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Common Fixed Point and Example for Type(β) Compatible Mappings with Implicit Relation in an Intuitionistic Fuzzy Metric Space

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Abstract

In this paper, we establish common fixed point theorem for type(β) compatible four mappings with implicit relations defined on an intuitionistic fuzzy metric space. Also, we present the example of common fixed point satisfying the conditions of main theorem in an intuitionistic fuzzy metric space.

Keywords: Type(β) compatible map, Fixed point, Implicit relation

1. Introduction

Zadeh [1] introduced the concept of fuzzy sets in 1965 and in the next decade, Grabiec [2] obtained the Banach contraction principle in setting of fuzzy metric spaces, Also, Altun and Turkoglu [3] proved some fixed theorems using implicit relations in fuzzy metric spaces. Furthermore, Park et al. [4] defined the intuitionistic fuzzy metric space, and Park et al. [5] proved a fixed point theorem of Banach for the contractive mapping of a complete intuitionistic fuzzy metric space. Recently, Park [6, 7], Park et al. [8] obtained a unique common fixed point theorem for type(α) and type(β) compatible mappings defined on intuitionistic fuzzy metric space. Also, authors proved the fixed point theorem using compatible properties in many articles [9–12].

In this paper, we will obtain a unique common fixed point theorem and example for this theorem under the type(β) compatible four mappings with implicit relations defined on intuitionistic fuzzy metric space.

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2. Preliminaries

We will give some definitions, properties of the intuitionistic fuzzy metric space X as following:

Let us recall (see [13]) that a continuous $t-$norm is a binary operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions:(a)$*$ is commutative and associative; (b)$*$ is continuous; (c)$a * 1 = a$ for all $a \in [0,1]$; (d)$a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Similarly, a continuous $t-$conorm is a binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions:

(a) $\diamond$ is commutative and associative;

(b) $\diamond$ is continuous;

(c) $a \diamond 0 = a$ for all $a \in [0, 1]$;

(d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

Definition 2.1. ([14]) The 5−tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space (IFMS) if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a) $M(x, y, t) > 0$,

(b) $M(x, y, t) = 1 \Longleftrightarrow x = y$,

(c) $M(x, y, t) = M(y, x, t)$,

(d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(e) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,

(f) $N(x, y, t) > 0$,

(g) $N(x, y, t) = 0 \Longleftrightarrow x = y$,

(h) $N(x, y, t) = N(y, x, t)$,

(i) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,

(j) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is called an IFM on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.2. ([6]) Let X be an IFMS.

(a) $\{x_n\}$ is said to be convergent to a point $x \in X$ if, for any $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that

$$M(x_n, x, t) > 1 - \epsilon, \quad N(x_n, x, t) < \epsilon$$

for all $n \geq n_0$.

(b) $\{x_n\}$ is called a Cauchy sequence if for any $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that

$$M(x_n, x_m, t) > 1 - \epsilon, \quad N(x_n, x_m, t) < \epsilon$$

for all $m, n \geq n_0$.

(c) X is complete if every Cauchy sequence converges in X.

Lemma 2.3. ([8]) Let X be an IFMS. If there exists a number $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(x, y, kt) \geq M(x, y, t), \; N(x, y, kt) \leq N(x, y, t),$$

then $x = y$.

Definition 2.4. ([7]) Let A, B be mappings from IFMS X into itself. The mappings are said to be type(β) compatible if for all $t > 0$,

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1,$$
$$\lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0,$$

whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Proposition 2.5. ([15]) Let X be an IFMS with $t * t \geq t$ and $t \diamond t \leq t$ for all $t \in [0, 1]$. A, B be type(β) compatible maps from X into itself and let $\{x_n\}$ be a sequence in X such that $Ax_n, Bx_n \to x$ for some $x \in X$. Then we have the following

(a) $BBx_n \to Ax$ if A is continuous at x,

(b) $AAx_n \to Bx$ if B is continuous at x,

(c) $ABx = BAx$ and $Ax = Bx$ if A and B are continuous at x.

Implicit relations on fuzzy metric spaces have been used in many articles ([3, 16]). Let $\Psi = \{\phi_M, \psi_N\}$, $I = [0, 1]$, $\phi_M, \psi_N : I^6 \to R$ be continuous functions and $*, \diamond$ be a continuous t-norm, t-conorm. Now, we consider the following conditions ([6]):

(I) ϕ_M is decreasing and ψ_N is increasing in sixth variables.

