_0 \sum_{t=0}^{\infty} \beta \left\{ \left(\frac{1+\varphi}{1-\alpha} \right) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_{p_H}} (\pi_{H,t})^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_{W,t})^2 \right\} + t.i.p.,$$

(45)

where t.i.p. collects various terms that are independent of policy. Thus, the average period welfare loss is

$$L = \frac{1-\delta}{2} \left[\left(\frac{1+\varphi}{1-\alpha} \right) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_{p_H}} var(\pi_{H,t}) + \frac{\epsilon_w (1-\alpha)}{\lambda_w} var(\pi_{W,t}) \right].$$

(46)

Note that the relative weight of each of the variances is a function of parameters underlying the considered model. The welfare loss in (46) resembles that derived in Erceg et al. (2000) except for the presence of degree of openness (δ) and its dependence on domestic price inflation. This is also similar to the welfare loss in Campolmi (2014) and Rhee and Turdaliev (2013).

Now, we characterize optimal policy for our small open economy when both wages and domestic prices are sticky. According to the welfare loss in (46), it can be implied that optimal policy should pursue a balance in stabilizing domestic price inflation, wage inflation, and the output gap. Hence, the central banks will seek to minimize (45) subject to the sequence of equilibrium constraints given by (38), (40), and (42). The resulting first-order conditions are:

$$(1-\delta)\left(\frac{1+\varphi}{1-\alpha}\right)\tilde{y}_t + \varsigma_{1t}\kappa_{p_H} + \varsigma_{2t}\kappa_w = 0, \tag{47}$$

$$(1-\delta)\frac{\varepsilon_p}{\lambda_{pH}}\pi_{H,t} - \varsigma_{1t} + \varsigma_{1t-1} - \varsigma_{3t} = 0, \tag{48}$$

$$(1-\delta)\frac{\varepsilon_W(1-\alpha)}{\lambda_w}\pi_{W,t} - \varsigma_{2t} + \varsigma_{2t-1} + \varsigma_{3t} = 0, \tag{49}$$

$$-\varsigma_{3t} + \lambda_{pH}\pi_{W,t} + \beta E_t\{\varsigma_{3t+1}\} = 0, \tag{50}$$

where ς_{1t}, ς_{2t}, and ς_{3t} are the Lagrange multipliers associated with the three constraints. The dynamical system describing the optimal monetary policy thus consists of (47)-(50) together with constraints (38)-(42).

2. Evaluation of monetary policy rules

In this section, we compare alternative policy rules and perform calibrations to quantitatively evaluate those monetary policy rules. In this section, three different simple rules are considered: domestic inflation-based Taylor rule, CPI inflation-based Taylor rule, and wage inflation-based Taylor rule. The domestic inflation-based Taylor rule requires that the domestic interest rate reacts systematically to domestic inflation while the CPI inflation-based Taylor rule assumes that the domestic interest rate responds to CPI inflation. In addition, the wage inflation-based Taylor rule requires that domestic interest rate responds to wage inflation.

First, the domestic inflation-based Taylor rule (DIT, for short) takes the form of

$$r_t = \rho + \emptyset_\pi \pi_{H,t} + \emptyset_y \tilde{y}_t. \tag{51}$$

The CPI inflation-based Taylor rule (CPIT, for short) is assumed to take the form of

$$r_t = \rho + \emptyset \pi_{C,t} + \emptyset_y \tilde{y}_t. \tag{52}$$

Finally, the wage inflation-based Taylor rule (WIT, for short) is structured by

$$r_t = \rho + \emptyset_\pi \pi_{W,t} + \emptyset_y \tilde{y}_t. \tag{53}$$

1) Calibration

The setting chosen for most of parameters in our calibration is standard. In the baseline calibration of the model, one period corresponds to one quarter of a year. We set $\sigma = \eta = 1$, which is consistent with the case considered in the next section. The discount factor β is set to 0.99, which generates a real interest rate of around 4% per annum. We set the value of δ (degree of openness) to 0.4, following most previou studies. Parameter α, the degree of decreasing returns to labor, is set to 0.25. The elasticity of substitution among goods ε_p is set to 9. This implies that at the steady state, the price markup is 12.5 percent, and with the calibration of α, labor income share is equal to two thirds. The domestic price and wage contract duration parameters are set as $\theta_{PH} = \theta_W = 0.75$, which is consistent with much of the micro-evidence.

