# THE GENERALIZED HYERS-ULAM STABILITY OF QUADRATIC FUNCTIONAL EQUATION WITH AN INVOLUTION IN NON-ARCHIMEDEAN SPACES 

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$$
\begin{aligned}
& \text { AbSTRACT. In this paper, using fixed point method, we prove the } \\
& \text { Hyers-Ulam stability of the following functional equation } \\
& (k+1) f(x+y)+f(x+\sigma(y))+k f(\sigma(x)+y)-2(k+1) f(x)-2(k+1) f(y)=0
\end{aligned}
$$

with an involution $\sigma$ for a fixed non-zero real number $k$ with $k \neq-1$.

## 1. Introduction and preliminaries

In 1940, Ulam [13] posed the following problem concerning the stability of functional equations: Let $G_{1}$ be a group and let $G_{2}$ a meric group with the metric $d(\cdot, \cdot)$. Given $\epsilon>0$, does there exist a $\delta>0$ such that if a mapping $h: G_{1} \longrightarrow G_{2}$ satisfies the inequality $d(h(x y), h(x) h(y))<\delta$ for all $x, y \in G_{1}$, then there exists a homomorphism $H: G_{1} \longrightarrow G_{2}$ with $d(h(x), H(x))<\epsilon$ for all $x \in G_{1}$ ?

Hyers [6] solved the Ulam's problem for the case of approximately additive functions in Banach spaces. Since then, the stability of several functional equations have been extensively investigated by several mathematicians [3, 5, 7, 8]. The Hyers-Ulam stability for the quadratic functional equation

$$
\begin{equation*}
f(x+y)+f(x-y)=2 f(x)+2 f(y) \tag{1.1}
\end{equation*}
$$

was proved by Skof [11] for a function $f: E_{1} \longrightarrow E_{2}$, where $E_{1}$ is a normed space and $E_{2}$ is a Banach space and later by Jung [10] on unbounded domains.

[^0]Let $X$ and $Y$ be real vector spaces. For an additive mapping $\sigma$ : $X \longrightarrow X$ with $\sigma(\sigma(x))=x$ for all $x \in X$, then $\sigma$ is called an involution of $X$. For a given involution $\sigma: X \longrightarrow X$, the functional equation

$$
\begin{equation*}
f(x+y)+f(x+\sigma(y))=2 f(x)+2 f(y) \tag{1.2}
\end{equation*}
$$

for all $x, y \in X$ is called the quadratic functional equation with an involution and a solution of (1.2) is called a quadratic mapping with an involution. The functional equation (1.2) has been studied by Stetkær [12] and the Hyers-Ulam-Rassias Theorem has been obtained by Bouikhalene et al. $[1,2,9]$.

In this paper, using fixed point method, we prove the generalized Hyers-Ulam stability of the following functional equation
$(k+1) f(x+y)+f(x+\sigma(y))+k f(\sigma(x)+y)-2(k+1) f(x)-2(k+1) f(y)=0$ for a fixed non-zero real number $k$ with $k \neq-1$.

A valuation is a function $|\cdot|$ from a field $K$ into $[0, \infty)$ such that for any $r, s \in K$, the following conditions hold: (i) $|r|=0$ if and only if $r=0$, (ii) $|r s|=|r||s|$, and (iii) $|r+s| \leq|r|+|s|$.

A field $K$ is called $a$ valued field if $K$ carries a valuation. The usual absolute values of $\mathbb{R}$ and $\mathbb{C}$ are examples of valuations. If the triangle inequality is replaced by $|r+s| \leq \max \{|r|,|s|\}$ for all $r, s \in K$, then the valuation $|\cdot|$ is called a non-Archimedean valuation and the field with a non-Archimedean valuation is called non-Archimedean field. If $|\cdot|$ is a non-Archimedean valuation on $K$, then clearly, $|1|=|-1|$ and $|n| \leq 1$ for all $n \in \mathbb{N}$.