(II) If, for some $k \in (0, 1)$, we have

$$\phi_M(u(kt), v(t), v(t), u(t), 1, u(\frac{t}{2}) * v(\frac{t}{2})) \geq 1,$$
$$\psi_N(x(kt), y(t), y(t), x(t), 0, x(\frac{t}{2}) \diamond y(\frac{t}{2})) \leq 1$$

or

$$\phi_M(u(kt), v(t), u(t), v(t), u(\frac{t}{2}) * v(\frac{t}{2}), 1) \geq 1,$$
$$\psi_N(x(kt), y(t), x(t), y(t), x(\frac{t}{2}) \diamond y(\frac{t}{2}), 0) \leq 1$$

for any fixed $t > 0$, any nondecreasing functions $u, v : (0, \infty) \to I$ with $0 < u(t), v(t) \leq 1$, and any nonincreasing functions $x, y : (0, \infty) \to I$ with $0 < x(t), y(t) \leq 1$, then there exists $h \in (0,1)$ with

$$u(ht) \geq v(t) * u(t), \quad x(ht) \leq y(t) \diamond x(t).$$

(III) If, for some $k \in (0,1)$, we have

$$\phi_M(u(kt), u(t), 1, 1, u(t), u(t)) \geq 1$$

for any fixed $t > 0$ and any nondecreasing function $u : (0,\infty) \to I$, then $u(kt) \geq u(t)$. Also, if, for some $k \in (0,1)$, we have

$$\psi_N(x(kt), x(t), 0, 0, x(t), x(t)) \leq 1$$

for any fixed $t > 0$ and any nonincreasing function $x : (0,\infty) \to I$, then $x(kt) \leq x(t)$.

Example 2.6. ([6]) Let $a*b = \min\{a,b\}$ and $a\diamond b = \max\{a,b\}$,

$$\phi_M(u_1, \cdots, u_6) = \frac{u_1}{\min\{u_2, \cdots, u_6\}},$$
$$\psi_N(x_1, \cdots, x_6) = \frac{x_1}{\max\{x_2, \cdots, x_6\}}.$$

Also, let $t > 0$, $0 < u(t), v(t), x(t), y(t) \leq 1$, $k \in (0, \frac{1}{2})$ where $u, v : [0,\infty) \to I$ are nondecreasing functions and $x, y : [0,\infty) \to I$ are nonincreasing functions. Now, suppose that

$$\phi_M(u(kt), v(t), v(t), u(t), 1, u(\frac{t}{2}) * v(\frac{t}{2})) \geq 1,$$
$$\psi_N(x(kt), y(t), y(t), x(t), 0, x(\frac{t}{2}) \diamond y(\frac{t}{2})) \leq 1,$$

then from Park [6], $\phi_M, \psi_N \in \Psi$.

3. Main Results and Example

Now, we will prove some common fixed point theorem for four mappings on complete IFMS as follows:

Theorem 3.1. Let X be a complete intuitionistic fuzzy metric space with $a*b = \min\{a,b\}$, $a\diamond b = \max\{a,b\}$ for all $a, b \in I$ and A, B, S and T be mappings from X into itself satisfying the conditions:

(a) $S(X) \subseteq B(X)$ and $T(X) \subseteq A(X)$,

(b) One of the mappings A, B, S, T is continuous,

(c) A and S as well as B and T are type(β) compatible

(d) There exist $k \in (0,1)$ and $\phi_M, \psi_N \in \Psi$ such that

$$\phi_M\begin{pmatrix} M(Sx,Ty,kt), M(Ax,By,t), \\ M(Sx,Ax,t), M(Ty,By,t), \\ M(Sx,By,t), M(Ty,Ax,t) \end{pmatrix} \geq 1,$$

$$\psi_N\begin{pmatrix} N(Sx,Ty,kt), N(Ax,By,t), \\ N(Sx,Ax,t), N(Ty,By,t), \\ N(Sx,By,t), N(Ty,Ax,t) \end{pmatrix} \leq 1,$$

for all $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point of X. Then from Theorem 3.1 of ([6]), we can construct a Cauchy sequence $\{y_n\} \subset X$. Since X is complete, $\{y_n\}$ converges to a point $x \in X$. Since $\{Ax_{2n+2}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$ and $\{Tx_{2n+1}\} \subset \{y_n\}$, we have

$$\begin{aligned}\lim_{n\to\infty} Ax_{2n+2} &= \lim_{n\to\infty} Bx_{2n+1} \\ &= \lim_{n\to\infty} Sx_{2n} \\ &= \lim_{n\to\infty} Tx_{2n+1} = x.\end{aligned}$$