Galí (2011b) argues that the introduction of unemployment into the standard New Keynesian model imposes some restriction on the calibration of the inverse Frisch elasticity of labor supply, φ, and the elasticity of substitution among labor services, ε_W, since the average markup is related to the natural rate of unemployment as $\frac{\varepsilon_w}{1-\varepsilon_w} = exp(\varphi u^n)$. Therefore, we set $\varphi=5$, which implies that the labor supply elasticity is taken as one fifth and $\varepsilon_W = 4.52$. The values of average wage markup, then, is 28 percent. Finally, we follow Galí and Monacelli (2005), and Galí (2011b) to specify the exogenous processes as follows:

$$a_t = 0.66a_{t-1} + \varepsilon_t^a, \quad \sigma_a = 0.0071,$$
$$\xi_t = 0.90\xi_{t-1} + \epsilon_t^\xi, \quad \sigma_\xi = 0.0075,$$

where ε_t^a and ϵ_t^ξ are white noises with variances σ_a and σ_ξ, respectively.

2) Impulse Response

Given the baseline calibration, Figure 1 shows the dynamic responses of major economic variables to a productivity shock under different policy rules. For the purpose of comparison, we also present the impulse responses under the optimal rule. First, we describe impulse responses under the optimal policy. We see that most variables remain stable to the shock under the optimal policy. However, the result implies that optimal policy is more accommodative of technological shock than any other alternative policies since the associated output is relatively more

Figure 1. Impulse Responses to a Technological Shock: Alternative Policy Rules
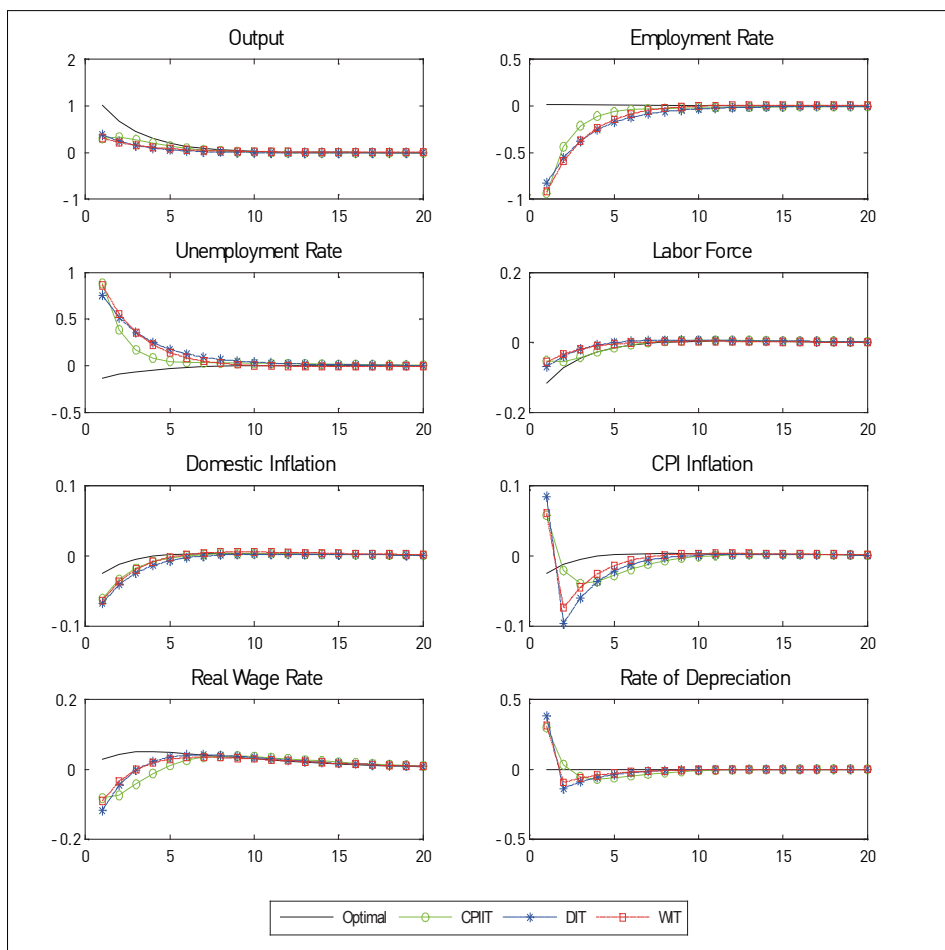
increasing. The accommodative policy reaction, then, leads to very stable unemployment rate. The response of CPI inflation is very weak due to muted response of the rate of depreciation. Due to the presence of nominal wage rigidity and the muted response of CPI inflation, real wage responds weakly. The stable movement of real wage under optimal policy should result in stable employment.