Definition 1.1. Let $X$ be a vector space over a scalar field $K$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\|: X \longrightarrow \mathbb{R}$ is called a non-Archimedean norm if satisfies the following conditions:
(a) $\|x\|=0$ if and only if $x=0$,
(b) $\|r x\|=|r|\|x\|$, and
(c) the strong triangle inequality (ultrametric) holds, that is,

$$
\|x+y\| \leq \max \{\|x\|,\|y\|\}
$$

for all $x, y \in X$ and all $r \in K$.
If $\|\cdot\|$ is a non-Archimedean norm, then $(X,\|\cdot\|)$ is called a nonArchimedean normed space. Let $(X,\|\cdot\|)$ be a non-Archimedean normed space. Let $\left\{x_{n}\right\}$ be a sequence in $X$. Then $\left\{x_{n}\right\}$ is said to be convergent if there exists $x \in X$ such that $\lim _{n \rightarrow \infty}\left\|x_{n}-x\right\|=0$. In that case, $x$ is called the limit of the sequence $\left\{x_{n}\right\}$, and one denotes it by $\lim _{n \rightarrow \infty} x_{n}=$
$x$. A sequence $\left\{x_{n}\right\}$ is said to be a Cauchy sequence if $\lim _{n \rightarrow \infty} \| x_{n+p}-$ $x_{n} \|=0$ for all $p \in \mathbb{N}$. Since

$$
\left\|x_{n}-x_{m}\right\| \leq \max \left\{\left\|x_{j+1}-x_{j}\right\| \mid m \leq j \leq n-1\right\} \quad(n>m)
$$

a sequence $\left\{x_{n}\right\}$ is Cauchy in $(X,\|\cdot\|)$ if and only if $\left\{x_{n+1}-x_{n}\right\}$ converges to zero in $(X,\|\cdot\|)$. By a complete non-Archimedean space we mean one in which every Cauchy sequence is convergent.

THEOREM 1.2. [4] Let $(X, d)$ be a complete generalized metric space and let $J: X \longrightarrow X$ be a strictly contractive mapping with some Lipschitz constant $L$ with $0<L<1$. Then for each given element $x \in X$, either $d\left(J^{n} x, J^{n+1} x\right)=\infty$ for all nonnegative integers $n$ or there exists a positive integer $n_{0}$ such that
(1) $d\left(J^{n} x, J^{n+1} x\right)<\infty$ for all $n \geq n_{0}$;
(2) the sequence $\left\{J^{n} x\right\}$ converges to a fixed point $x^{*}$ of $J$;
(3) $x^{*}$ is the unique fixed point of $J$ in the set $Y=\left\{y \in X \mid d\left(J^{n_{0}} x, y\right)<\right.$ $\infty\}$ and
(4) $d\left(y, y^{*}\right) \leq \frac{1}{1-L} d(y, J y)$ for all $y \in Y$.

Throughout this paper, we assume that $X$ is a non-Archimedean normed space and $Y$ is a complete non-Archimedean normed space.

## 2. Solutions of (1.3)

In this section, we investigate solutions of (1.3). We start with the following lemma.

Lemma 2.1. Let $f: X \longrightarrow Y$ be mapping. Then $f$ satisfies (1.3) if and only if $f$ is a quadratic mapping with an involution.

Proof. Suppose that $f$ satisfies (1.3). Letting $x=y=0$ in (1.3), we have $f(0)=0$. Letting $x=x+\sigma(x), y=x+\sigma(x)$ in (1.3), we have

$$
2(k+1) f(2(x+\sigma(x)))=4(k+1) f(x+\sigma(x))
$$

for all $x \in X$ and since $k \neq-1$,

$$
\begin{equation*}
f(2(x+\sigma(x)))=2 f(x+\sigma(x)) \tag{2.1}
\end{equation*}
$$

for all $x \in X$. Letting $x=x+\sigma(y), y=\sigma(x)+y$ in (1.3), we get

$$
\begin{align*}
& (k+1) f(x+y+\sigma(x+y))+f(2(x+\sigma(y)))+k f(2(\sigma(x)+y))  \tag{2.2}\\
& =2(k+1) f(x+\sigma(y))+2(k+1) f(\sigma(x)+y))
\end{align*}
$$