Now, let A is continuous. Then

$$\lim_{n\to\infty} ASx_{2n} = Ax.$$

By Proposition 2.5,

$$\lim_{n\to\infty} SSx_{2n} = Ax.$$

Using condition (d), we have, for any $t > 0$,

$$\phi_M\begin{pmatrix} M(SSx_{2n}, Tx_{2n+1}, kt), M(ASx_{2n}, Bx_{2n+1}, t), \\ M(SSx_{2n}, ASx_{2n}, t), M(Tx_{2n+1}, Bx_{2n+1}, t), \\ M(SSx_{2n}, Bx_{2n+1}, t), M(Tx_{2n+1}, ASx_{2n}, t) \end{pmatrix}$$
$$\geq 1,$$

$$\psi_N\begin{pmatrix} N(SSx_{2n}, Tx_{2n+1}, kt), N(ASx_{2n}, Bx_{2n+1}, t), \\ N(SSx_{2n}, ASx_{2n}, t), N(Tx_{2n+1}, Bx_{2n+1}, t), \\ N(SSx_{2n}, Bx_{2n+1}, t), N(Tx_{2n+1}, ASx_{2n}, t) \end{pmatrix}$$
$$\leq 1$$

and by letting $n \to \infty$, ϕ_M, ψ_N are continuous, we have

$$\phi_M\begin{pmatrix} M(Ax,x,kt), M(Ax,x,t), 1, \\ 1, M(Ax,x,t), M(Ax,x,t) \end{pmatrix} \geq 1,$$

$$\psi_N\begin{pmatrix} N(Ax,x,kt), N(Ax,x,t), 0, \\ 0, N(Ax,x,t), N(Ax,x,t) \end{pmatrix} \leq 1.$$

Therefore, by (III), we have

$$M(Ax, x, kt) \geq M(Ax, x, t),$$
$$N(Ax, x, kt) \leq N(Ax, x, t).$$

Hence $Ax = x$ from Lemma 2.3. Also, we have, by condition (d),

$$\phi_M \begin{pmatrix} M(Sx, Tx_{2n+1}, kt), M(Ax, Bx_{2n+1}, t), \\ M(Ax, Sx, t), M(Tx_{2n+1}, Bx_{2n+1}, t), \\ M(Sx, Bx_{2n+1}, t), M(Tx_{2n+1}, Ax, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(Sx, Tx_{2n+1}, kt), N(Ax, Bx_{2n+1}, t), \\ N(Ax, Sx, t), N(Tx_{2n+1}, Bx_{2n+1}, t), \\ N(Sx, Bx_{2n+1}, t), N(Tx_{2n+1}, Ax, t) \end{pmatrix} \leq 1$$

and, let $n \to \infty$, we have

$$\phi_M \begin{pmatrix} M(Sx, x, kt), 1, M(Sx, x, t), \\ 1, M(Sx, x, t), 1 \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(Sx, x, kt), 0, N(Sx, x, t), \\ 0, N(Sx, x, t), 0 \end{pmatrix} \leq 1.$$

On the other hand, since

$$M(Sx, x, t) \geq M(Sx, x, \frac{t}{2}) = M(Sx, x, \frac{t}{2}) * 1,$$
$$N(Sx, x, t) \leq N(Sx, x, \frac{t}{2}) = N(Sx, x, \frac{t}{2}) \diamond 0,$$

ϕ_M is nonincreasing and ψ_N is nondecreasing in the fifth variable, we have, for any $t > 0$,

$$\phi_M \begin{pmatrix} M(Sx, x, kt), 1, M(Sx, x, t), \\ 1, M(Sx, x, \frac{t}{2}) * 1, 1 \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(Sx, x, kt), 0, N(Sx, x, t), \\ 0, N(Sx, x, \frac{t}{2}) \diamond 0, 0 \end{pmatrix} \leq 1$$

which implies that $Sx = x$. Since $S(X) \subseteq B(X)$, there exists a point $y \in X$ such that $By = x$. Using condition (d), we have

$$\phi_M \begin{pmatrix} M(x, Ty, kt), 1, 1, \\ M(Ty, x, t), 1, M(Ty, x, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(x, Ty, kt), 0, 0, \\ N(Ty, x, t), 0, N(Ty, x, t) \end{pmatrix} \leq 1$$

which implies that $x = Ty$. Since $By = Ty = x$ and B, T are type(β) compatible, we have $TTy = BBy$. Hence $Tx = TTy = BBy = Bx$. Therefore, from (d), we have, for any $t > 0$,

$$\phi_M \begin{pmatrix} M(x, Tx, kt), M(x, Tx, t), 1, \\ 1, M(x, Tx, t), 1, M(x, Tx, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(x, Tx, kt), N(x, Tx, t), 0, \\ 0, N(x, Tx, t), 0, N(x, Tx, t) \end{pmatrix} \leq 1.$$