Same figure shows the corresponding impulse responses under alternative policy rules. The main finding is that the CPI inflation-based Taylor rule is more accommodative of the productivity shock than any other policy rules since the associated output increases more and employment remains relatively stable. As a

Figure 2. Impulse Responses to Labor Supply Shock: Alternative Policy Rules
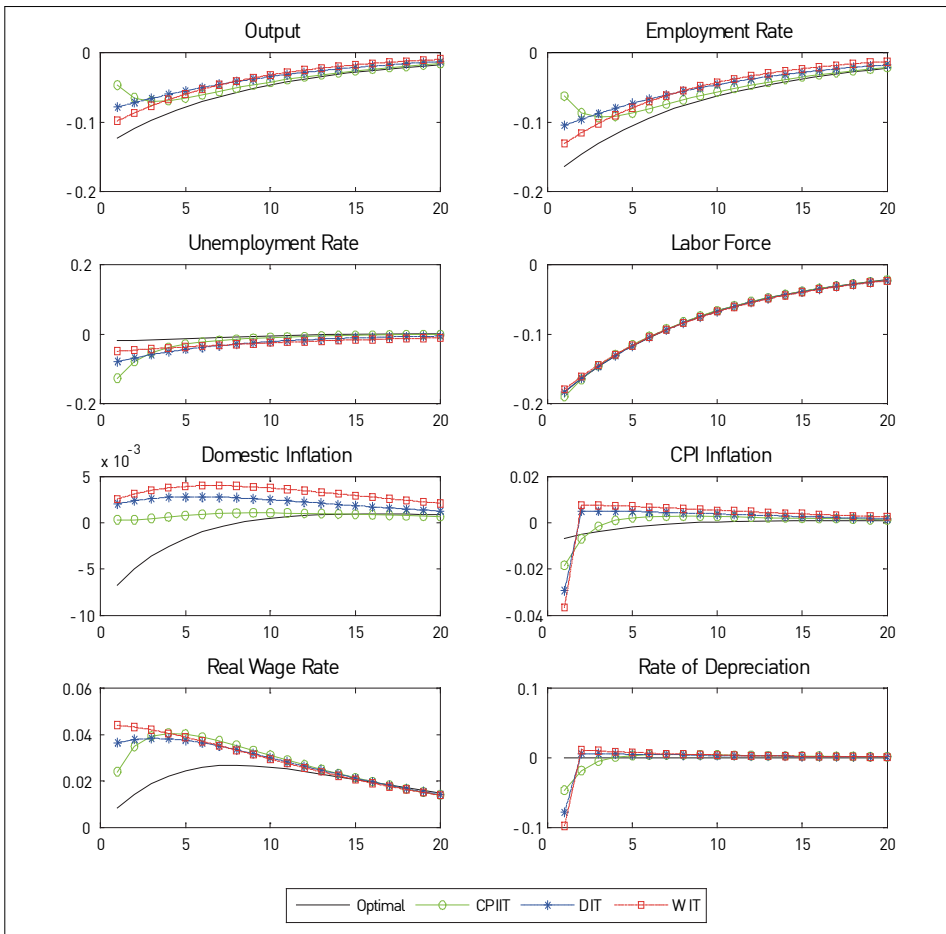
result, the responses of key variables are considerably muted under the CPI inflation-based Taylor rule comparing to other alternative policies. This is mainly due to the stabilization of the exchange rate and CPI inflation. The stable CPI inflation generated more muted response of real wage, leading to relatively small change in unemployment rate and employment.

Figure 2 displays the response of the same variables to a labor supply shock under the optimal policy and the alternative policy rules. Notice that response of labor force is almost identical under different policy rules. The optimal policy stabilizes unemployment rate almost perfectly by fully accommodating the labor supply shock. Hence, the response of employment is very close to the labor force. Other variables, especially real wage, show relatively muted responses under optimal policy except domestic inflation. The responses of the same variables are similar under three different policy regimes except that the CPI inflation-based Taylor rule generates more muted responses of CPI and domestic inflation.

