Regular paper http://dx.doi.org/10.13067/JKIECS.2014.9.2.187

매크로 다이버시티 결합의 확률 기하 이론 기반 Outage 확률 분석

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Outage Probability Analysis of Macro Diversity Combining Based on Stochastic Geometry

Ewaldo Zihan* · Kae-Won Choi**

요 약

본 논문에서는 다른 모바일 단말에서의 통합적 간섭을 고려하여 셀룰러 네트워크에서 매크로 다이버시티 결합을 사용하였을 경우의 Outage 확률을 분석한다. Outage 확률을 분석한 타 논문과 달리 본 논문에서는 상대적으로 간섭이 적은 기지국을 선택하였을 때의 다이버시티 이득을 분석한다. 분석을 위해 모바일 단말이 포아송 포인트 프로세스에 따라 확률적으로 분포한다고 가정하였다. 다수의 기지국에 가해지는 통합적 간섭의 다변수 분포를 다변수 로그노멀 분포로 근사시켜 분석을 수행하였다.

ABSTRACT

In this paper, we analyze the outage probability of macro diversity combining in cellular networks in consideration of aggregate interference from other mobile stations (MSs). Different from existing works analyzing the outage probability of macro diversity combining, we focus on a diversity gain attained by selecting a base station (BS) subject to relatively low aggregate interference. In our model, MSs are randomly located according to a Poisson point process. The outage probability is analyzed by approximating the multivariate distribution of aggregate interferences on multiple BSs by a multivariate lognormal distribution.

키워드

Stochastic Geometry, Aggregate Interference, Poisson Point Process, Selection Combining, Outage Probability

확률 기하 이론, 통합적 간섭, 포아송 포인트 프로세스, 선택적 다이버시티 결합, Outage 확률

Ⅰ. Introduction

Recently, cellular systems evolve in the direction of minimizing the distance between base stations (BSs) and mobile stations (MSs) by deploying small cells (e.g., femtocell and picocell) overlaid on macro cells. A cellular network with a large number of small-cell BSs has different characteristics from the traditional cellular network. First, the relationship between a BS and an MS becomes many-to-many from one-to-many. That is, a number of BSs cooperatively serve multiple MSs rather than one BS serves many MSs. Second, more MSs can simultaneously be served (schedu-

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접수일자 : 2013. 11. 15 심사(수정)일자 : 2013. 12. 23 게재확정일자 : 2014. 02. 11

led) within the same area due to densely located BSs, resulting in BSs subject to high interference from MSs.

In this paper, we focus our attention on the uplink of the cellular networks with small-cell BSs as described in Fig. 1. In this network, signals from an MS are received by multiple BSs and are combined together to achieve a diversity gain. For simplicity, we consider selection combining which selects a signal from the BS with the highest signal-to-interference-and-noise ratio (SINR).

Typically, a diversity gain of selection combining comes from variations in signal strength to different BSs due to channel fading. However, in the cellular networks with small-cell BSs, interference from other MSs plays more important role than signal strength does since multiple nearby MSs can simultaneously be served. In this environment, a BS with low interference is preferred for receiving a signal. For example, in Scenario A in Fig. 1, BS 1 is selected for receiving a signal from MS 1 since there is MS 2, which emits interference, close to BS 2. On the other hand, in Scenario B in Fig. 1, BS 1 is avoided since MS 2 is close to BS 1 in this scenario. By selecting a BS with low interference, another type of diversity gain, which can be called interference diversity, is attained.

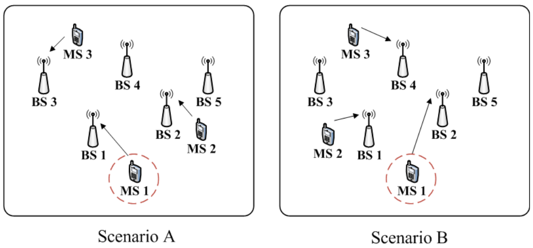

Fig. 1 Example scenarios of cellular networks with small-cell BSs

In this paper, we analyze the outage probability of selection combining in a cellular network with small-cell BSs in consideration of exploiting interference diversity. There have been many existing literatures deriving the outage probability of selection combining in various fading models (e.g., [1]). Different from these literatures, we take account of interference from randomly located MSs, which is modeled by a Poisson point process, as well as the effect of channel fading. Despite previous efforts to analyze the outage probability under the aggregate interference from a Poisson field of interferers (e.g., [2], [3]), no work has been done so far for deriving the outage probability of diversity combining in this environment.

