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구간시변 지연을 가지는 비선형시스템의 H_∞ 필터링

H_∞ Filtering for a Class of Nonlinear Systems with Interval Time-varying Delay

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Abstract - In this paper, a delay-dependent H_∞ filtering problem is investigated for discrete-time delayed nonlinear systems which include a more general sector nonlinear function instead of employing the commonly used Lipschitz-type function. By using the Lyapunov-Krasovskii functional approach, a less conservative sufficient condition is established for the existence of the desired filter, and then, the corresponding solvability condition guarantee the stability of the filter with a prescribed H_∞ performance level. Finally, two simulation examples are given to show the effectiveness of the proposed filtering scheme.

Key Words : Discrete-time nonlinear system, Robust filtering, General sector nonlinear function, Stability, Interval time-varying delay

1. Introduction

During the past decades, the filter design problem has been widely studied due to its extensive applications in control systems and signal processing. The purpose of the filtering design is to estimate the unavailable state variables of a given system through noisy measurements. There are basically two approaches to the problem: the Kalman filtering approach and the H_∞ filtering approach. The Kalman filtering approach is based on the assumption that a linear system model is required and all noise terms and measurements have Gaussian distributions [1]. However, the prior knowledge of noise may not always be precisely known. In order to overcome this problem, H_∞ filtering method, which provides both a guaranteed noise attenuation level and robustness against unmodeled dynamics, has been proposed. Compared with the Kalman filtering method, the main advantage of H_∞ filtering method is that the noise sources are supposed to be arbitrary signal with bounded energy, and no exact statistics are required to be known. On the other hand, since time delay is commonly encountered in various engineering systems and is frequently a source of instability and poor performance, the problem of H_∞ filtering for time delay systems have been received increasing attention in the last decades[2-17].

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It is well-known that the main objective of H_∞ filtering is to design a suitable filter such that the bound of induced L_2 norm of the operator from the noise signals to the filtering error is less than a prescribed level. In order to get less conservative results, that is, obtain a smaller H_∞ disturbance attenuation lever γ, various methods are utilized. For example, in [3], a new finite sum inequality was employed to get a sufficient condition for the existence of a suitable filter. In [8], the input-output(IO) approach is used to get the less conservative result than [3]. But some delay terms are neglected for estimating the derivative bound of the constructed Lyapunov functional in the IO approach. Therefore, the results in [8] are conservative to some extent, and there is much room to improve the result in [8]. Moreover, it should be noted that the results in [2-9] are considered for only linear system.

It is well known that nonlinearities exist universally in practical systems, so the H_∞ filtering problem for nonlinear dynamical system have been investigated by many researchers [10-17]. Xu [15] was concerned with the problem of robust H_∞ filtering for a class of discrete-time nonlinear systems with state delay and norm-bounded parameter uncertainty. In [11], a stable full or reduced order filter with the same repeated scalar nonlinearities was designed to guarantee the induced L_2 or generalized H_∞ performance. In [14], the problem of H_∞ filtering for systems with repeated scalar nonlinearities under unreliable communication was investigated. In [15], a robust H_∞ filtering problem for a class of discrete-time nonlinear systems was considered. But it should be pointed out that the time delay was not taken into consideration in [11,14,15], and only constant time delay was considered in [10]. To the best of our knowledge, there are few results on the problem of the H_∞ filtering for a nonlinear system with time-varying delay.

In this paper, we consider the H_∞ filtering problem for a class of discrete-time systems involving sector nonlinearities and interval time-varying delay. Inspired by the work [13], the sector nonlinearities considered in the paper are more general than usual Lipschitz conditions. By using a new Lyapunov functional and Linear matrix inequality technique, delay dependent conditions are obtained for designing a filter with an H_∞ disturbance attenuation level γ. When the involved LMIs are feasible, a set of the parameters of a desired filter can be obtained. Two numerical examples are provided to show the usefulness and effectiveness of the proposed design method.

Notation: Throughout the paper, R^n denotes the n-dimensional Euclidean space, $\mathrm{R}^{m\times n}$ denotes the set of m by n real matrix. For symmetric matrices X, $X>0$ and $X<0$, mean that X is a positive/negative definite symmetric matrix, respectively. I and 0 denote the identity matrix and zero matrix with appropriate dimension. ★ represents the elements below the main diagonal of a symmetric matrix. $diag...$ denotes the diagonal matrix. $\|.\|$ refers to the induced matrix 2-norm. L_2 means the space of square integral vector functions on $[0,\infty)$ with norm

$$\|.\|_2=\left(\int_0^\infty \|.\|^2 dt\right)^2.$$

2. Problem Statements

Consider a discrete-time nonlinear system with time-varying delay and disturbance:

