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Modified MMSE Estimator based on Non-Linearly Spaced Pilots for OFDM Systems

Latif Ullah Khan

Department of Electrical Engineering, University of Engineering & Technology / Peshawar, Pakistan
latifullahkhan@nwfpuet.edu.pk

***** Corresponding Author: Latif Ullah Khan

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** Short Paper*

Abstract**:** This paper proposes a Modified Minimum Mean Square Error (M-MMSE) estimator for an Orthogonal Frequency Division Multiplexing (OFDM) System over fast fading Rayleigh channel. The proposed M-MMSE estimator considered the effects of the efficient placement of pilots based on the channel energy distribution. The pilot symbols were placed in a non-linear manner according to the density of the channel energy. Comparative analysis of the MMSE estimator for a comb-type pilot arrangement and M-MMSE estimator for the proposed pilot insertion scheme revealed significant performance improvement of the M-MMSE estimator over the MMSE estimator.

Keywords**:** OFDM, Modified MMSE estimator, Comb-type channel estimation

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a spectrally efficient multicarrier digital communication technique adopted by newly developed wireless communication standards. 802.11-based wireless Local Area Networks (WLANs), power line communication and Long Term Evolution (LTE) use OFDM. Efficient use of the spectrum is due to the overlapping nature of the orthogonal subcarriers in the frequency domain. Channel estimation and equalization in OFDM is used to cancel out the impairments introduced by the multipath fading channel. Pilot arrangement in the OFDM symbol determines the performance of the channel estimation technique used to estimate the channel impulse response at the pilot tones. The Least Square (LS) estimator and Minimum Mean Square Error (MMSE) estimator can be used to estimate the channel impulse response at the pilot tones. The MMSE estimator outperforms the LS estimator at the cost of the high computational complexity [1]. A zero forcing (ZF) estimator was used to estimate the channel frequency response at the pilot tones to cancel out the Inter Carrier Interference (ICI) due to the Doppler shifts induced by the fading channel [2]. The performance of the ZF estimator is degraded compared to the MMSE estimation but the MMSE estimator achieves improved performance at the cost of computational complexity associated with matrix inversion and the use of the prior channel statistics.

Non-overlapping and overlapping techniques were used to partition the channel auto correlation matrix into the sub matrices to reduce the complexity associated with the linear MMSE estimator [3]. The proposed simplified MMSE estimator outperformed the LS estimator but showed degraded performance compared to the MMSE estimator. The MMSE estimator was realized using the DFT-based estimator [4]. An estimation of the noise variance and channel auto-correlation matrix was performed using the channel impulse response estimated in the DFT-based channel estimation. A performance comparison of the proposed MMSE estimator with the LS and MMSE estimator showed improved performance over the LS estimator but degraded performance compared to the MMSE estimator. On the other hand, the computational complexity associated with the proposed MMSE estimator was less than the MMSE estimator because it does not require a power delay profile for its operation. Previous studies discussed the design of 2-D pilot patterns for the frequency selective fading channels [5-7]. The disadvantage of 2-D pilot patterns is their high complexity compared to 1-D pilot patterns, which makes them unattractive for practical implementation [8].

This paper proposes a novel Non-Linearly Spaced Pilot Insertion (NSPI) scheme based on the efficient placement of the pilots according to channel energy is proposed for the OFDM system over the fast fading Rayleigh channel. The proposed NSPI scheme uses the channel energy distribution to insert the pilots in OFDM symbol in a non-linear manner. Modifications in the MMSE estimator for the proposed NSPI scheme are also discussed. Comparative analysis of the Modified MMSE (M-MMSE) estimator for the NSPI scheme with a MMSE estimator for a comb-type pilot arrangement revealed significant performance improvement.

The remainder of the paper is organized as follows. Section II introduces the OFDM system model. A MMSE estimation for a comb-type pilot arrangement is discussed in section III. Section IV describes the proposed channel estimation algorithm. The simulation results are presented in section V and the paper is concluded in section VI.

Notations: The matrices are denoted by bold face italic upper case letters and the vectors are denoted by lower case italic letters. Superscripts ^{H} and ^{T} are used to denote the Hertmitian transpose and Transpose respectively.