The remainder of this paper is organized as follows. In Section II, we introduce the system model. Section III, the outage probability of selection combining under aggregate interference is analyzed. Section IV provides numerical results and the paper is concluded in Section V.

II. System Model

2.1. Network Model

Consider K base stations (BSs) which are capable of selection combining. A BS is indexed by $k(=1,...,K)$. The location of BS k is fixed at the coordinate c_k on the two-dimensional plane R^2. Mobile stations (MSs) transmit signals to the BSs in the uplink direction. Differently from BSs, we refer to an MS by its coordinate. That is, MS x indicates an MS located at the coordinate x. We define Π as the set of the coordinates of all MSs.

The set of the coordinates of MSs, Π, follows the Poisson point process [3]. Therefore, the number and the locations of MSs are completely random. Let us define $\Phi(X)$ as the number of MSs in the area X. From the definition of the Poisson point process [3], $\Phi(X)$ follows a Poisson distribution, and $\Phi(X)$ and $\Phi(Y)$ are independent of each other for any disjoint sets X and Y. In addition, the density of MSs at the coordinate x is denoted by $\lambda(x)$. We assume that an MS can appear only within a specific area A, that is, $\lambda(x)=0$ for $x \not\in A$. If $\Lambda(X)$ denotes the expected number of MSs within X, we have $\Lambda(X)=E[\Phi(X)]=\int_{x\in X}\lambda(x)dx$.

2.2. Channel Model

For a channel model, we consider the following two cases: a non-fading case and a fading case. Let $g_k(s)$ denote the power attenuation from MS s to BS k. In the fading case, the power attenuation includes the shadowing and the multi-path fading as well as the path-loss. Therefore, the channel gain from MS x to BS k is $g_k(x)=\omega_k(x)\delta_k(x)\eta_k(x)$, where $\omega_k(x)$, $\delta_k(x)$, and $\eta_k(x)$ respectively denote the path-loss, the log-normal shadow fading, and the multi-path fading components from MS x to BS k. In the non-fading case, the channel attenuation includes only the path-loss component, that is, $g_k(x)=\omega_k(x)$.

With the bounded path-loss model [4], the path-loss component is given as $\omega_k(x)=(1+\| x-c_k \|^{\alpha})^{-1}$, where α s the path-loss exponent. The log-normal shadow fading component $\delta_k(x)$ in a dB scale (i.e., $10\log_{10}\delta_k(x)$) follows the normal distribution with zero mean and standard deviation γ_δ. The multi-path fading component $\eta_k(x)$ follows the exponential distribution of mean μ_η. The ith moment of the shadow fading and the multi-path fading components are $\mu_\eta^{(i)}=E[\delta_k(x)^i]=10^{(\ln(10)\cdot i^2\cdot\gamma_\delta^2/200)}$ and $\mu_\eta^{(i)}=E[\eta_k(x)^i]=i!\cdot(\mu_\eta)^i$, respectively [5]. We assume that the power attenuation of a channel is independent of that of other channels. An MS transmits a signal to BSs with the fixed transmission power ρ. Therefore, the received power from MS x on BS k is given by $g_k(x)\rho$.

2.3. Selection Combining

The signals received by BSs from an MS are combined by using selection combining. For selection combining, one BS receiving the signal with the highest SINR is selected among all BSs. The SINR of the signal from MS s on BS k is

$$\zeta_k(s)=\frac{g_k(s)\rho}{\sum_{x\in\Pi\backslash\{s\}}g_k(x)\rho+N_oW}=\frac{g_k(s)\rho}{I_k(s)+N_oW} \quad (1)$$

where $\Pi\backslash\{s\}$ is the set of the coordinates of all MSs but MS s, N_o is the noise spectral density on a BS, W is the system bandwidth, and $I_k(s)$ is the aggregate interference on BS k from all MSs but MS s, i.e., $I_k(s)=\sum_{x\in\Pi\backslash\{s\}}g_k(x)\rho$.

After selection combining, the SINR of MS s is the maximum of the SINRs on all BSs such that $\xi(s)=\max_{k=1,\ldots,K}\zeta_k(s)$. To properly decode a received signal, the SINR should exceed the required SINR, denoted by γ. An outage occurs if the SINR after selection combining does not exceed a required SINR (i.e., if $\xi(s)\le\gamma$). We will analyze the outage probability in the next section.

