# 불확실 선형 시스템을 위한 적분 가변구조 지점에서 지점으로 레귤레이션 제어기

### A Variable Structure Point-to-Point Regulation Controller for Uncertain General Linear Systems

이 정 훈\* (Jung-Hoon Lee)

Abstract - In this paper, an alternative variable structure controller is designed for the point-to-point regulation control of uncertain general linear plants so that the output of plants can be controlled from an arbitrarily given initial point to an arbitrarily given reference point in the state space. By using the error between the steady state value of the output and an arbitrarily given reference point and those integral, a transformed integral sliding surface is defined, in advance, as the surface from an initial state to an arbitrarily given reference point without the reaching phase problems. A corresponding control input is suggested to satisfy the existence condition of the sliding mode on the preselected transformed integral sliding surface against matched uncertainties and disturbances. Therefore, the output controlled by the proposed controller is completely robust and identical to that of the preselected transformed integral sliding surface. Through an example, the effectiveness of the suggested controller is verified.

Key Words: Point-to-point regulation control, Sliding mode control, Vairable structure system, Strong robustness

#### 1. Introductions

The theory of the variable structure system (VSS) or sliding mode control (SMC) can provide the effective means to the problem of controlling uncertain dynamical under parameter variations disturbances[1-5,23]. One of its essential advantages is the invariance of the controlled system to the variations of parameters and disturbances in the sliding mode on the predetermined sliding surface, s(t)=0. The proper design of the sliding surface can determine the almost output dynamics and its performances. Many design algorithms including the linear(optimal control[6][7], eigenstructure assignment[8][9], geometric approach[10], differential geometric approach[11], Lyapunov approach[29]) and nonlinear[12][22] techniques reported. Moreover, an integral action have been augmented by the two groups[7][13]-[15]. One is to improve the steady state performance[7][13][14] against the external disturbances possibly in the implementation of the VSS, and the other aims to reduce the chattering problems by means of filtering the Unfortunately, most of these existing VSS's have the reaching phase and are applied to canonical plants. During the reaching phase, the controlled systems may be sensitive to the parameter variations and disturbances because the sliding mode can not be realized[18]. And it is difficult to find the designed performance from the real output, that is, the output is not predictable in the design stage. Moreover, introducing the integral to the VSS without removing the reaching phase can inevitably cause the overshoot problems.

One alleviation method for the reaching phase problems is the use of the high-gain feedback[1]. This has the drawbacks related to the high-gain feedback, for example, the sensitivity to the unmodelled dynamics and actuator saturation[18]. An adaptive rotating or shifting of the sliding surface is suggested to reduce the reaching phase problems in [2] and [20], and the segmented sliding surface connected from a given initial condition to the origin is also suggested in [21]. But these changing techniques and segmented sliding surfaces are applicable to only second order systems and those outputs are not predictable. In [22] and [23], the exponential term is added to the conventional linear sliding surface in order to make the sliding surface be zero at t=0. But, its resultant sliding dynamics becomes nonlinear. In [30], Park attempted to remove the reaching phase problems

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discontinuous input through the integral action[15].

<sup>\*</sup> Corresponding Author : ERI, Dept. of Control & Instrumentation Eng. Gyeongsang Nat. University, Korea E-mail : jhleew@gnu.ac.kr

for general uncertain systems. However, the developed algorithms for regulation problems use a complex sliding surface including the control input term and need mathematical accuracy in the formulation of the algorithm. Utkin suggested the two transformation(diagonalization) methods, i.e., transformation(diagonalization) of the sliding surface and transformation(diagonalization) of the control through the invariant theorem in [1] and that theorem is proved for multi input linear systems in [39]. Using transformation(diagonalization) of the control input, a new integral variable structure controller(IVSC) without the reaching phase problems is suggested for the point-to-point regulation control[32] of uncertain general systems to an any given predetermination/prediction of output response in [38].