2. OFDM System Model

In the OFDM system shown in Fig. 1, the input bit sequence is first mapped using the digital modulation scheme (such as BSPK) to yield the frequency domain sequence of the symbols. The frequency domain sequence of the symbols is divided into blocks with a size equal to the data subcarriers per OFDM symbol for the comb-type channel estimation and the proposed NSPI scheme. The pilot symbols are inserted into each group of symbols at the dedicated subcarriers after converting the serialized sequence into the parallelized sequence for the comb-type and proposed NSPI scheme. The i^{th} group of the symbols is $\boldsymbol{x}^{(i)} = [x^{(i)}(0), x^{(i)}(1), \dots, \ x^{(i)}(N-1)]^T$. The conversion of the frequency domain sequence of the symbols into the time domain sequence of symbols is performed using the Inverse Discrete Fourier Transform (IDFT). Let $[\boldsymbol{F}]n,n = e^{j2\pi(n-1)(n-1)}/_N$ for $n = 0, 1, 2, \dots, N-1$. The i^{th} group of symbols in time domain can be expressed as

$$\boldsymbol{d}^{(i)} = \boldsymbol{F}\boldsymbol{x}^{(i)} \tag{1}$$

The cyclic prefix of length, L_{CP}, is added to the OFDM symbol to cancel the ISI. The cyclic prefix added to thei^{th} group of the OFDM symbol is $\boldsymbol{d}_{CP}{}^{(i)}[d^{(i)}(N-L_{CP}), d^{(i)}(N-L_{CP}+1), \dots, d^{(i)}(N-2L_{CP}+1), d^{(i)}(0), d^{(i)}(1), \dots, d^{(i)}(N-1)]^T$ .. Finally, the cyclic prefix added i^{th} group of the OFDM symbol is passed through the fading channel in the presence of AWGN.

$$\boldsymbol{y}_{CP}{}^{(i)} = \tilde{\boldsymbol{d}}_{CP}{}^{(i)} \otimes \boldsymbol{h}^{(i)} + \boldsymbol{w}^{(i)} \tag{2}$$

where $\boldsymbol{w}^{(i)}$ and $\boldsymbol{h}^{(j)}$ are the AWGN noise vector and the time domain channel impulse response vector, respectively. At the receiver side, the cyclic prefix is removed from the received signal to cancel out the ISI. The cyclic prefix is removed by dropping the first L_{CP} symbols from the $\boldsymbol{y}_{CP}{}^{(i)}$ to yield $\boldsymbol{y}^{(i)}$. The signal was then converted to the frequency domain using DFT.

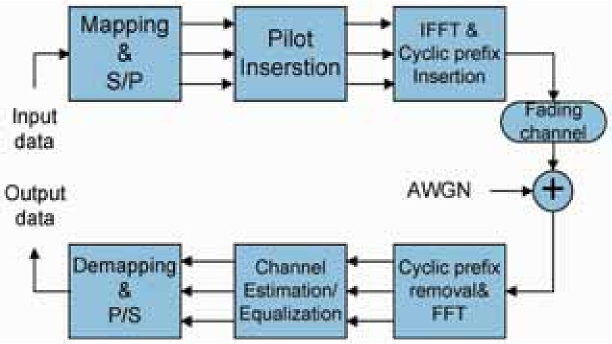


Fig. 1. OFDM System Model.

$$\boldsymbol{r}^{(i)} = \boldsymbol{F}^H \boldsymbol{y}^{(i)} \tag{3}$$

A channel estimation is carried out after converting the signal from the time domain to the frequency domain. Finally, after equalization, the signal is demapped to yield the output bits.

3. MMSE Estimator Based on the Comb-Type Pilot Arrangement

A comb-type channel estimation was used for the fast fading channels, where the variations in the channel impulse response are rapid. Fig. 2 shows the arrangement of pilots in comb-type and block-type channel estimation. Let $\boldsymbol{r}_P^{(i)}$ represent the received signal vector for the pilot symbols in the frequency domain for the i^{th} OFDM symbol. In the frequency domain, the MMSE estimates of the time domain channel vector $\boldsymbol{h}_P$ of the Gaussian distribution and non-correlation with noise, is given [9]:

$$\boldsymbol{h}_{P,MMSE} = \boldsymbol{F}_N{}^H \boldsymbol{R}_{HY} \boldsymbol{R}_{YY}{}^{-1} \boldsymbol{r}_P \tag{4}$$