for all $x, y \in X$. Letting $x=\sigma(x)+y, y=x+\sigma(y)$ in (1.3), we get

$$
\begin{align*}
& (k+1) f(x+y+\sigma(x+y))+f(2(\sigma(x)+y))+k f(2(x+\sigma(y)))  \tag{2.3}\\
& =2(k+1) f(\sigma(x)+y)+2(k+1) f(x+\sigma(y)))
\end{align*}
$$

for all $x, y \in X$. From (2.2) and (2.3),

$$
f(2(x+\sigma(y)))=f(2(\sigma(x)+y))
$$

for all $x, y \in X$. By (2.1),

$$
\begin{equation*}
f(x+\sigma(y))=f(\sigma(x)+y) \tag{2.4}
\end{equation*}
$$

for all $x, y \in X$. Substituting (2.4) by (1.3), we get

$$
f(x+y)+f(x+\sigma(y))=2 f(x)+2 f(y)
$$

Therefore $f$ be quadratic with an involution.
Assume that $f(x+y)+f(x+\sigma(y))=2 f(x)+2 f(y)$. Letting $x=y=0$ in (1.3), we have $f(0)=0$. Letting $x=0$ in (1.3), we have

$$
\begin{equation*}
f(y)=f(\sigma(y)) \tag{2.5}
\end{equation*}
$$

for all $y \in X$. By (2.5)

$$
\begin{equation*}
f(x+y)+f(\sigma(x)+y)=2 f(x)+2 f(y) \tag{2.6}
\end{equation*}
$$

for all $x, y \in X$. From (1.3) and (2.6), $f$ satisfies (1.3).
Remark 2.2. The mapping $f: X \longrightarrow Y$ satisfying (1.3) for the case $k=-1$ is not quadratic. In fact, for $a \in Y$ with $a \neq 0$, the constant mapping $f(x)=a$ satisfies (1.3) but it is not quadratic.

## 3. The generalized Hyers-Ulam stability for (1.3)

Using the fixed point methods, we will prove the generalized HyersUlam stability of the quadratic functional equation (1.3) with an involution $\sigma$ in non-Archimedean normed space.

Theorem 3.1. Assume that $\phi: X^{2} \longrightarrow[0, \infty)$ is a mapping and there exists a real number $L$ with $0<L<1$ such that

$$
\begin{equation*}
\phi(2 x, 2 y) \leq|4| L \phi(x, y), \phi(x+\sigma(x), y+\sigma(y)) \leq|4| L \phi(x, y) \tag{3.1}
\end{equation*}
$$

for all $x, y \in X$. Let $f: X \longrightarrow Y$ be a mapping such that $f(0)=0$ and

$$
\begin{align*}
& \|(k+1) f(x+y)+f(x+\sigma(y)) \\
& \quad+k f(\sigma(x)+y)-2(k+1) f(x)-2(k+1) f(y) \| \leq \phi(x, y) \tag{3.2}
\end{align*}
$$

for all $x, y \in X$ and a fixed real number $k$ with $k \neq-1$. Then there exists a unique quadratic mapping $Q: X \longrightarrow Y$ with an involution such that

$$
\begin{equation*}
\|f(x)-Q(x)\| \leq \frac{1}{|4(k+1)|(1-L)} \phi(x, x) \tag{3.3}
\end{equation*}
$$

for all $x \in X$.
Proof. Consider the set $S=\{g \mid g: X \longrightarrow Y\}$ and the generalized metric $d$ in $S$ defined by $d(g, h)=\inf \{c \in[0, \infty) \mid\|g(x)-h(x)\| \leq$ $c \phi(x, x)$ for all $x \in X\}$. Then $(S, d)$ is a complete metric space(See [9]). Define a mapping $J: S \longrightarrow S$ by $J g(x)=\frac{1}{4}\{g(2 x)+g(x+\sigma(x))\}$ for all $x \in X$ and all $g \in S$. Let $g, h \in S$ and $d(g, h) \leq c$ for some non-negative real number $c$. Then by (3.1), we have