$$\begin{aligned} x(k+1) &= Ax(k)+A_dx(k-d(k))+Ff(x(k))+B_w w(k), \\ y(k) &= Cx(k)+C_dx(k-d(k))+H_1h(x(k)) \\ &\quad +H_2h(x-d(k))+D_w w(k), \\ z(k) &= Lx(k)+L_dx(k-d(k))+G_w w(k), \end{aligned} \tag{1}$$

where $x(k)\in\mathrm{R}^n$ is the state, $w(k)\in\mathrm{R}^q$ is a disturbance input belongs to $L_2[0,\infty)$, $y(k)\in\mathrm{R}^m$ is the signal to be estimated, $f(.)$ and $h(.)$ are known vector-valued nonlinear functions $A, A_{d,} F, B_w, C, C_d, H_1, H_2, D_w, L, L_d, G_w$ are known constant matrices of appropriate dimensions, and $d(k)$ is the time varying delay satisfying

$$d_1\le \mathrm{d(k)}\le \mathrm{d}_2$$

where $d_1>0$ and $d_2>0$ demote the lower and upper bounds of the delay, respectively.

In this paper, without loss of generality, we always assume that $f(0)=0, h(0)=0$ and for vector-valued functions f, h, we assume

$$[f(x)-f(y)-U_1(x-y)][f(x)-f(y)-U_2(x-y)]\le 0, x,y\in R^n, \tag{2}$$

$$[h(x)-h(y)-V_1(x-y)][h(x)-h(y)-V_2(x-y)]\le 0, x,y\in R^m, \tag{3}$$

where U_1, U_2, V_1, V_2 are known real constant matrices, and $U_2 - U_1, V_2-V_1$ are positive definite matrices.

Remark 1 Eq. (2) and (3) are the so-called sector-bounded conditions [18], which are more general than the Lipschitz conditions, and have been widely adopted in the literature [19-20]. The reason is that if we use the Lipschitz condition, the matrix U_1, U_2, V_1, V_2 are diagonal matrix, it is a special case included in our considered condition.

The objective of this paper is to estimate the system states $x(k)$. In this paper, we consider a full-order linear asymptotically stable filter for system (1) with state-space realization of the form

$$\begin{aligned} \tilde{x}(k+1) &= A_F\tilde{x}(k)+B_Fy(k), \\ \tilde{z}(k) &= C_F\tilde{x}(k)+D_Fy(k), \tilde{x}(0)=0. \end{aligned} \tag{4}$$

where $\tilde{x}(k)$ is the filter state vector and $A_F B_{F,} C_F D_F$ are appropriately dimensioned filter gains to be determined.

Denote

$$\zeta(k)=\begin{bmatrix}\mathrm{x(k)}\\ \tilde{\mathrm{x}}\mathrm{(k)}\end{bmatrix},$$
$$e(k)=z(k)-\tilde{z}(k),$$

Then the following error system is obtained

$$\begin{aligned} \zeta(k+1) &= \overline{A}\zeta(k)+\overline{A_d}\zeta(k-d(k))+\overline{F}f(x(k))+\overline{H_1}h(x(k)) \\ &\quad +\overline{H_2}h(x(k-d(k))))+\overline{B_w}w(k), \\ e(k) &= \overline{C}\zeta(k)+\overline{C_d}\zeta(k-d(k))+\overline{D_1}h(x(k)) \\ &\quad +\overline{D_2}h(x(k-d(k)))+\overline{D_w}w(k) \end{aligned} \tag{5}$$

where

$$\overline{A}=\begin{bmatrix}A & 0\\ B_FC & A_F\end{bmatrix}, \overline{A_d}=\begin{bmatrix}A & 0\\ B_FC & A_F\end{bmatrix}, \overline{F}=\begin{bmatrix}F\\ 0\end{bmatrix},$$
$$\overline{H_1}=\begin{bmatrix}0\\ B_FH_1\end{bmatrix}, \overline{H_2}=\begin{bmatrix}0\\ B_FH_2\end{bmatrix}, \overline{B_w}=\begin{bmatrix}B_w\\ B_FD_w\end{bmatrix}$$
$$\overline{C}=[L-D_FC-C_F],\ \overline{C_d}=[L_d-D_FC_d\ 0].$$

The aim of this paper is to design the H_∞ filter satisfying that the filtering error system (5) with $w(k)=0$ is asymptotically stable and H_∞ performance

$$J=\sum_{k=0}^{\infty}\{e^T(k)e(k)-\gamma^2 w^T(k)w(k)\}<0 \tag{6}$$

is guaranteed under zero-initial conditions.