3) Second moments and welfare losses

To perform another quantitative comparison, we calculate the standard deviations of economic variables and the loss incurred by the shocks. Table 1 shows main variables' standard deviation due to the shocks. The left panel of table 1 compares the standard deviations of the main variables conditional on technology shock. The results confirm that key macroeconomic variables are relatively stable under the optimal policy even though output is more volatile. Especially, it seems to be that

Table 1. Statistical Properties of Alternative Policy Regimes

	Technology Shock				Labor Supply Shock			
	Optimal	CPIT	DIT	WIT	Optimal	CPIT	DIT	WIT
Real Wage	0.0496	0.0871	0.1225	0.0836	0.0213	0.0275	0.0438	0.0419
CPI Inf	0.0200	0.0576	0.1140	0.0772	0.0069	0.0138	0.0250	0.0311
Output	0.8246	0.2945	0.3527	0.2516	0.1266	0.0512	0.0815	0.0882
Labor Force	0.0941	0.0512	0.0644	0.0449	0.1909	0.1437	0.1879	0.1623
Unemp Rate	0.1094	0.6046	0.7112	0.6948	0.0223	0.0904	0.0796	0.0450
Employment	0.0204	0.6462	0.7728	0.7395	0.1687	0.0682	0.1087	0.1176
Rate of Dep	0.0200	0.2584	0.2904	0.2017	0.0069	0.0510	0.0841	0.0857
Domestic Inf	0.0200	0.0447	0.0624	0.0521	0.0069	0.0004	0.0029	0.0032
Wage Inf	0.0038	0.0168	0.0317	0.0237	0.0016	0.0041	0.0074	0.0068

note: Standard deviations expressed in percent

the unemployment rate is very stable under the optimal policy. Comparing other rules, we find that CPI inflation-based Taylor rule generates relatively stable real wage and unemployment rate.

The right panel of table 1 shows the relevant second moments conditional on labor supply shock. Under the optimal policy, unemployment rate is stable even though output and employment are more unstable than those of alternative policies. From table 1, we can see that the volatility of unemployment rate incurred by both technology and labor supply shocks is minimized under the optimal policy. The conventional simple interest rate rules, however, do not respond to unemployment rate. Therefore, stabilizing unemployment rate by introducing the unemployment rate as an another argument into the Taylor-rule type interest rate rule may reduce the welfare loss.

To provide a quantitative evaluation of alternative policy rules, we compute the unconditional losses incurred by both technology and labor supply shocks under the three alternative policy rules (DIT, CPIT and WIT). In this exercise, the parameters other than the Frisch elasticity of labor supply (φ) are set at their baseline values. Table 2 reports the variance of output gap, domestic price and wage inflation, and the welfare losses associated with the three alternative policy rules (DIT, CPIT and WIT). We display the effects of changing the inverse of the Frisch elasticity of labor supply (as implied by changes in φ). The top panel reports statistics corresponding to the benchmark calibration of the elasticity of labor supply, namely, $\varphi=5$. Among the three alternative rules, the one that targets CPI inflation generates the smallest volatilities of the three welfare-relevant variables, and thus implies that welfare loss is minimized. Relative to that benchmark, second panel assumes a lower inverse of the Frisch elasticity of labor supply ($\varphi=1$), while the third panel reports results for a higher inverse of the Frisch elasticity of labor supply ($\varphi=10$). Under these calibrations considered, the ranking among alternative policy rules is not affected. The main findings of this exercise are consistent with the quantitative evaluation of the standard deviation conducted in table 1, that CPI inflation-based Taylor rule generates relatively small welfare losses. This result is similar to those of Campolmi (2014), and Rhee and Turdaliev (2013).

The terms of trade and nominal exchange rate for the CPI inflation-based Taylor rule is smoother than for the domestic inflation-based rule. In the sequel, smooth terms of trade and nominal exchange rate can mute response of the real wage under the CPI inflation-based rule. In an open economy, controlling of CPI inflation, by reducing the volatility of the exchange rate, reduces unemployment fluctuation by stabilizing real wage.