$$
\begin{aligned}
& \|J g(x)-J h(x)\| \\
& =\frac{1}{|4|}\|g(2 x)+g(x+\sigma(x))-h(2 x)-h(x+\sigma(x))\| \\
& \leq \frac{1}{|4|} \max \{\|g(2 x)-h(2 x)\|,\|g(x+\sigma(x))-h(x+\sigma(x))\|\} \\
& \leq c L \phi(x, x)
\end{aligned}
$$

for all $x \in X$. Hence we have $d(J g, J h) \leq L d(g, h)$ for any $g, h \in S$ and so $J$ is a strictly contractive mapping.

Next, we claim that $d(J f, f)<\infty$. Putting $y=x$ in (3.2) and dividing both sides by $|4(k+1)|$, we get
$\left\|\frac{1}{4}\{f(2 x)+f(x+\sigma(x))\}-f(x)\right\|=\|J f(x)-f(x)\| \leq \frac{1}{|4(k+1)|} \phi(x, x)$
for all $x \in X$ and hence

$$
\begin{equation*}
d(J f, f) \leq \frac{1}{|4(k+1)|}<\infty \tag{3.4}
\end{equation*}
$$

By Theorem 1.2, there exists a mapping $Q: X \longrightarrow Y$ which is a fixed point of $J$ such that $d\left(J^{n} f, Q\right) \rightarrow 0$ as $n \rightarrow \infty$. By induction, we can easily show that

$$
\left(J^{n} f\right)(x)=\frac{1}{2^{2 n}}\left\{f\left(2^{n} x\right)+\left(2^{n}-1\right) f\left(2^{n-1}(x+\sigma(x))\right)\right\}
$$

for all $x \in X$ and $n \in \mathbb{N}$. Since $d\left(J^{n} f, Q\right) \rightarrow 0$ as $n \rightarrow \infty$, there exists a sequence $\left\{c_{n}\right\}$ in $\mathbb{R}$ such that $c_{n} \rightarrow 0$ as $n \rightarrow \infty$ and $d\left(J^{n} f, Q\right) \leq c_{n}$ for every $n \in \mathbb{N}$. Hence, it follows from the definition of $d$ that

$$
\left\|\left(J^{n} f\right)(x)-Q(x)\right\| \leq c_{n} \phi(x, x)
$$

for all $x \in X$. Thus for each fixed $x \in X$, we have

$$
\lim _{n \rightarrow \infty}\left\|\left(J^{n} f\right)(x)-Q(x)\right\|=0
$$

and

$$
\begin{equation*}
Q(x)=\lim _{n \rightarrow \infty} \frac{1}{2^{2 n}}\left\{f\left(2^{n} x\right)+\left(2^{n}-1\right) f\left(2^{n-1}(x+\sigma(x))\right)\right\} . \tag{3.5}
\end{equation*}
$$

It follows from (3.2) and (3.5) that

$$
\begin{aligned}
& \|(k+1) Q(x+y)+Q(x+\sigma(y))+k Q(\sigma(x)+y)-2(k+1) Q(x)-2(k+1) Q(y)\| \\
& \leq \lim _{n \rightarrow \infty} \frac{1}{|4|^{n}} \max \left\{\phi\left(2^{n} x, 2^{n} y\right),\left|2^{n}-1\right| \phi\left(2^{n-1}(x+\sigma(x)), 2^{n-1}(y+\sigma(y))\right)\right\} \\
& \leq \lim _{n \rightarrow \infty} \frac{1}{|4|^{n}} \max \left\{|4|^{n} L^{n} \phi(x, y),\left|2^{n}-1\right||4|^{n} L^{n} \phi(x, y)\right\}=0
\end{aligned}
$$

for all $x, y \in X$, because $|4| L<1$ and $\left|2^{n}-1\right|<1$. Hence $Q$ satisfies (1.3) and by Lemma 2.1, $Q$ is a quadratic mapping with an involution. By (4) in Theorem 1.2 and (3.4), $f$ satisfies (3.3).