Lemma 1 [5] For any matrix $M>0$, integers γ_1 and γ_2 satisfying $\gamma_2>\gamma_1$, and vector function $w: N[\gamma_1,\gamma_2]\to R^n$ such that the sums concerned are well defined, then

$$-(\gamma_2-\gamma_1-1)\sum_{\alpha=\gamma_1}^{\gamma_2} a_2 w^T(\alpha)w(\alpha)\le\sum_{\alpha=\gamma_1}^{\gamma_2} w^T(\alpha)M\sum_{\alpha=\gamma_1}^{\gamma_2} w(\alpha)$$

Lemma 2 [5] For any matrix $\begin{bmatrix} M & S \\ \star & M\end{bmatrix}\ge 0$, scalars $\alpha_1(k)>0, \alpha_2(k)>0$ satisfying $\alpha_1(k)+\alpha_2(k)=1$, vector functions $\delta_1(k)$ and $\delta_2(k): N\to R^n$, the following inequality holds

$$-\frac{1}{\alpha_1(k)}\delta_1^T(k)M\delta_1(k)-\frac{1}{\alpha_2(k)}\delta_2^T(k)M\delta_2(k)$$
$$\le\begin{bmatrix}\delta_1(k)\\ \delta_2(k)\end{bmatrix}^T\begin{bmatrix} M & S\\ \star & M\end{bmatrix}\begin{bmatrix}\delta_1(k)\\ \delta_2(k)\end{bmatrix}$$

3. Main Results

In this section, first of all, let us give a sufficient condition, which ensure system (1) to be asymptotically stable with H_∞ performance level γ. In convenience, we define

$$\delta(k)=x(k+1)-x(k),\ \ I_1=[I 0],$$
$$\eta^T(k)=[\zeta^T(k)\ \ x^T(k-d_1)\ \ \zeta^T(k-d(k))\ \ x^T(k-d_2)$$
$$f^T(x(k))\ \ h^T(x(k))\ \ h^T(x(k-d(k)))\ \ w^T(k)],$$
$$Y=\left[I_1^T(A-I)^T 0\ \ I_1^T A_d^T\ \ 0\ \ F^T 0\ \ B_w^T\right]^T,$$
$$R=d_L^2 R_1+(d_H-d_L)^2 R_2,$$
$$\overline{U_1}=\frac{U_1^T U_2+U_2^T U_1}{2},\ \overline{U_2}=-\frac{U_1^T+U_2^T}{2},$$
$$\overline{V_1}=\frac{V_1^T V_2+V_2^T V_1}{2},\ \overline{V_2}=-\frac{V_1^T+V_2^T}{2},$$
$$\overline{W_1}=\frac{W_1^T W_2+W_2^T W_1}{2},\ \overline{U_2}=-\frac{W_1^T+W_2^T}{2},$$
$$\Omega_{11}=-P-I_1^T R_1 I_1+Q_1+I_1^T Q_2 I_1-I_1^T\overline{U_1}I_1-I_1^T\overline{V_1}I_1,$$
$$\Omega_{33}=-Q_1-2I_1^T R_1 I_1+I_1^T T_1 I_1+(I_1^T T_1 I_1)^T-2I_1^T R_2 I_1$$
$$+I_1^T T_2 I_1+(I_1^T T_2 I_1)^T-I_1^T\overline{W_1}I_1,$$
$$\Omega_{34}=I_1^T R_1-I_1^T T_1+I_1^T R_2-I_1^T T_2,$$
$$\Omega_{44}=-R_1-R_2-Q_2,$$

$$\Omega=\begin{bmatrix}
\Omega_{11} & 0 & I_1^T R_1 I_1-I_1^T T_1 I_1 & I_1^T T_1 & -I_1^T\overline{U_2} & -I_1^T\overline{V_2} & -I_1^T\overline{W_2} & 0\\
\star & -R_2 & R_2 I_1-T_2 I_1 & T_2 & 0 & 0 & 0 & 0\\
\star & \star & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0\\
\star & \star & \star & \Omega_{44} & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -I & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -I & 0 & 0\\
\star & \star & \star & \star & \star & \star & -I & 0\\
\star & \star & \star & \star & \star & \star & \star & -\gamma^2 I
\end{bmatrix},$$

$$\overline{\Omega}=\Omega+\Gamma^T R\Gamma,$$
$$\Phi=[A\ 0\ A_d\ 0\ \overline{F}\ \overline{H_1}\ \overline{H_2}\ \overline{B_w}],$$
$$\Psi=[C\ 0\ C_d\ 0\ 0\ \overline{D_1}\ \overline{D_2}\ \overline{D_w}],$$
$$\Xi=\begin{bmatrix} ZA+\overline{B}C & \overline{A}\ 0 & ZA_d+\overline{B}C_d & 0\ 0 & ZF & \overline{B}H_1 & \overline{B}H_2 & ZB_w+\overline{B}D_w\\ M^TA+\overline{B}C & \overline{A}\ 0 & M^TA_d+\overline{B}C_d & 0\ 0 & ZF & \overline{B}H_1 & \overline{B}H_2 & M^TB_w+\overline{B}D_w\end{bmatrix},$$
$$V=\begin{bmatrix} Z & M\\ \star & M\end{bmatrix}\ge 0.$$

The following theorem provides a sufficient condition, which ensures the system (5) to be asymptotically stable with H_∞ performance γ.