In this paper, as an alternative approach of [38], using the transformation(diagonalization) of the sliding surface, a modified integral variable structure controller(MIVSC) without the reaching phase problems is suggested for the point-to-point regulation control of uncertain general linear systems to an any given reference point with the same performance of [38]. A used integral sliding surface is transformed by means of the transformation(diagonalization) of the sliding surface in the invariant theorem of Utkin. The reaching phase is completely removed by only introducing an integral action of the state error with a special non-zero initial value to the conventional sliding surface. After obtaining the dynamic representation of the integral sliding surface, those coefficients are designed by the point-to-point regulation controller design. A corresponding control input is proposed to completely guarantee the designed output in the sliding surface from any initial condition to a given desired reference point for all the parameter variations and disturbances. The stability of the suggested algorithm together with the existence condition of the sliding mode is investigated in Theorem 1. Finally, an example is presented to show the effectiveness of the algorithm.

#### 2. Alternative Variable Structure Systems

#### 2.1 Description of plants

An n-th order uncertain non-canonical general linear system is described by

$$\dot{z} = (A_0 + \Delta A)z(t) + (B_0 + \Delta B)u(t) + Df(t), \quad z(t_0)$$

$$u = Ez(t)$$

$$(1)$$

where  $z(\bullet) \in \mathbb{R}^n$  is the original state,  $u(\bullet) \in \mathbb{R}^1$  is the control input,  $f \in \mathbb{R}^r$  is the external disturbance,  $\Delta A$ ,  $\Delta B$ , and D are the bounded system matrix

uncertainty, the bounded input matrix uncertainty, and the disturbance matrix, and those satisfy the matching condition:

$$\begin{split} R(\Delta A) \subset R(B_0) \\ R(\Delta B) \subset R(B_0) \\ R(D) \subset R(B_0) \end{split} \tag{2}$$

#### Assumption

**A1**:It is assumed that the following equation is satisfied for a non zero element coefficient vector  $C_{z1} \subseteq R^{1 \times n}$ 

$$\left| \left( C_{z1} B_0 \right)^{-1} C_{z1} \Delta B \right| = \left| \Delta I \right| \le \eta < 1 \tag{3}$$

where  $\eta$  is a positive constant less than 1, which menas that the amount of the uncertainty  $\Delta B$  is less than that of the nominal value  $B_0$ . Because of that, the assumption A1 is practical. The purpose of the controller design is to control the output(state) of a plant (1) to track the predetermined intermediate dynamics from an arbitrarily given initial point finally to any arbitrary reference value  $y_r(z_r,y_r=Ez_r)$  for all the uncertainties and disturbances by using the integral sliding mode control. Generally, this type point-to-point regulation control is following a non zero reference command from an arbitrarily given initial point[32]. By state transformation, x=Pz a weak canonical form of (1) is obtained as,

$$\dot{x} = Ax(t) + \Gamma u(t) + \Pi h(t), \qquad x(t_0)$$

$$y = EP^{-1}x(t)$$
(4)

where

$$A = PA_0 P^{-1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -\alpha_1 - \alpha_2 - \alpha_2 & \cdots & -\alpha_n \end{bmatrix} \text{ and } \Gamma = PB_0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}$$
 (5)

where  $x(t_0)$  is the initial condition transformed from  $z(t_0)$  and h(t) is the lumped uncertainty in the transformed system as

$$h(t) = \Delta A'P^{-1}x(t) + \Delta B'u(t) + D'f(t)$$
. (6) In (5), if  $b=1$ , it is the standard canonical form, otherwise, it is called as a weak canonical form. Hence it is more general in view of the degree of freedom in the MIVSC design and the approach in this note can be an extension of [38].