From (III), we have

$$M(x, Tx, kt) \geq M(x, Tx, t),$$
$$N(x, Tx, kt) \leq N(x, Tx, t).$$

Therefore, we have $x = Tx = Bx$ from Lemma 2.3. Hence x is a common fixed point of A, B, S and T. The same result holds if we assume that B is continuous instead of A.

Now, suppose that S is continuous. Then

$$\lim_{n \to \infty} SAx_{2n} = Sx.$$

By Proposition 2.5,

$$\lim_{n \to \infty} AAx_{2n} = Sx.$$

Using (d), we have for any $t > 0$,

$$\phi_M \begin{pmatrix} M(SAx_{2n}, Tx_{2n+1}, kt), M(AAx_{2n}, Bx_{2n+1}, t), \\ M(SAx_{2n}, AAx_{2n}, t), M(Tx_{2n+1}, Bx_{2n+1}, t), \\ M(SAx_{2n}, Bx_{2n+1}, t), M(Tx_{2n+1}, AAx_{2n}, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(SAx_{2n}, Tx_{2n+1}, kt), N(AAx_{2n}, Bx_{2n+1}, t), \\ N(SAx_{2n}, AAx_{2n}, t), N(Tx_{2n+1}, Bx_{2n+1}, t), \\ N(SAx_{2n}, Bx_{2n+1}, t), N(Tx_{2n+1}, AAx_{2n}, t) \end{pmatrix} \leq 1,$$

and by $n \to \infty$, since $\phi_M, \psi_N \in \Psi$ are continuous, we have

$$\phi_M \begin{pmatrix} M(Sx, x, kt), M(Sx, x, t), 1, \\ 1, M(Sx, x, t), M(Sx, x, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(Sx, x, kt), N(Sx, x, t), 0, \\ 0, N(Sx, x, t), N(Sx, x, t) \end{pmatrix} \leq 1.$$

Thus, we have, from (III),

$$M(Sx, x, kt) \geq M(Sx, x, t),$$

$$N(Sx, x, kt) \leq N(Sx, x, t).$$

Hence $Sx = x$ by Lemma 2.3. Since $S(X) \subseteq B(X)$, there exists a point $z \in X$ such that $Bz = x$. Using (d), we have

$$\phi_M \begin{pmatrix} M(SAx_{2n}, Tz, kt), M(AAx_{2n}, Bz, t), \\ M(SAx_{2n}, AAx_{2n}, t), M(Tz, Bz, t), \\ M(SAx_{2n}, Bz, t), M(Tz, AAx_{2n}, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(SAx_{2n}, Tz, kt), N(AAx_{2n}, Bz, t), \\ N(SAx_{2n}, AAx_{2n}, t), N(Tz, Bz, t), \\ N(SAx_{2n}, Bz, t), N(Tz, AAx_{2n}, t) \end{pmatrix} \leq 1,$$

letting $n \to \infty$, we get

$$\phi_M \begin{pmatrix} M(x, Tz, kt), 1, 1, \\ M(x, Tz, t), 1, M(x, Tz, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(x, Tz, kt), 0, 0, \\ N(x, Tz, t), 0, N(x, Tz, t) \end{pmatrix} \leq 1$$

which implies that $x = Tz$. Since $Bz = Tz = x$ and B, T are type(β) compatible, we have $TBz = BBz$ and so $Tx = TBz = BBz = Bx$. Thus, we have

$$\phi_M \begin{pmatrix} M(Sx_{2n}, Tx, kt), M(Ax_{2n}, Bx, t), \\ M(Sx_{2n}, Ax_{2n}, t), M(Tx, Bx, t), \\ M(Sx_{2n}, Bx, t), M(Tx, Ax_{2n}, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(Sx_{2n}, Tx, kt), N(Ax_{2n}, Bx, t), \\ N(Sx_{2n}, Ax_{2n}, t), N(Tx, Bx, t), \\ N(Sx_{2n}, Bx, t), N(Tx, Ax_{2n}, t) \end{pmatrix} \leq 1,$$

letting $n \to \infty$,

$$\phi_M \begin{pmatrix} M(x, Tx, kt), M(x, Tx, t), 1, \\ 1, M(x, Tx, t), M(x, Tx, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(x, Tx, kt), N(x, Tx, t), 0, \\ 0, N(x, Tx, t), N(x, Tx, t) \end{pmatrix} \leq 1.$$