Assume that $Q_{1}: X \longrightarrow Y$ is another solution of (1.3) satisfying (3.3). We know that $Q_{1}$ is a fixed point of $J$. Due to (3) in Theorem 1.2 , we get $Q=Q_{1}$. This proves the uniqueness of $Q$.

Theorem 3.2. Assume that $\phi: X^{2} \longrightarrow[0, \infty)$ is a mapping and there exists a real number $L$ with $0<L<1$ such that

$$
\begin{equation*}
\phi(x, y) \leq \frac{L}{|4|} \phi(2 x, 2 y), \phi(x+\sigma(x), y+\sigma(y)) \leq \frac{L}{|2|} \phi(4 x, 4 y) \tag{3.6}
\end{equation*}
$$

for all $x, y \in X$. Let $f: X \longrightarrow Y$ be a mapping satisfying (3.2) and $f(0)=0$. Then there exists a unique quadratic mapping $Q: X \longrightarrow Y$ with an involution such that

$$
\begin{equation*}
\|f(x)-Q(x)\| \leq \frac{L}{|4(k+1)|(1-L)} \phi(x, x) \tag{3.7}
\end{equation*}
$$

for all $x \in X$.
Proof. Consider the set $S=\{g \mid g: X \longrightarrow Y\}$ and the generalized metric $d$ in $S$ defined by $d(g, h)=\inf \{c \in[0, \infty) \mid\|g(x)-h(x)\| \leq$ $c \phi(x, x)$ for all $x \in X\}$. Then $(S, d)$ is a complete metric space. Define a mapping $J: S \longrightarrow S$ by

$$
J g(x)=4\left\{g\left(\frac{x}{2}\right)-\frac{1}{2} g\left(\frac{x+\sigma(x)}{4}\right)\right\}
$$

for all $x \in X$ and all $g \in S$. Let $g, h \in S$ and $d(g, h) \leq c$ for some non-negative real number $c$. Then by (3.13), we have

$$
\begin{aligned}
& \|J g(x)-\operatorname{Jh}(x)\| \\
& =|4|\left\|g\left(\frac{x}{2}\right)-\frac{1}{2} g\left(\frac{x+\sigma(x)}{4}\right)-h\left(\frac{x}{2}\right)+\frac{1}{2} h\left(\frac{x+\sigma(x)}{4}\right)\right\| \\
& \leq|4| \max \left\{\left\|g\left(\frac{x}{2}\right)-h\left(\frac{x}{2}\right)\right\|, \frac{1}{|2|}\left\|g\left(\frac{x+\sigma(x)}{4}\right)-h\left(\frac{x+\sigma(x)}{4}\right)\right\|\right\} \\
& \leq c L \phi(x, x)
\end{aligned}
$$

for all $x \in X$. Hence $d(J g, J h) \leq L d(g, h)$ for any $g, h \in S$ and so $J$ is a strictly contractive mapping.

Next, we claim that $d(J f, f)<\infty$. Putting $x=\frac{x}{2}$ and $y=\frac{x}{2}$ in (3.2) and dividing both sides by $|k+1|$, we get

$$
\begin{equation*}
\left\|f(x)+f\left(\frac{x+\sigma(x)}{2}\right)-4 f\left(\frac{x}{2}\right)\right\| \leq \frac{1}{|k+1|} \phi\left(\frac{x}{2}, \frac{x}{2}\right) \tag{3.8}
\end{equation*}
$$

and putting $x=\frac{x+\sigma(x)}{4}$ and $y=\frac{x+\sigma(x)}{4}$ in (3.2) and dividing both sides by $|2(k+1)|$, we get