#### 2.2 Design of Transformed Integral Sliding Surfaces

To design the MIVSC, a transformed integral sliding surface in error coordinate system is suggested to the following form having an integral of the state errors as

$$\begin{split} s(x,t) &= \left( C_{x1} I \right)^{-1} C_{x0} \left[ \int_{0}^{t} (x - x_r) dt + \int_{-\infty}^{0} (x - x_r) dt \right] \\ &+ \left( C_{x1} I \right)^{-1} C_{x1} (x - x_r) &= 0 \end{split} \tag{7}$$

$$s(z,t) = \left(C_{z1}B_0\right)^{-1}C_{z0}\left[\int_0^t (z-z_r)dt + \int_{-\infty}^0 (z-z_r)dt\right] + \left(C_{z1}B_0\right)^{-1}C_{z1}(z-z_r) = 0$$
(8)

where the coefficient matrices and the initial conditions for the integral states are expressed as shown

$$\begin{split} &C_{x1} = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \in R^{1 \times n}, & c_n = 1 \\ &C_{z1}B_0 = C_{x1}\Gamma, & C_{z0} = C_{x0}P, & C_{z1} = C_{x1}P \in R^{1 \times n} \\ &\int_{-\infty}^{0} (x_i - x_{ri})dt = C_{x1i}(x_{ri} - x_i(t_0))/C_{x0i} \,, \end{split} \tag{9}$$

$$\int_{-\infty}^{0} (z_i - z_{ri}) dt = C_{z_{1i}}(z_{ri} - z_i(t_0)) / C_{z_{0i}}$$
(10)

The initial conditions (10) for the integral states in (7) and (8) are selected so that transformed integral sliding surfaces are zeros at t=0 for removing the reaching phase from the beginning. From  $\dot{s}(x,t) = 0$  , (4), (7), and the invariant theorem of Utkin[1][39], the differential equation for  $x_n$  is obtained as

$$\begin{split} \dot{x_n}(t) = & -C_{x0}(x-x_r) - \left[0\,c_1\,c_2\cdots c_{n-1}\right] x(t) \\ = & -C_x x(t) + C_{x0} x_r \end{split}$$

where

$$C_x = \left[ c_{x1} c_{x2} \cdots c_{xn} \right] = C_{x0} + \left[ 0 c_1 c_2 \cdots c_{n-1} \right]$$
 (12)

Finally combing (11) with the first n-1 differential equation in the systems (4) leads to the ideal sliding dynamics:

$$\dot{x}_s(t) = \Lambda_c x_s(t) + \Gamma C_{x0} x_r, \qquad x_s(t_0) = x(t_0)$$
 (13)

$$\dot{z}_s(t) = P^{-1} \Lambda_c P z_s(t) + P^{-1} \Gamma C_{r0} P z_r, \quad z_s(t_0) = z(t_0)$$
 (14)

$$\Lambda_c = \begin{bmatrix} O^{(n-1)\times 1} & I^{(n-1)\times (n-1)} \\ -C_x \end{bmatrix}$$
(15)

which can be considered as a dynamic representation of the transformed integral sliding surface (7) or (8). Because of the reference command in the system (13), the design problems becomes the point-to-point regulation controller design also[32]. In order to apply well-studied linear regulator theories to choosing the coefficient matrices of the transformed integral sliding surfaces, (13) and (14) are transformed to the nominal system from of (1)

$$\begin{split} \dot{x_s}(t) &= Ax_s(t) + \Gamma u_s(x_s,t) + \Gamma C_{x0}x_r \\ u_s(x_s,t) &= -Gx_s(t) \end{split} \tag{16}$$

where

$$\Lambda_c = \Lambda - \Gamma G \tag{17}$$

and expressed with the original state as

$$\dot{z}_{o}(t) = A_{0}z_{o}(t) + B_{0}u_{o}(z_{o},t) + B_{0}C_{r0}Pz_{r}$$

$$u_s(z_s,t) = - \mathit{GPz}_s(t) = - \mathit{Kz}_s(t) \tag{18}$$

where

$$P^{-1}\Lambda_{c}P = A_{0} - B_{0}K \tag{19}$$

When one determines the continuous gain, the condition on the gain should be satisfied to be  $y = y_r$  in the steady state

$$\Gamma G = \Lambda + \Gamma C_{x0}$$
 and  $B_0 K = A_0 + B_0 C_{x0} P$  (20)