Thus, $x = Tx = Bx$. Since $T(X) \subseteq A(X)$, there exists $w \in X$ such that $Aw = x$. Thus, from (d),

$$\phi_M \begin{pmatrix} M(Sw, x, kt), 1, M(Sw, x, t), \\ 1, M(Sw, x, t), 1 \end{pmatrix} \geq 1,$$

$$\psi_N \begin{pmatrix} N(Sw, x, kt), 0, N(Sw, x, t), \\ 0, N(Sw, x, t), 0 \end{pmatrix} \leq 1.$$

Hence we have $x = Sw = Aw$. Also, since A, S are type(β) compatible,

$$x = Sx = SSw = AAw = Ax.$$

Hence x is a common fixed point of A, B, S and T. The same result holds if we assume that T is continuous instead of S.

Finally, suppose that A, B, S and T have another common fixed point u. Then we have, for any $t > 0$,

$$\phi_M \begin{pmatrix} M(x, u, kt), M(x, u, t), 1, \\ 1, M(x, u, t), M(x, u, t) \end{pmatrix} \geq 1,$$
$$\psi_N \begin{pmatrix} N(x, u, kt), N(x, u, t), 0, \\ 0, N(x, u, t), N(x, u, t) \end{pmatrix} \leq 1.$$

Therefore, from (III), $x = u$. This completes the proof.

Example 3.2. Let X be a intuitionistic fuzzy metric space with $X = [0, 1]$, $*, \diamond$ be t-norm and t-conorm defined by

$$a * b = \min\{a, b\}, \quad a \diamond b = \max\{a, b\}$$

for all $a, b \in X$. Also, let M, N be fuzzy sets on $X^2 \times (0, \infty)$ defined by

$$M(x, y, t) = [\exp(\frac{|x-y|}{t})]^{-1},$$
$$N(x, y, t) = [(\exp(\frac{|x-y|}{t})) - 1][\exp(\frac{|x-y|}{t})]^{-1}.$$

Let $\phi_M, \psi_N : X^6 \to R$ be defined as in Example 2.6 and define the maps $A, B, S, T : X \to X$ by $Ax = x$, $Bx = \frac{x}{2}$, $Sx = \frac{x}{4}$ and $Tx = \frac{x}{8}$. Then, for some $k \in [\frac{1}{2}, 1)$, we have

$$\begin{aligned} &M(Sx, Ty, kt) \\ &\quad = [\exp(\frac{|\frac{x}{4} - \frac{x}{8}|}{kt})]^{-1} \\ &\quad \geq \left[\exp(\frac{|x - \frac{x}{2}|}{t})\right]^{-1} \\ &\quad = M(Ax, By, t) \\ &\quad \geq \min\{M(Ax, By, t), M(Sx, Ax, t), \\ &\qquad\qquad M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t)\}, \\ &N(Sx, Ty, kt) \\ &\quad = [(\exp(\frac{|\frac{x}{4} - \frac{x}{8}|}{t})) - 1][\exp(\frac{|\frac{x}{4} - \frac{x}{8}|}{kt})]^{-1} \\ &\quad \leq \left[(\exp(\frac{|x - \frac{x}{2}|}{t})) - 1\right]\left[\exp(\frac{|x - \frac{x}{2}|}{t})\right]^{-1} \\ &\quad = N(Ax, By, t) \end{aligned}$$

$$\leq \max\{N(Ax, By, t), N(Sx, Ax, t),$$
$$N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t)\}.$$

Thus the condition (d) of Theorem 3.1 is satisfied. Also, it is obvious that the other conditions of the theorem are satisfied. Therefore 0 is the unique common fixed point of A, B, S and T.

4. Conclusion

Park et al. [4, 5] defined an IFMS and proved uniquely existence fixed point for the mappings satisfying some properties in an IFMS. Also, Park et al. [8] studied the type(α) compatible mapping, and Park [7] proved some properties of type(β) compatibility in an IFMS.

In this paper, we obtain a unique common fixed point and example for type(β) compatible mappings under implicit relations in an IFMS. This paper attempted to develop a proof method according to some conditions based on the fundamental properties and results in this space. I think that this results will be extended and applied to the other spaces, and further research should be conducted to determine how to combine the collaborative learning algorithm with our proof method in the future.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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