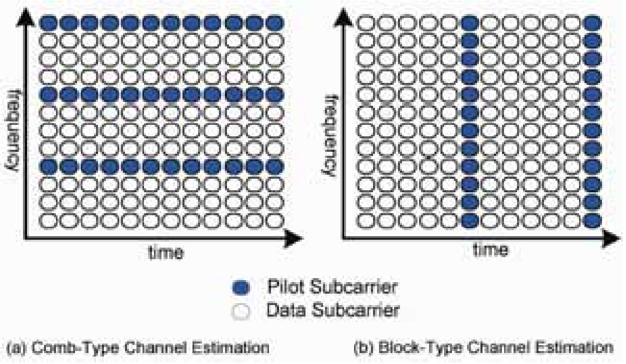
where $\boldsymbol{F}_N$ is the PXP matrix $[\boldsymbol{F}_N]p,p = e^{j2\pi(p-1)(p-1)/P}$

$\boldsymbol{R}_{HY} = E\{\boldsymbol{h}_P \boldsymbol{r}_P\} = \boldsymbol{R}_{HH} \boldsymbol{F}_N \boldsymbol{x}_P{}^H$ is the cross covariance matrix of the received signal vector $\boldsymbol{r}_P$ and the time domain channel vector $\boldsymbol{h}_P$ for pilot symbols.

$\boldsymbol{R}_{YY} = E\{\boldsymbol{r}_P \boldsymbol{r}_P{}^H\} = \boldsymbol{x}_P \boldsymbol{F}_N{}^H \boldsymbol{R}_{HH} \boldsymbol{F}_N \boldsymbol{x}_P{}^H + \sigma^2 \boldsymbol{i}_P$ is the auto-covariance matrix of the received signal vector $\boldsymbol{r}_P$.

$\boldsymbol{R}_{HH}$ is the auto-covariance matrix of the time domain channel vector $\boldsymbol{h}_P$ and σ^2 is the variance of noise $\boldsymbol{w}_P$. $\boldsymbol{R}_{HH}$ and σ^2 are quantities that are known.

Eq. (4) can be re-written as



(a) Comb-Type Channel Estimation (b) Block-Type Channel Estimation

Fig. 2. Pilot Arrangement in the Comb-Type and Block-type channel estimation.

$$\boldsymbol{h}_{P,MMSE} = \boldsymbol{F}_N{}^H \boldsymbol{M}_{MMSE} \boldsymbol{F}_N \boldsymbol{x}_P{}^H \boldsymbol{r}_P \tag{5}$$

where

$$\begin{aligned}\boldsymbol{M}_{MMSE} = \boldsymbol{R}_{HH}[(\ \boldsymbol{F}_N \boldsymbol{x}_P{}^H \boldsymbol{x}_P \boldsymbol{F}_N{}^H)^{-1}\sigma^2 \\ + \boldsymbol{R}_{HH}]^{-1}(\ \boldsymbol{F}_N \boldsymbol{x}_P{}^H \boldsymbol{x}_P \boldsymbol{F}_N{}^H)^{-1}\end{aligned}$$

The estimated channel frequency response vector $\boldsymbol{h}_{CE}$ for all subcarriers was obtained using the interpolation technique. In this study, linear, low pass and spline cubic one dimensional interpolation techniques were used. The reason for considering the one-dimensional interpolation techniques is their low complexity compared to the two dimensional interpolation techniques [8].

4. Modified MMSE Estimator based on Proposed NSPI Scheme

This section describes the modifications in the MMSE estimator to work for the proposed NSPI scheme. The channel impulse response was distributed in the frequency domain and concentrated in the time domain [4]. No energy leakage was observed in case of the sample-spaced channels and all the impulses were located at integer multiples of the system sampling rate [10]. Figs. 3 and 4 present the channel energy distribution in the frequency domain and time domain, respectively, for the channel order $L = 8$ and $L = 16$. Figs. 3 and 4 show that the energy distribution of the channel is non-uniform in the frequency domain. The energy lies mainly in the beginning and end subcarriers. The significant subcarriers (shown by highlighting in Figs. 3 and 4) have more energy than the less significant subcarriers (subcarriers in the middle). Figs. 3 and 4 show that the locations of the significant subcarriers are different for the channels with different orders. This non-uniform distribution of channel energy motivates the insertion of pilots in a non-linear manner. The proposed NSPI considers the non-uniform distribution of the channel energy for the non-linear placement of the pilot symbols in the frequency domain. Let $\boldsymbol{i}_d = [i_d(1), i_d(2), i_d(3), \ldots, i_d(N_d - 1)]^T$ and $\boldsymbol{i}_p = [i_P(1), i_P(2), i_P(3), \ldots, i_P(P - 1)]^T$ be vectors denoting the indices of the data and pilot subcarriers, respectively. The estimated channel frequency can be expressed as