III. OUTAGE PROBABILITY ANALYSIS

3.1. Outage Probability of Single MS

We first consider the case that there is an MS at a specific coordinate s (i.e., the case that $s\in\Pi$) and calculate the outage probability of MS s. The outage probability of MS s given that $s\in\Pi$ is denoted by $\psi(s)$. Then, we have

$$\begin{aligned}\psi(s)&=\Pr[\xi(s)\le\gamma|s\in\Pi]\\&=\Pr[\zeta_k(s)\le\gamma\ \text{for}\ all\ k=1,\ldots K|s\in\Pi]\end{aligned} \quad (2)$$

To express the inequality in this equation in a vector form, we define a "received power vector" as $R(s)=(r_1(s),\ldots,r_K(s))^T$ where $r_k(s)=g_k(s)\rho$ and define an "interference and noise vector" as

$V(s)=(v_1(s),...,v_K(s))^T$,where $v_k(s)=I_k(s)+N_oW$. In addition, $\Gamma=(\gamma,...,\gamma)^T$ is defined as a column vector all K components of which are γ. From (1) and (2), we can rewrite the outage probability as

$$\psi(s)=\Pr[\ln R(s)-\ln V(s)\leq \ln\Gamma|s\in\Pi] \quad (3)$$

where the logarithm of a vector $X=(x1,...,x_K)^T$ is $\ln X=(\ln x_1,...,\ln x_K)^T$ and the notation $\leq$ is a component-wise inequality.

To calculate the outage probability in (3), we should derive the multivariate distributions of the received power vector $R(s)$ and the interference and noise vector $V(s)$. Fortunately, the distributions of these vectors can well be approximated by the multivariate log-normal distribution. We will derive the approximate istribution of the received power vector in Section III-B and the approximate distribution of the interference and noise vector in Section III-C.

3.2. Multivariate Distribution of Received Power Vector

To derive the distribution of the received power vector, we first focus on one component in the vector, i.e., the received power from MS s on BS k, i.e., $g_k(s)\rho$. In the fading case, the received power is $g_k(s)\rho=\omega_k(s)\delta_k(s)\eta_k(s)\rho$ which follows the composite log-normal and exponential distribution since $\delta_k(s)$ follows a log-normal distribution and $\eta_k(s)$ follows an exponential distribution. It is well known that the composite log-normal and exponential distribution approximately follows the log-normal distribution since the log-normal component is dominant in a wireless channel. From [6], the probability density function (pdf) of the received power $r_k(s)$ in the fading case is

$$f_{r_k(s)}(x) = \frac{10/\ln 10}{\gamma_{r_k(s)}\sqrt{2\pi}\,x}exp(-\frac{(10\log_{10}x-\psi_{r_k(s)})^2}{2(\gamma_{r_k(s)})^2}) \quad (4)$$

where $\psi_{r_k(s)}$ and $\gamma_{r_k(s)}$ a are respectively the mean and the standard deviation of the received power in a dB scale (i.e., $10\log_{10}r_k(s)$). Then $\psi_{r_k(s)}=10\log_{10}\rho+10\log_{10}\omega_k(s)+10\log_{10}\mu_\eta-2.5$ and $\gamma_{r_k(s)}=\sqrt{\gamma_\delta^2+5.57^2}$. In the non-fading case, the received power has a deterministic value such that $g_k(s)\rho=\omega_k(s)\rho$. Therefore, the received power in a dB scale in the non-fading case follows the normal distribution with the mean $\psi_{r_k(s)}=10\log_{10}\omega_k(s)\rho$ and the standard deviation $\gamma_{r_k(s)}=0$.

From (4), the multivariate distribution of $R(s)$ can be approximated by a multivariate log-normal distribution. The pdf of the K-dimensional multivariate log-normal distribution is given as

$$f_{R(s)}(X)=\frac{1}{(2\pi)^{K/2}|U|^{1/2}\prod_{k=1}^{K}x_k} \times\exp(-\frac{1}{2}(\ln X-Q)^TU^{-1}(\ln X-Q)), \quad (5)$$

where $Q=(q_1,...,q_K)^T$ is the mean vector and $U=(u_{k,m})_{k,m=1,...K}$ is the covariance matrix of the multivariate normal distribution which the logarithm of the random vector $R(s)$ follows. Since the pdf in (5) is not based on the received power in a dB scale but on the natural logarithm of the received power, we have $q_k=\frac{\ln 10}{10}\psi_{r_k(s)}$ for all k. In addition, $u_{k,k}=(\frac{\ln 10}{10}\gamma_{r_k(s)})^2$ for all k and $u_{k,m}=0$ for $k\neq m$ since the power attenuations to all BS are independent of each other.