After determining K or G to have a desired ideal sliding dynamics, the coefficient matrix of the new surface (7) or (8) can be directly determined from the relationship:

$$C_{x} = \begin{bmatrix} c_{x0_{1}} & c_{x0_{2}} + c_{x1_{1}} & \cdots & c_{x0_{n}} + c_{x1_{n-1}} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{1} & \alpha_{2} & \cdots & \alpha_{n} \end{bmatrix} + bG$$

$$= \begin{bmatrix} \alpha_{1} & \alpha_{2} & \cdots & \alpha_{n} \end{bmatrix} + bKP^{-1}$$

$$C_{x0} = C$$
(22)

which is derived from (17) and (20). If the point-to-point regulation control using the nominal plant (16) or (18) is designed, then the sliding surface having exactly the same output performance can be effectively chosen using (21) or (22). As a consequence, the output of (16) or (18) becomes the state set of the chosen sliding surface, called as the ideal sliding output meaning the nominal output design in the sliding surface or the desired performance design.

#### 2.3 Corresponding Control Input and Stability Analysis

Now, as the second design phase of the MIVSC, a following corresponding control input to generate the sliding mode on the pre-selected transformed integral sliding surface is proposed as composing of the continuous and switching terms

$$u(t) = -\left(C_{z_1}B_0\right)^{-1} \left[C_{z_0}(z - z_r) + C_{z_1}A_0z(t)\right]$$

$$-\sum_{i=1}^{n} \beta_{1i}|z_i - z_{ri}|sgn(s) - \sum_{i=1}^{n} \beta_{2i}z_i - \beta_3sgn(s)$$
(23)

with the gains satisfying the inequalities:

 $\min\{I + \Delta I\}$ 

$$\beta_{1i} > |c_{z0i}| \frac{\max \left\{ \Delta I(C_{z1}B_0)^{-1} \right\}}{\min \left\{ I + \Delta I \right\}}$$

$$\beta_{2i} = \begin{cases} > \frac{\max \left\{ (C_{z1}B_0)^{-1}C_{z1}\Delta A_i - \Delta I(C_{z1}B_0)^{-1}C_{z1}A_{0i} \right\}}{\min \left\{ I + \Delta I \right\}} & \text{for } (sz_i) > 0 \\ < - \frac{\min \left\{ (C_{z1}B_0)^{-1}C_{z1}\Delta A_i - \Delta I(C_{z1}B_0)^{-1}C_{z1}A_{0i} \right\}}{\min \left\{ I + \Delta I \right\}} & \text{for } (sz_i) < 0 \end{cases}$$

$$\beta_3 > \frac{\max \left\{ (C_{z1}B_0)^{-1}C_{z1}Df(t) \right\}}{\min \left\{ I + \Delta I \right\}}$$

$$(24)$$

The continuous term in (23) is directly determined according to choosing the transformed integral sliding surface. Only the switching gains are the design parameters for the robustness problems. As a result, the controller design is separated into the performance design and robustness design. Because of the feasibility of the control, the necessary constraint is imposed by the Assumption A1 on the uncertain bound of  $\Delta B(t)$  with respect to the coefficient  $C_{z1}$  and the nominal input matrix  $B_0$ . By means of the derived algorithm until now, the obtainable performance including the stability of the closed loop systems can be stated in next theorem.

**Theorem 1**: The proposed feasible variable structure controller with the input (23) and the modified sliding surface (8) can exhibit the asymptotic stability and the

(24)

ideal output of the sliding mode dynamics for all the uncertainties exactly defined by the modified sliding surface (8).

Proof: Take a Lyapunov candidate function as

$$V(t) = \frac{1}{2}s^{2}(z,t) \tag{25}$$

Differentiating (25) with time leads to

$$\dot{V} = s(z,t) \cdot \dot{s}(z,t) \tag{26}$$

Now, the real dynamics of the transformed integral sliding surface by the new corresponding control input can be obtained as

$$\begin{split} \dot{s}(z,t) &= (C_{z1}B_0)^{-1}C_{z0}(z-z_r) + (C_{z1}B_0)^{-1}C_{z1}\dot{z}(t) \\ &= (C_{z1}B_0)^{-1}C_{z0}(z-z_r) \\ &+ (C_{z1}B_0)^{-1}C_{z1}\Big[(A_{0+}\Delta A)z(t) \\ &+ (B_0+\Delta B)u(z,t) + Df(t)\Big] \end{split} \tag{27}$$

