

A Philosophical Implication of Rough Set Theory^{*}

Chang Kyun Park

【Abstract】 Human being has attempted to solve the problem of imperfect knowledge for a long time. In 1982 Pawlak proposed the rough set theory to manipulate the problem in the area of artificial intelligence. The rough set theory has two interesting properties: one is that a rough set is considered as distinct sets according to distinct knowledge bases, and the other is that distinct rough sets are considered as one same set in a certain knowledge base. This leads to a significant philosophical interpretation: a concept (or an event) may be understood as different ones from different perspectives, while different concepts (or events) may be understood as a same one in a certain perspective. This paper claims that such properties of rough set theory produce a mathematical model to support critical realism and theory ladenness of observation in the philosophy of science.

【Key Words】 rough set theory, knowledge bases, critical realism, theory ladenness of observation

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I. Overview

Gödel's first incompleteness theorem says that every sufficiently strong axiomatic theory is either incomplete or inconsistent. The Gödel's theorem is usually interpreted that there are true statements that cannot be proven. Furthermore, the interpretation is supposed to show that the world of truth is bigger than the world of proof, though it is apprehended that the theorem might be abused.

What is proved mathematically has strength which does not allow a divergent view because there is no room for a refutation. However, rigorousness of mathematical proof can have a weak point because most of the statements which are not mathematical cannot be mathematically formalized.

This paper tries to show mathematically that perception or understanding can be different according to the perspective. It doesn't aim to prove it but to show it through a mathematical model using rough set theory (RST). The fact that perception can be different according to fore-understanding is considered to be too natural to be proved, but in fact it is rather more difficult to be proved because of its "naturalness." This paper aims to present a mathematical model to settle the unclearness, claiming that RST philosophically implies that perception of objects depends on perspectives and the perspectives may be fallible. It introduces basic concepts of rough set theory, and discusses philosophical meanings of RST.

II. Rough Set Theory¹⁾

The RST is a mathematical approach to vagueness. A rough set is a set having objects which cannot be clearly classified. RST was suggested by Pawlak 1982 and has been applied in computer science, AI, medicine, engineering, and financial analysis.

Suppose that the universe $U \neq \emptyset$ is finite set of objects. Any subset of U is called a concept or a category in U . Let $C = \{X_1, X_2, \dots, X_n\}$, $X_i \subseteq U$, $X_i \neq \emptyset \forall i \in \{1, 2, \dots, n\}$ be a partition of a universe U . Then $X_i \cap X_j = \emptyset$ for $i \neq j$, $i, j = 1, \dots, n$ and $\bigcup_{i=1}^n X_i = U$. By a knowledge base we mean a relational system $K = (U, R)$ where U is a universe and R is a family of equivalence relations over U . Suppose that P is a non-empty subset of R . Then intersection of all equivalence relations belonging to P is also an equivalence relation which is called an “indiscernibility relation” over P and denoted by $\text{IND}(P)$. So $U/\text{IND}(P)$ (simply U/P) denotes knowledge associated with the family of equivalence relations P , called P -basic knowledge about U in K . And the family of all equivalence relations defined in a knowledge base can be denoted by $\text{IND}(K) = \{\text{IND}(P) : P \neq \emptyset, P \subseteq R\}$.

Let $X \subseteq U$ and R be an equivalence relation. We have the following definitions. X is R -definable if X is the union of R -basic categories. Otherwise, X is R -undefinable. If X is

¹⁾ This chapter is a summary of the rough set theory from Zdzisław Pawlak, pp. 1-29.

R -definable, then it is an exact set. If X is R -undefinable, then we say that it is a R -rough.

Now we come to introduce the fundamental concepts of RST. In case of rough sets we need to approximate them by some exact sets which we called the upper and the lower approximations. Suppose $K=(U, R)$ is a knowledge base, $X \subseteq U$ and $R \in \text{IND}(K)$ is an equivalence relation. Then with each subset $X \subseteq U$ we can associate two subsets:

$$\underline{RX} = \cup \{Y \in U / R : Y \subseteq X\}, \quad \overline{RX} = \cup \{Y \in U / R : Y \cap X \neq \emptyset\}$$

\underline{RX} and \overline{RX} are called the lower and the upper approximation of X respectively. So \underline{RX} is the set of all elements of U which certainly belong to X , and \overline{RX} is the set of elements of U which possibly belong to X . Obviously $\underline{RX} \subseteq X \subseteq \overline{RX}$. Let $BN(X) = \overline{RX} - \underline{RX}$. Then $BN(X)$ is called the boundary of X . \underline{RX} is also called the positive region of X . $U - \overline{RX}$ is called the negative region of X . $BN(X)$ is called borderline region of X .

We can introduce an interesting classification of rough sets employing the notion of lower and upper approximations.

- ① If $\underline{RX} \neq \emptyset$ and $\overline{RX} \neq U$, then X is roughly definable.
- ② If $\underline{RX} = \emptyset$ and $\overline{RX} \neq U$, then X is internally undefinable.
- ③ If $\underline{RX} \neq \emptyset$ and $\overline{RX} = U$, then X is externally undefinable.
- ④ If $\underline{RX} = \emptyset$ and $\overline{RX} = U$, then X is totally undefinable.

Now let's introduce the concept of equality of rough sets. In

the conventional set theory, two sets are equal if they have exactly the same elements. But in RST two sets are equal if their lower and upper approximations are same.

Consider two following examples to check the philosophical meaning of RST.