3.3. Multivariate Distribution of Interference and Noise Vector

Now, we derive the approximate distribution of the interference and noise vector $V(s)$. In the interference and noise vector, the aggregate interference $I_k(s)$ is the sum of interferences coming from all MSs but MS s, i.e., MS X such that $X \in \Pi \backslash \{s\}$. From Slivnyak's theorem [3], the point process $\Pi \backslash \{s\}$ given that $s \in \Pi$ also follows the same Poisson point process as does. Therefore, the joint distribution of $I_1(s),...,I_K(s)$ given that $s \in \Pi$ is the same as the joint distribution of $I_1,...,I_K$, where I_k is the interference on BS k from all MSs such that

$$I_k = \sum_{X \in \Pi} g_k(X)\rho = \int_{X \in A} g_k(X)\rho\Phi(dX) \quad (6)$$

Therefore, if we let $V = (v_1,...,v_K)^T$ a component of which is $v_k = I_k + N_o W$ for all k, the distribution of V is the same as that of $V(s)$. Since there is no known closed-form expression for the joint distribution of the aggregate interferences $I_1,...,I_K$, we proposed in our previous work [5] to approximate the distribution by a multivariate log-normal distribution and showed that the approximated distribution is very close to the true one. Although the interference and noise vector is composed of the noise as well as the aggregate interference, this vector can also be approximated by a multivariate log-normal distribution. This is because a log-normal variable plus a deterministic variable can be regarded as a sum of log-normal variables which can be approximated by another log-normal variable [6]. Moreover, noise is almost negligible compared to interference in the interference-limited scenario.

To derive the parameters of the log-normal distribution, we match the mean and the covariance of the log-normal distribution to those of the interference and noise vector V. To calculate the mean and the covariance of V, we need to derive the cumulant generating function of V defined as $C_V(B) = \ln(E[\exp(B^T V)])$, where $B = (\beta_1,...,\beta_K)^T$ From [5], we can calculate $C_V(B)$ as

$$C_V(B) = -\int_{X \in A} (1 - E[\exp(B^T g(X)\rho)])\lambda(X)dX + N_o W B^T 1 \quad (7)$$

where $g(X) = (g_1(X),...,g_K(X))^T$ and 1 is the column vector of all ones. The mean of v_k is calculated as

$$E[v_k] = \frac{dC_V(B)}{d\beta_k}|_{\beta=0} = N_o W + \rho\int_{X \in A} E[g_k(X)]\lambda(X)dX, \quad (8)$$

where $E[g_k(X)] = \omega_k(X)$ in the non-fading case and $E[g_k(X)] = \omega_k(X)\mu_\delta^{(1)}\mu_\eta^{(1)}$ in the fading case. Similarly, the covariance of v_k and v_m is calculated as

$$Cov[v_k, v_m] = \frac{d^2 C_V(B)}{d\beta_k d\beta_m}|_{\beta=0} = \rho^2 \int_{X \in A} E[g_k(X)g_m(X)]\lambda(X)dX, \quad (9)$$

where $E[g_k(X)g_m(X)] = \omega_k(X)\omega_m(X)$ in the non-fading case. In the fading case, $E[g_k(X)g_m(X)] = \omega_k(X)\omega_m(X)(\mu_\delta^{(1)})^2(\mu_\eta^{(1)})^2$ if $k \neq m$ and $E[g_k(X)^2] = \omega_k(X)^2\mu_\delta^{(2)}\mu_\eta^{(2)}$ otherwise. Note that the calculation of the mean and the covariance involves numerical integration. The distribution of the interference and noise vector V is approximated by a multivariate log-normal distribution such that

$$f_V(X) = \frac{1}{(2\pi)^{K/2}|H|^{1/2}\prod_{k=1}^{K} x_k} \times \exp(-\frac{1}{2}(\ln X - D)^T H^{-1}(\ln X - D)), \quad (10)$$

where $D=(d_1,...,d_K)^T$ and $H=(h_{k,m})_{k,m=1,...,K}$. From [5], we can derive the parameters D and H by matching the mean and the covariance of the approximate log-normal distribution to those of V. Then, we have