Substituting (23) into (27) leads to

$$\begin{split} \dot{s}(z,t) &= (C_{z1}B_0)^{-1}C_{z1}\Delta Az(t) \\ &+ (C_{z1}B_0)^{-1}C_{z1}\Delta B(C_{z1}B_0)^{-1}\{C_{z0}(z-z_r) + C_{z1}A_0z(t)\} \\ &- (C_{z1}B_0)^{-1}C_{z1}(B_0 + \Delta B)\sum_{i=1}^n \beta_{1i}|z_i - z_{ri}|sgn(s) \\ &- (C_{z1}B_0)^{-1}C_{z1}(B_0 + \Delta B)\sum_{i=1}^n \beta_{2i}z_i \\ &- (C_{z1}B_0)^{-1}C_{z1}(B_0 + \Delta B)\beta_3sgn(s) \\ &= \Delta I(C_{z1}B_0)^{-1}C_{z0}(z-z_r) \\ &- (I + \Delta I)\sum_{i=1}^n \beta_{1i}|z_i - z_{ri}|sgn(s) \\ &+ (C_{z1}B_0)^{-1}C_{z1}\Delta Az(t) \\ &- \Delta I(C_{z1}B_0)^{-1}C_{z1}A_0z(t) - (I + \Delta I)\sum_{i=1}^n \beta_{2i}z_i \\ &+ (C_{z1}B_0)^{-1}C_{z1}Ap(t) - (I + \Delta I)\beta_ssgn(s) \end{split}$$

To stabilize the dynamics of the sliding surface in (28), the Assumption A1 is naturally necessary. By menas of the inequalities of gain (24), it can be easily shown that the derivative of the Lyapunov candidate function (25) and the existence condition of the sliding mode

$$s(z,t) \cdot \dot{s}(z,t) < 0 \tag{29}$$

is satisfied, which completes the proof.

From the above theorem, the control input in this paper can generate the sliding mode at every point on the modified transformed integral sliding surface from an arbitrary given initial point to an arbitrary given reference point. Therefore, the output trajectory by the proposed controller can be identical to that of the ideal sliding mode dynamics from a given initial state to a given reference point identically defined by the new sliding surface because of the insensitivity the controlled system to uncertain parameters and disturbances in the sliding mode of the VSS[31].

#### 3. Design Examples and Simulation Studies

Consider a following plant with uncertainties and disturbance

$$\begin{split} \dot{z}_1 &= (-2 + \Delta a_1) z_1(t) + (2 + \Delta b_1) u(t) + f(t) \\ \dot{z}_2 &= \Delta a_1 z_1(t) - 3 z_2(t) + (2 + \Delta b_2) u(t) + f(t) \\ y &= \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T \end{split} \tag{30}$$

where

$$\Delta a_1 = 3\sin(9t), \quad \Delta b_1 = \Delta b_2 = 0.3\cos(6t), \quad f(t) = 0.5\cos(7t)$$
  
 $|\Delta a_1| \le 3, \quad |\Delta b_1| = |\Delta b_2| \le 0.3, \quad |f(t)| \le 0.5$  . (31)

The MIVSC controller aims to drive the output of the plant (30) to any given  $y_r$  from any given initial state. In the steady state, the state should be  $x_r = \begin{bmatrix} 1 & 0 \end{bmatrix}^T y_r$  and  $z_r = \begin{bmatrix} 3/8 & 1/4 \end{bmatrix}^T y_r$  due to the steady state condition of (20). The transformation matrix to a controllable weak canonical form and the resultant transformed system matrices are

$$P = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}, \ \Lambda = \begin{bmatrix} 0 & 1 \\ -6 - 5 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$
 (32)