Table 2. Contribution to Welfare Losses

	Technology Shock				Labor Supply Shock			
	Optimal	CPIT	DIT	WIT	Optimal	CPIT	DIT	WIT
				$\varphi =5$				
Var(ỹ)	0.0006	0.0036	0.0045	0.0041	0.0028	2.0E-5	0.0005	0.0008
Var(π_H)	2.2E-5	0.0005	0.0011	0.0008	0.0001	0.0018	0.0033	0.0036
Var(π_W)	0.0003	0.4310	0.3870	0.4750	0.0012	0.3330	0.1350	0.0503
Loss	0.0857	1.6900	1.9400	2.0000	0.4200	1.5200	2.0300	2.3200
				$\varphi =1$				
Var(ỹ)	0.0024	0.4360	0.3890	0.4980	0.0089	0.2440	0.1260	0.0420
Var(π_H)	0.0004	0.0034	0.0043	0.0039	0.0015	0.0001	0.0007	0.0012
Var(π_W)	0.0002	0.0002	0.0006	0.0004	0.0012	0.0024	0.0046	0.0052
Loss	0.1510	1.5600	1.7300	1.8300	0.7020	1.5200	2.1700	2.2700
				$\varphi =10$				
Var(ỹ)	9.6E-5	0.4070	0.3840	0.4710	0.0004	0.2710	0.1380	0.0568
Var(π_H)	0.0006	0.0036	0.0046	0.0042	0.0029	4.0E-5	0.0005	0.0011
Var(π_W)	6.3E-6	0.0006	0.0012	0.0009	4.5E-5	0.0014	0.0031	0.0035
Loss	0.0862	1.6500	1.9800	2.0400	0.3920	1.2200	1.6000	1.6300

note: Entries are percentage units of natural output

4) Optimal simple rules

Despite alternative interest rate rules discussed in the previous section, most central banks implement simple feedback interest rate rules. For this reason, we study the optimal operational interest rate rules. Such a rule can be derived by searching, within the class of Taylor-type rules, for the parameters that minimize the unconditional period utility given by

$$(1-\delta)\left[\left(\frac{1+\varphi}{1-\alpha}\right)var\left(\tilde{y}_t\right) + \frac{\varepsilon_p}{\lambda_{PH}}var\left(\pi_{H,t}\right) + \frac{\varepsilon_w\left(1-\alpha\right)}{\lambda_W}var\left(\pi_{W,t}\right)\right].$$

We consider the following specification of the interest rate rules:

$$r_t = \rho + \emptyset_y y_t + \emptyset_{\pi H}\pi_{H,t} + \emptyset_{\pi C}\pi_{C,t} + \emptyset_{\pi W}\pi_{W,t} + \emptyset_u u_t,$$

where we include the unemployment rate as an argument relative to the alternative policy rules considered in the previous section.

Table 3 reports the optimized coefficients and the corresponding welfare loss of the simple interest rate rules specified above. Rows (a)-(f) show the optimized coefficients and the resulting welfare loss for the following specification of the interest rate rules: a) DIT without unemployment rate, (b) DIT with unemployment rate, (c) CPIT without unemployment rate, (d) CPIT with unemployment, (e) WIT without unemployment rate, and (f) WIT with unemployment rate.

Notice that for all cases, the inflation coefficients are positive and above one, whereas the output coefficients are negative. The negativity of output coefficient is opposed to the conventional Taylor rule in which the optimized weight to output is positive. The possible explanation is that the optimized simple rule calls for the further accommodation of shock in computation. However, the optimal response to output is very small, especially when unemployment is introduced as an argument, and this results are also consistent with the findings of Galí (2011b) in a closed economy environment. When interest rate responds to unemployment rate, the output coefficients increase significantly in absolute values but the inflation coefficients change slightly. The unemployment coefficients are negative and relatively larger than output coefficients in absolute value. The welfare losses are reduced significantly once the interest rate is allowed to respond to the unemployment rate. This result points to the desirability of unemployment stabilization in monetary policy, which is in line with the findings of Blanchard and Galí (2010) and Faia (2009).

Table 3. Optimal Simple Rules

	Technology Shock						Labor Supply Shock					
	$\emptyset_y$	$\emptyset_{\pi H}$	$\emptyset_{\pi C}$	$\emptyset_{\pi W}$	$\emptyset_u$	Loss	$\emptyset_y$	$\emptyset_{\pi H}$	$\emptyset_{\pi C}$	$\emptyset_{\pi W}$	$\emptyset_u$	Loss
(a)	-0.04	1.47				6.75	-0.007	1.54				5.82
(b)	-0.62	1.14			-1.84	0.40	-0.093	1.11			-0.17	0.08
(c)	-0.07		1.95			2.60	-0.027		1.11			4.82
(d)	-0.37		1.14		-0.66	0.65	-0.050		1.10		-0.12	0.26
(e)	-0.03			1.01		5.26	-0.031			1.90		25.8
(f)	-0.47			1.12	-0.91	0.45	-0.011			1.12	-0.25	0.14