$$h_{k,m}=\ln\left(\frac{Cov[v_k,v_m]}{E[v_k]E[v_m]}+1\right) \quad (11)$$

$$q_k=\ln(E[v_k])-\frac{h_{k,k}}{2} \quad (12)$$

3.4. Deriving Outage Probability of Single MS and Whole Network

The outage probability in (3) can be rewritten as

$$\psi(s)=\Pr[Z(s)\le n\Gamma|s\in\Pi]=\int_{y\le\ln\Gamma} f_{Z(s)}(y)dy \quad (13)$$

where $Z(s)=\ln R(s)-\ln V$ and $f_{z(s)}$ is the pdf of $Z(s)$. Since we approximate $R(s)$ and V by the multivariate log-normal distributions, the distribution of $Z(s)$ given $s\in\Pi$ follows the multivariate normal distribution the mean vector and the covariance matrix of which are $Q-D$ and $U+H$ respectively. Therefore, the pdf of $Z(s)$ is

$$f_{Z(s)}(X)=\frac{1}{(2\pi)^{K/2}|U+H|^{1/2}}\times\exp\left(-\frac{1}{2}(X-(Q-D))^T(U+H)^{-1}(X-(Q-D))\right). \quad (14)$$

To evaluate the outage probability $\psi(s)$ in (3), a cumulative density function (cdf) of the multivariate normal distribution should be calculated. For doing this, we can use various methods developed to calculate the cdf of a multivariate normal distribution. For example, we can approximate the covariance matrix $U+H$ by a Green's matrix and derive a closed-form expression for the cdf as in [1].

From the outage probability of a single MS s, i.e., $\psi(s)$, we can calculate the outage probability of the whole network, denoted by $\overline{\psi}$. Let $O(X)$ denote the outage indicator of MS X, which is 1 an outage occurs (i.e., $\xi(X)<\gamma$); 0 otherwise. The outage probability $\overline{\psi}$ is defined as the average number of MSs undergoing an outage divided by the average number of all MSs. That is,

$$\overline{\psi}=\frac{E[\sum_{x\in\Pi}O(X)]}{E[|\Pi|]} \quad (15)$$

where $|\Pi|=\Phi(A)$ is the number of all MSs.

In (15), the average number of all MSs is calculated as $E[|\Pi|]=\Lambda(A)=\int_{X\in A}\lambda(X)dX$. On the other hand, the average number of Mss undergoing an outage is

$$E[\sum_{x\in\Pi}O(X)]=\int_{X\in A}\psi(X)\lambda(X)dX. \quad (16)$$

Then, the outage probability of the whole network is calculated as

$$\overline{\psi}=\frac{\int_{X\in A}\psi(X)\lambda(X)dX}{\int_{X\in A}\lambda(X)dX} \quad (17)$$

Ⅴ. Simulation Result

In Fig. 2, we present a comparison between the Monte Carlo simulation results and the Log-normal approximation based on our analysis. This figure shows the complementary cumulative distributed function (ccdf) of the multivariate distribution of the aggregate interference on two receivers (i.e., BSs). We can see that the analysis results well match the simulation results.

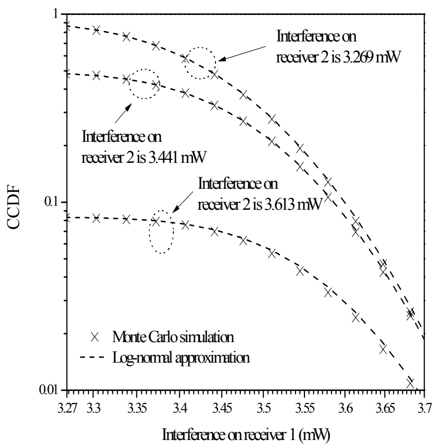

Fig. 2 Comparison between the monte carlo simulation results and the log-normal approximation

Ⅵ. Conclusion

In this paper, we have analyzed the outage probability of selection combining when MSs follow a Poisson point process. In future work, the analytic framework in this paper can be extended for analyzing the outage probability of diversity combining in the multiuser distributed antenna system (DAS) and coordinated multipoint (CoMP) [7]. This work can also be extended for wireless personal area networks [8][9].

감사의 글

이 연구는 서울과학기술대학교 교내 학술연구비 지원으로 수행되었습니다.

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