Example 1. (modifying Pawlak, p. 17)

$$\begin{aligned} K &= (U, R), \quad U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \\ E, F &\subseteq R, \quad E = \{E_1, E_2, E_3, E_4\}, \quad E_1 = \{x_1, x_4, x_8\}, \\ E_2 &= \{x_2, x_5, x_7\}, \quad E_3 = \{x_3\}, \quad E_4 = \{x_6\} \\ F &= \{F_1, F_2, F_3\}, \quad F_1 = \{x_1, x_4, x_8\}, \quad F_2 = \{x_2, x_5, x_7\} \quad \text{and} \\ F_3 &= \{x_3, x_6\} \end{aligned}$$

Consider $X = \{x_3, x_5\}$.

Then $\underline{R}X = E_3 = \{x_3\}$, $\overline{R}X = E_2 \cup E_3 = \{x_2, x_3, x_5, x_7\}$,

$BN(X) = E_2 = \{x_2, x_5, x_7\}$

while $\underline{R}X = \emptyset$, $\overline{R}X = F_2 \cup F_3 = \{x_2, x_3, x_5, x_6, x_7\}$,

$BN(X) = F_2 \cup F_3 = \{x_2, x_3, x_5, x_6, x_7\}$.

The set X is considered as distinct sets according to the distinct knowledge bases E and F .

Example 2. (Pawlak, p. 25)

Let $K = (U, R)$, $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$.

$R \in R$, $R = \{E_1, E_2, E_3, E_4\}$: $E_1 = \{x_2, x_3\}$, $E_2 = \{x_1, x_4, x_5\}$,

$E_3 = \{x_6\}$, $E_4 = \{x_7, x_8\}$

Consider $X_1 = \{x_1, x_2, x_6\}$ and $X_2 = \{x_3, x_4, x_6\}$.

Then $\underline{R}X_1 = E_3 = \{x_6\}$, $\overline{R}X_1 = E_1 \cup E_2 \cup E_3 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$,

and $\underline{R}X_2 = E_3 = \{x_6\}$, $\overline{R}X_2 = E_1 \cup E_2 \cup E_3 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

Though two sets X_1 and X_2 are different in this example, they are equal in the sense of RST.

III. Philosophical Implication of RST

Basic properties of RST examined above can be summarized as follows:

Whether an element belongs to a set is not decided by the object property of the element but by the basic knowledge about it. Equality of sets is not determined by absolute sense, either, but by a basic knowledge. In other words, all properties of rough sets are not absolute but related to what we know about them. In this sense, it can be said that RST is premised on relativism.

Contemporary philosophy of science tends to deny superiority of facts. Kuhn's "paradigm," Hanson's "theory ladenness of observation," or Polanyi's "fiduciary framework" does, too. They seem to argue that facts or reality can be effected as objects of perception only through the media of languages, conceptual frame, frame of interpretation, and perspectives. It means that a scientific knowledge can't be formed without a frame of belief. Gadamer asserts that preconception or tradition does not interrupt understanding but enables it.

(Wo)man interprets and understands the world in which s/he is

located by her or his own way. Understanding is a universal way of existence of (wo)man, and in this sense it is universal. However, Gadamer argues that the universality is always related to specific conditions, traditions, and fore-understandings.

In RST the knowledge base reflects the structure of our fore-understanding, tradition, and preconception. (Wo)man pursues new understanding and judgment on the knowledge base. Basic properties and the knowledge base of RST mentioned above are comprehended as fore-understanding or preconception. Three philosophical meanings can be conferred on RST as follows:

Firstly, RST affirms that objects are recognized differently according to fore-understandings or perspectives. The first example shows that objects are grasped differently according to the perspective E or F. Secondly, the fact that actually different sets are considered as same sets in the second example shows fallibility of perception. The second example might defend critical realism which admits existence of an objectively knowable reality while acknowledges limitation of perception and cognition. Lastly, It can be said that postmodern philosophy of the late twentieth century such as philosophical hermeneutics and new philosophy of science is extending the boundary region in the context of RST. Accordingly it leads to the result that the certain region, whether it is the positive region or the negative one, is significantly reduced. Postmodern philosophers of the late twentieth century seem to see truth and knowledge as an “internally undefinable” or “totally undefinable” rough set among four classifications of the rough set.

IV. Concluding Remarks

In RST membership of an element and equality of sets depend on preexisting knowledge. Once we admit preexisting knowledge as our perspectives or fore-understandings, it opens a way for philosophical analysis. It would be interesting to investigate what kinds of fore-understanding philosophers have had in major subjects such as being and truth, and to classify them according to four categories of RST.

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서경대학교 인문대학 철학전공

Seokyeong University

ckpark4g@gmail.com

러프집합론의 철학적 함의

박 창 균

불완전한 지식의 문제는 오랫동안 인간이 해결하려는 것이었다. 인공지능에서 불완전한 지식의 문제를 다루기 위해 파블락은 러프 집합론을 1982년에 제안하였다. 러프집합론은 다음과 같은 두 가지 흥미있는 성질을 가지고 있다. 먼저 하나의 러프집합은 지식기반에 따라 같은 집합이 아닌 다른 집합으로 간주된다는 것이다. 그리고 서로 다른 러프집합도 어떤 지식 기반에서 보면 서로 같은 집합으로 여겨진다는 것이다. 이러한 성질은 의미있는 철학적 해석을 낳는다. 즉 하나의 개념이나 사건은 다른 철학적 관점에서 다른 것으로 이해되기도 하고, 서로 다른 개념이나 사건도 어떤 관점에 따라서는 같은 것으로 간주될 수 있다는 것이다. 본고에서는 이러한 러프집합의 성질은 비판적 실재론이나 과학철학에서 관찰의 이론적재성을 지지하는 수학적 모델로 취급될 수 있다고 주장한다.

주요어: 러프집합론, 지식기반, 비판적 실재론, 관찰의 이론의존성