The optimized simple rule for the specification of CPI inflation-based Taylor generates relatively small welfare lose when unemployment is not allowed in the policy. When unemployment rate is augmented, the optimized CPI inflation-based Taylor rule is not the best welfare loss-minimizing rule. The merit of CPI

inflation-based Taylor rule is that it reduces unemployment fluctuation by stabilizing real wage. Once unemployment rate is controlled, it might not be needed to dampen fluctuation of real wage to stabilize unemployment. Instead, CPI inflation-based Taylor rule generates the excess smoothness of both terms of trade and the nominal exchange rate. Thus, it hinders adjustment that might have occurred through exchange rate movement. Therefore, stabilizing power of CPI inflation-based Taylor rule is diminished.

V. Conclusion

In this paper, we extend Galí's (2011a, b) New Keynesian model with unemployment to a small open economy. By utilizing this framework, we seek for the optimal monetary policy rule and compare alternative policy rules, and also compute optimized simple rules. Our main findings can be summarized as threefold. First, the optimal policy is to minimize variance of domestic price inflation, wage inflation, and the output gap when both domestic price and wage are sticky. Second, a policy that responds to an unemployment rate is welfare enhancing. Last, controlling CPI inflation induces relatively small welfare losses.

However, the following limitations remain for future research. First, as pointed out by Galí (2011b), the only source of unemployment is the positive wage markup from noncompetitive labor market. However, as shown in the text, the wage markup is easily fixed by simple fiscal policy (employment subsidy). Therefore, introducing certain forms of real frictions into the labor market would improve the model's performance and provide more reality for the model.

The cyclical movements of CPI inflation and real wage rate in our model do not well correspond to real data. Such a different pattern between model and data is mainly due to the assumption of complete exchange rate pass-through of nominal exchange rate on import prices. However, imperfect pass-through is more often observed in real economy. Therefore, it may be possible to fix this anomaly by incorporating the assumption of imperfect pass-through. Recently, a large literature introduces traded-goods as well as non-traded-goods into small open economy New Keynesian models (e.g. Kam, 2007; Kuralbayeva, 2011). Therefore, it would be interesting to study how the introducing non-traded goods to domestic goods market can affect the major findings in the paper. Another shortcoming of our study is that, for the simple interest rate rule, the optimized weight to output is negative. This result is also found in many literature without any detailed discussion. This issue should be discussed in more detail in the future research.

Appendix

This appendix presents a second-order approximation to the the representative household utility around an efficient steady state. In doing so, we assume $\sigma = \eta = 1$ and take an advantage of the following fact:

$$\frac{X_t - X}{X} = x_t + \frac{1}{2} x_t^2,$$

where x_t is the log deviation from steady state for the variable X_t. The second-order Taylor approximation of the household i's period t utility, $U_t(i)$, around a steady state and intergrating across households yields

$$\int_0^1 (U_t(i) - Y)di$$
$$\cong U_c C \left[c_t + (\frac{1}{2} + \frac{C}{2} \frac{U_{CC}}{U_C}) c_t^2 \right] + U_{N} N \left[n_t + (\frac{1}{2} + \frac{N}{2} \frac{U_{NN}}{U_N}) n_t^2 \right] + t.i.p.,$$

where t.i.p. collects all the terms independent of policy.

Using the fact $\frac{c}{2} \frac{U_{CC}}{U_C} = -\frac{1}{2}$ and $\frac{1}{2} + \frac{N}{2} \frac{U_{NN}}{U_N} = \frac{1+\varphi}{2}$ and the market clearing condition $c_t = (1-\delta)y_t + \delta y_t^*$,

$$\int_0^1 (U_t(i) - U)di \cong U_c C(1-\delta)y_t + U_N N \left[\int_0^1 n_t(i)di + \frac{1+\varphi}{2} \int_0^1 n_t^2(i)di \right] + t.i.p.,$$

define aggregate employment as $N_t = \int_0^1 N_t(i)di$, or, in terms of log deviations from the steady state and up to a second-order approximation,

$$n_t + \frac{1}{2} n_t^2 \cong \int_0^1 \widetilde{n_t}(i)di + \frac{1}{2} \int_0^1 \widetilde{n_t}(i)^2 di.$$