$$
\begin{equation*}
\left\|f\left(\frac{x+\sigma(x)}{2}\right)-2 f\left(\frac{x+\sigma(x)}{4}\right)\right\| \leq \frac{1}{|2(k+1)|} \phi\left(\frac{x+\sigma(x)}{4}, \frac{x+\sigma(x)}{4}\right) \tag{3.9}
\end{equation*}
$$

for all $x \in X$. Combining (3.8) and (3.9), by (3.6), we deduce that
$\|J f(x)-f(x)\|=\left\|4 f\left(\frac{x}{2}\right)-2 f\left(\frac{x+\sigma(x)}{4}\right)-f(x)\right\| \leq \frac{L}{|4(k+1)|} \phi(x, x)$
for all $x \in X$ and hence

$$
d(J f, f) \leq \frac{L}{|4(k+1)|}<\infty .
$$

By Theorem 1.2, there exists a mapping $Q: X \longrightarrow Y$ which is a fixed point of $J$ such that $d\left(J^{n} f, Q\right) \rightarrow 0$ as $n \rightarrow \infty$. By induction, we can easily show that

$$
\left(J^{n} f\right)(x)=2^{2 n}\left\{f\left(\frac{x}{2^{n}}\right)+\left(\frac{1}{2^{n}}-1\right) f\left(\frac{x+\sigma(x)}{2^{n+1}}\right)\right\}
$$

for each $n \in \mathbb{N}$. Since $d\left(J^{n} f, Q\right) \rightarrow 0$ as $n \rightarrow \infty$, there exists a sequence $\left\{c_{n}\right\}$ in $\mathbb{R}$ such that $c_{n} \rightarrow 0$ as $n \rightarrow \infty$ and $d\left(J^{n} f, Q\right) \leq c_{n}$ for every $n \in \mathbb{N}$. Hence, it follows from the definition of $d$ that

$$
\left\|\left(J^{n} f\right)(x)-Q(x)\right\| \leq c_{n} \phi(x, x)
$$

for all $x \in X$. Thus for each fixed $x \in X$, we have

$$
\lim _{n \rightarrow \infty}\left\|\left(J^{n} f\right)(x)-Q(x)\right\|=0
$$

and

$$
Q(x)=2^{2 n}\left\{f\left(\frac{x}{2^{n}}\right)+\left(\frac{1}{2^{n}}-1\right) f\left(\frac{x+\sigma(x)}{2^{n+1}}\right)\right\} .
$$

Analogously to the proof of Theorem 3.1, we can show that Q is a unique quadratic mapping with an involution satisfying (3.7).

As examle of $\phi(x, y)$ in Theorem 3.1 and Theorem 3.2, we can take $\phi(x, y)=\theta\left(\|x\|^{p}+\|y\|^{p}\right)$ for all $x, y \in X$. Then we have the following corollary.

Corollary 3.3. Let $\theta \geq 0$ and $p$ be a positive real number with $p \neq 2$. Suppose that $|2|<1$. Let $f: X \longrightarrow Y$ be a mapping satisfying

$$
\begin{align*}
& \|(k+1) f(x+y)+f(x+\sigma(y))  \tag{3.10}\\
& \quad+k f(\sigma(x)+y)-2(k+1) f(x)-2(k+1) f(y) \| \leq \theta\left(\|x\|^{p}+\|y\|^{p}\right)
\end{align*}
$$

and $\|x+\sigma(x)\|^{p} \leq|2|^{p}\|x\|^{p}$ for all $x, y \in X$. Then there exists a unique mapping $Q: X \longrightarrow Y$ with an involution such that $Q$ is a solution of the functional equation (1.3) and the inequality

$$
\|f(x)-Q(x)\| \leq \begin{cases}\left.\frac{2}{|1+k|\left(|4|-|2|^{p}\right)} \theta| | x\right|^{p}, & \text { if } p>2, \\ \frac{2}{|1+k|\left(|2|^{p}-|4|\right)} \theta| | x| |^{p}, & \text { if } 0 \leq p<2\end{cases}
$$

holds for all $x \in X$.

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