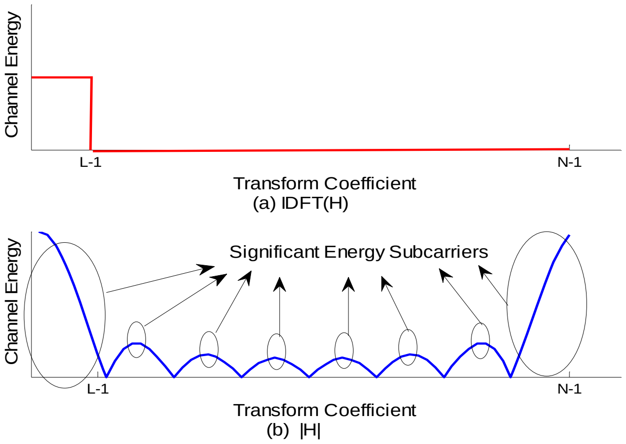


Fig. 3. Channel Energy distribution for the sample spaced channel with L=8 and N=64.

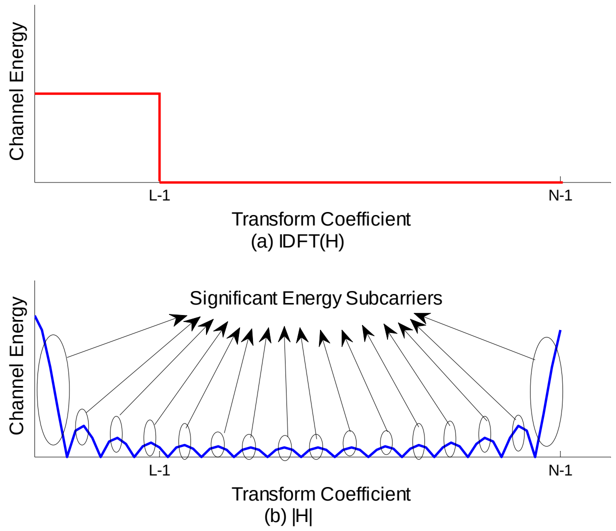


Fig. 4. Channel Energy distribution for the sample spaced channel with L=16 and N=64.

$$\boldsymbol{h}_{M-MMSE} = \boldsymbol{F}_P{}^H \boldsymbol{h}_{M-P} \tag{6}$$

where $[\boldsymbol{F}_P]l, p = e^{j2\pi(l-1)(p-1)/N}$ and $\boldsymbol{h}_{M-P}$ is the estimated channel impulse response using the M-MMSE estimator in the time domain at pilot sub carriers for the proposed NSPI scheme, respectively. Define a matrix $\boldsymbol{Q}_P$ containing the columns of $\boldsymbol{F}_P$ corresponding to the pilot locations. The M-MMSE estimate of the channel in the time domain is given by

$$\boldsymbol{h}_{M-P} = \boldsymbol{R}^H_{M-yh} \boldsymbol{R}^{-1}_{M-yy} \boldsymbol{r}_P \tag{7}$$

where

$$\boldsymbol{R}_{M-hh} = E\{\boldsymbol{h}_{M-P} \boldsymbol{h}_{M-P}{}^H\} = diag\ \{\sigma_h^2(0), \sigma_h^2(1), \ldots, \sigma_h^2(N-1)\}$$

is the auto-covariance matrix of the time domain channel vector $\boldsymbol{h}_{M-P}$ and σ^2 is the variance of noise $\boldsymbol{w}_P$.

$\boldsymbol{R}_{M-yy} = E\{\boldsymbol{r}_P \boldsymbol{r}_P^H\} = \boldsymbol{x}_P \boldsymbol{Q}_P^H \boldsymbol{R}_{M-hh} \boldsymbol{Q}_P \boldsymbol{x}_P^H + \boldsymbol{N}_0 \boldsymbol{i}_P$ is the auto-covariance matrix of the received signal vector $\boldsymbol{r}_P$ for the proposed NSPI scheme.

$\boldsymbol{R}_{M-yh} = E\{\boldsymbol{r}_P \boldsymbol{h}_{M-P}{}^H\} = \boldsymbol{x}_P \boldsymbol{Q}_P^H \boldsymbol{R}_{M-hh}$ is the cross covariance matrix of the received signal vector $\boldsymbol{r}_P$ and the time domain channel vector $\boldsymbol{h}_{M-P}$ for the proposed NSPI scheme.Putting the value of $\boldsymbol{h}_{M-P}$ in Eq. (6) yielded the channel frequency response at all subcarriers.