Note also that

$$\int_0^1 n_t(i)^2 di = \int_0^1 (n_t(i) - n_t + n_t)^2 di$$

$$= \widetilde{n_t^2} - 2n_t \varepsilon_w \int_0^1 (w_t(i) - w_t) di + \varepsilon_t^2 \int_0^1 (w_t(i) - w_t)^2 di$$

$$= n_t^2 + \varepsilon_w^2 var_i\{w_t(i)\},$$

where we have used the labor demand function $n_t(i) - n_t = -\varepsilon_w(w_t(i) - w_t)$, and the fact that $\int_0^1 (w_t(i) - w_t) di = 0$ and that $\int_0^1 (w_t(i) - w_t)^2 di = var_i\{w_t(i)\}$ is of second order.

Next, we turn to a relationship between aggregate employment and output:

$$N_t = \int_0^1 \int_0^1 N_t(z,i) di\, dz = \int_0^1 N_t(z) \int_0^1 \frac{N_t(z,i)}{N_t(z)} di\, dz$$

$$= \triangle_{w,t} \int_0^1 N_t(z) dz = \triangle_{w,t} (\frac{Y_t}{A_t})^{\frac{1}{1-a}} \int_0^1 (\frac{Y_t(z)}{A_t})^{\frac{1}{1-\alpha}}$$

$$= \triangle_{w,t} \triangle_{pH} \int_0^1 (\frac{Y_t(z)}{Y_t})^{\frac{1}{1-\alpha}} dz,$$

where $\triangle_{w,t} = \int_0^1 (\frac{w_t(i)}{w_t})^{-\varepsilon_w} di$ and $\triangle_{PH,t} = \int_0^1 \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\epsilon_p} dz$. We therefore obtain the following second-order approximation:

$$n_t = \frac{1}{1-\alpha}(\widetilde{y}_t - a_t) + d_{w,t} + d_{p_{H,t}}$$

where $d_{w,t} = \log \int_0^1 \left(\frac{w_t(i)}{w_t}\right)^{-\varepsilon_w} di$ and $d_{p_{H,t}} = \log \int_0^1 \left(\frac{P_{H,t}(Z)}{P_{H,t}}\right)^{-\epsilon_w} dz$.

Lemma 1: $d_{P_{H,t}} = \frac{\varepsilon_p(1-\alpha+\alpha\varepsilon_p)}{2(1-\alpha)^2} var_{z\{P_{H,t}\}}$.

Proof. See Galí and Monacelli (2005).

Lemma 2: $d_{w,t} = \dfrac{\varepsilon_w}{2} var_i\{w_t(i)\}$.

Proof. See Erceg et al. (2000).

Then, one-period aggregate welfare can be expressed as follows

$$\int_0^1 \frac{U_t(i) - U}{U_C C} di =$$
$$-\frac{(1-\delta)}{2}\left[(\frac{1+\varphi}{1-\alpha})\widetilde{y}_t^2 + \frac{\varepsilon_p(1-\alpha+\alpha\varepsilon_{P)}}{(1-\alpha)} var_{z\{p_{H,t}\}} + \varepsilon_w(1-\alpha)[1+\varphi\varepsilon_w]var_i\{w_t(i)\}\right] + t.i.p.$$

Lemma 3:

$$\sum_{t=0}^{\infty} \beta^t var_z\{p_{H,t}\} = \frac{\theta_{p_H}}{(1-\beta\theta_{p_H})(1-\theta_{p_H})}\sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2 \,,$$
$$\sum_{t=0}^{\infty} \beta^t var_i\{w_t(i)\} = \frac{\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2 .$$

Proof. See Woodford (2003, Chapter 6).

Then, we can collect all the previous results and derive the following second-order approximation to the small open economy's aggregate welfare function:

$$W = \frac{1-\delta}{2} E_0 \sum_{t=0}^{\infty} \beta\left\{(\frac{1+\varphi}{1-\alpha})\widetilde{y}_t^2 + \frac{\varepsilon_p}{\lambda_{pH}}(\pi_{H,t})^2 + \frac{\varepsilon_w(1-\alpha)}{\lambda_w}(\pi_{W,t})^2\right\} + t.i.p.$$

where $\lambda_{pH} \equiv \dfrac{(1-\theta_{pH})(1-\beta\theta_{pH)}}{\theta_{pH}(1-\alpha+\alpha\varepsilon_p)}(1-\alpha)$ and $\lambda_W = \dfrac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$.

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