By means of Ackermanns formula, the continuous static gain is obtained

$$K=[1\ 0]$$
 and  $G=[3\ 1]$  (33) so that the closed loop eigenvalues of  $\varLambda_e$  are located at  $-3$  and  $-4$ . By the relationship (21) and (22), the coefficient matrices of the new modified transformed integral sliding surface directly becomes

$$\begin{split} &C_x = [\alpha_1 \quad \alpha_2] + G = [6 \quad 5] + 2[3 \quad 1] = [12 \quad 7] = C_{x0} \\ &C_{x0} = [12 \quad 7], \quad C_{x1} = [0 \quad 1] \\ &C_{z0} = C_{x0} P = [-2 \ 9], \quad C_{z1} = C_{x1} P = [-2 \ 3] \quad , \end{split} \tag{34}$$
 
$$C_{z1} B_0 = C_{x1} \Gamma = 2$$

As a result, the new transformed integral sliding surface becomes

$$\begin{split} s(z,t) = & -2/2 \int_{0}^{t} (z_{1} - z_{r_{1}}) dt \\ & + 2/2 (z_{r_{1}} - z_{1}(t_{0})) \\ & + 9/2 \int_{0}^{t} (z_{2} - z_{r_{2}}) dt \\ & - (3/2) (z_{r_{2}} - z_{r_{2}}) \\ & - 2/2 (z_{1} - z_{r_{1}}) + 3/2 (z_{2} - z_{r_{2}}) \end{split} \tag{36}$$

For the second design phase of the MIVSC, the equation (3) in the Assumption A1 is calculated

Thus the Assumption A1 is satisfied in this design. The inequalities for the switching gains in discontinuous input term, (24) becomes

$$\begin{split} \beta_{11} > & (2) \frac{0.075}{0.85} = 0.1765 \qquad , \\ \beta_{12} > & (9) \frac{0.075}{0.85} = 0.7941 \\ \beta_{21} = & \begin{cases} > \frac{1.8}{0.85} = 2.12 \text{ for } (sz_1) > 0 \\ < -\frac{1.8}{0.85} = -2.12 \text{ for } (sz_1) < 0 \end{cases} \end{split}$$

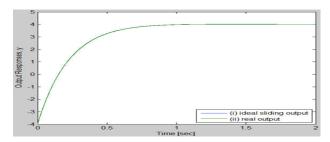


Fig. 1 Ideal and real output responses

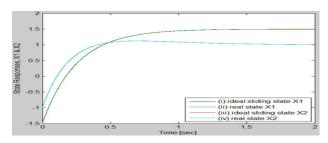


Fig. 2 Ideal and real state responses

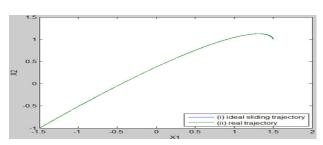


Fig. 3 Phase portrait

$$\begin{split} \beta_{22} = &\begin{cases} > \frac{0.675}{0.85} = 0.7941 \;\; \text{for} \;\; (sz_2) > 0 \\ < -\frac{0.675}{0.85} = -0.7941 \;\; \text{for} \;\; (sz_2) < 0 \end{cases} \\ \beta_3 > &\frac{0.25}{0.85} = 0.2941 \end{split} \end{split} \tag{38}$$

The selected control gains are

$$\begin{split} \beta_{11} &= 0.5, \quad \beta_{12} = 1 \\ \beta_{21} &= \begin{cases} 3.0 & \text{for } (sz_1) > 0 \\ -3.0 & \text{for } (sz_1) < 0 \end{cases} \\ \beta_{22} &= \begin{cases} 1.5 & \text{for } (sz_2) > 0 \\ -1.5 & \text{for } (sz_2) < 0 \end{cases} \\ \beta_{3} &= 2 \end{split}$$

and finally, the following control input is obtained to satisfy the existence condition of the sliding mode (29) as

$$\begin{split} u(z,t) = & -\frac{1}{2} \big\{ -2(z_1 - z_{r1}) + 9(z_2 - z_{r2}) + 4z_1 - 9z_2 \big\} \\ & -0.5|z_1 - z_{r1}|sgn(s) - |z_2 - z_{r2}|sgn(s) \\ & -\beta_{21}z_1 - \beta_{22}z_2 - 2sgn(s) \end{split} \tag{40}$$

The simulations are carried out with 0.1[msec] sampling time and Fortran software. Fig. 1 shows the ideal output and real output from an initial condition  $y(0) = -4 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} -1.5 - 1 \end{bmatrix}^T$  to a given command  $y_r = 4$ .