5. Simulation Results

This section presents the results of the MATLAB® simulation to evaluate the performance of the OFDM system for the M-MMSE and MMSE estimator. Table 1 lists the values of the parameters used for the performance evaluation. The channel used for the performance evaluation consisted of L independent taps of the Gaussian distribution and zero mean. Constant and exponential power delay profiles were used for the performance evaluation. The variance of each tap in the case of the exponential power delay profile can be expressed as

$$\sigma_l{}^2 = e^{-\frac{l}{45}} \quad l = 0,1,2 \ldots L-1 \tag{8}$$

The vector containing the indices of the pilots for channel order 8 is expressed as

$$\boldsymbol{i}_P = [0, 1, 2, 3, 6, 10, 19, 28, 37, 45, 53, 58, 61, 62]^T \tag{9}$$

The legends, ‘Low Pass’, ‘Linear’, ‘spline’, ‘Proposed’, ‘C’ and ‘E’, refer to the low pass interpolation, linear interpolation, spline interpolation, proposed channel estimation algorithm, constant power delay profile, and exponential power delay profile, respectively. Fig. 5 shows the performance of the OFDM system for the M-MMSE and MMSE estimator. The performance of M-MMSE for the proposed NSPI scheme was improved significantly compared to the MMSE estimator for comb-type pilot arrangement using low pass, spline and linear interpolation techniques. The performance of the proposed channel estimation algorithm had the same form for both types of power delay profiles but the performance of the MMSE estimator with different one-dimensional interpolation techniques was slightly better for the exponential power delay profile compared to the constant power delay profile because the higher channel taps are less significant in the case of the exponential power delay profile.

Table 1. Simulation Parameters.

Parameters	Values
Number of Subcarriers	64
Channel Estimation	MMSE and M-MMSE
Channel	Fast Fading Rayleigh
Modulation scheme	BPSK
Channel Order, L	8

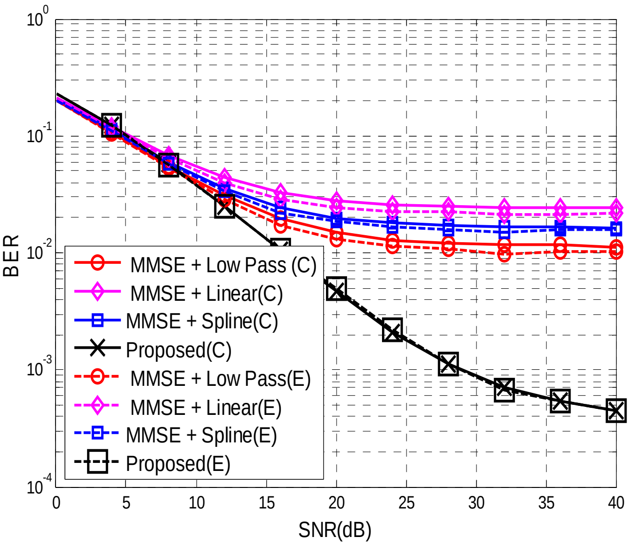


Fig. 5. BER Performance of the OFDM system for the BPSK modulation scheme.

6. Conclusions

A novel NSPI scheme was proposed for OFDM systems over fast fading Rayleigh channels. Modifications were also proposed in the MMSE estimator to work for the proposed NSPI scheme. The proposed channel estimation algorithm significantly improved the performance of the OFDM system significantly compared to the MMSE estimator for the comb-type pilot arrangement. Future studies should examine whether the proposed channel estimation algorithm can be made dynamic in that the pilot positions can be changed with the variations in the channel.

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Latif Ullah Khan is currently working as a Lecturer at Electrical Engineering department, University of Engineering & technology Peshawar, Pakistan. He received his B.Sc degree in Electrical Engineering from the University of Engineering &Technology Peshawar and is currently pursuing his M.Sc degree in Electrical Engineering from the same University. He has more than 20 Publications in reputable Engineering Conferences and Journals. He is a member of IEEE USA and had the honor to win the best paper award in the 15th IEEE International Conference on Advanced Communication Technology, (ICACT-2013) in South Korea. His research interests include channel estimation in OFDM, channel coding, intelligent system design and routing protocols in Mobile Ad-hoc Networks (MANETs).