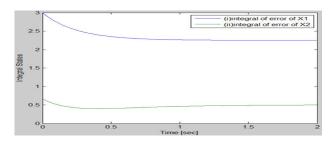


Fig. 4 Integral states

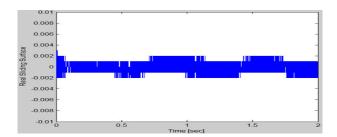


Fig. 5 Sliding surface

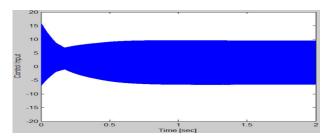


Fig. 6 Control input

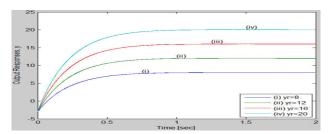


Fig. 7 Four outputs for different commands  $y_r = 8, \ 12,$   $16, \ {\rm and} \ 20$ 

The ideal and real state responses by the proposed SMC are shown in Fig. 2. As can be seen, the trajectories identically equal to those of the ideal sliding output. The phase portrait from  $[-1.5 \ 1]^T$  to  $[1.5 \ 1]^T$  is presented in Fig. 3. There is no reaching phase. The integral states of the output errors are depicted in Fig. 4. The fact of no reaching phase can be also found in Fig. 5 showing that the value of the new sliding surface chatters from the beginning without any reaching action. And this is fundamentally resulted from the switching of the

implemented control input from the initial time as shown in Fig. 6 as designed. Fig. 7 shows the output responses to the four different commands,  $y_r=8$ , 12, 16, and 20 . from an initial condition y(0)=-3.0 ..

#### 3. Conclusions

In this paper, an alternative design of an MIVSC is presented for the point-to-point regulation control of uncertain general linear systems under persistent disturbances. This algorithm basically concerns with the transformation of the integral sliding surface without the reaching phase and application to the point-to-point regulation control problem of uncertain non-canonical linear systems. To successfully remove the reaching phase problems, a sliding surface is augmented by an integral of the state error in order to define the hyper plane from any given initial condition and transformed by means of one transformation method in the invariant theorem of Utkin's. And for its design, the system is transformed to a weak canonical form and the ideal sliding dynamics is obtained in form of the nominal system. After choosing the desired output performance by means of the point-to-point regulation controller design with the nominal system, the coefficient of the integral sliding surface is determined effectively. A corresponding control input is also designed for completely guaranteeing the ideal sliding output in spite of the uncertainties and disturbances. The robustness of the ideal sliding output itself is proved under all the persistent disturbances in Theorem 1 together with the existence condition of the sliding mode of the MIVSC and the asymptotic stability of the proposed MIVSC. Therefore, the designed controller can drive uncertain systems to any arbitrary desired value with the predetermined identical sliding output as designed in the sliding surface. The two design concepts of the performance and robustness are perfectly separated in the suggested point-to-point regulation MIVSC. In the point-to-point regulation control area, the robustness problem is completely solved. Through simulation studies, the usefulness of the proposed controller is verified.

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## 저 자 소 개



#### 이 정 훈 (李 政 勳)

1966년 2월 1일생. 1988년 경북대학교 전자공학과 졸업(공학사), 1990년 한국과학기술원 전기 및 전자공학과 졸업(석사). 1995년 한국과학기술원 전기 및 전자공학과 졸업(공박). 1995-현재 경상대학교공과대학 제어계측공학과 교수.

1997-1999 경상대학교 제어계측공학과 학과장. 마르퀘스 사의 Who's Who in the world 2000년 판에 등재. American Biographical Institute(ABI)의 500 Leaders of Influence에 선정.

Tel: +82-55-772-1742 Fax: +82-55-772-1749 E-mail: jhleew@gnu.ac.kr