Theorem 1. *For given* $d_2>d_1>0$, $\gamma>0$ and *matrix* $U_1, U_2, V_1, V_2, W_1, W_2$, *the nonlinear filtering error system (5) is asymptotically stable with* H_∞ *performance* γ, *if there exist positive definite symmetric matrix* P, Q_1, Q_2, R_1, R_2 *and appropriate dimension matrix* $\overline{A}, \overline{B}, \overline{C}, \overline{D}, T_1, T_2$, *satisfying the following LMIs*

$$\begin{bmatrix}\Omega & \Gamma^{\mathrm{T}}\mathrm{R} & \Xi^{\mathrm{T}} & \Psi^{\mathrm{T}}\\ \star & -\mathrm{R} & 0 & 0\\ \star & \star & \mathrm{P}-\mathrm{V}-\mathrm{V}^{\mathrm{T}} & 0\\ \star & \star & \star & -\mathrm{I}\end{bmatrix}<0 \tag{7}$$

$$\begin{bmatrix} R_1 & T_1\\ \star & R_1\end{bmatrix}\ge 0,\ \begin{bmatrix} R_2 & T_2\\ \star & R_2\end{bmatrix}\ge 0, \tag{8}$$

Proof. Consider the following L-K functional candidate as

$$V(k)=V_1(k)+V_2(k)+V_3(k)+V_4(k), \tag{9}$$

where

$$V_1(k)=\zeta^T(k)P\zeta(k),$$
$$V_2(k)=\sum_{s=k-d(k)}^{k-1}\zeta^T(k)Q_1\zeta(k),$$
$$V_3(k)=\sum_{s=k-d_2}^{k-1}x^T(k)Q_2x(k),$$
$$V_4(k)=d_2\sum_{s=1-d_2}^{0}\sum_{l=k-1+s}^{k-1}\delta^T(l)R_1\delta(l),$$
$$V_5(k)=(d_2-d_1)\sum_{s=1-d_2}^{-d_1}\sum_{l=k-1+s}^{k-1}\delta^T(l)R_2\delta(l),$$

with the cost function (6).

Calculating the difference of $V_1(k), V_2(k)$ and $V_3(k)$, we have

$$\Delta V_1(k)=V_1(k+1)-V_1(k)=\zeta(k+1)^T P\zeta(k+1)-\zeta(k)^T P\zeta(k)$$
$$=\begin{bmatrix}\zeta(k)\\ \zeta(k-d(k))\\ f(x(k))\\ h(x(k))\\ h(x(k-d(k))\\ w(k)\end{bmatrix}^T\begin{bmatrix}\overline{A^T}\\ \overline{A_d^T}\\ \overline{F^T}\\ \overline{H_1^T}\\ \overline{H_2^T}\\ \overline{B_w^T}\end{bmatrix}P[\overline{A}\ \overline{A_d}\ \overline{F}\ \overline{H_1}\ \overline{H_2}\ \overline{B_w}]\begin{bmatrix}\zeta(k)\\ \zeta(k-d(k))\\ f(x(k))\\ h(x(k))\\ h(x(k-d(k))\\ w(k)\end{bmatrix}$$
$$-\zeta^T(k)P\zeta(k)=\eta^T(k)\Phi^T(k)P\Phi\eta(k)-\zeta^T(k)P\zeta(k) \tag{10}$$
$$\Delta V_2(k)=\zeta^T(k)Q_1\zeta(k)-\zeta^T(k-d(k))Q_1\zeta(k-d(k)), \tag{11}$$
$$\Delta V_3(k)=x^T(k)Q_2x(k)-x^T(k-d_2)Q_1x(k-d_2). \tag{12}$$

Since

$$\sum_{i=k-d_2}^{k-1}\delta(i)=x(k)-x(k-d_2), \tag{13}$$

an upper bound of the difference of $V_4(k)$ is obtained from Lemma 2

$$\begin{aligned}
\Delta V_4(k) &= d_2^2\delta^T(k)R_1\delta(k)-d_2\sum_{l=k-d_H}^{k-1}\delta^T(l)R_1\delta(l)\\
&= d_2^2\delta^T(k)R_1\delta(k)-d_2\sum_{l=k-d_2}^{k-d(k)-1}\delta^T(l)R_1\delta(l)-d_2\sum_{l=k-d(k)}^{k-1}\delta^T(l)R_1\delta(l)\\
&= d_2^2\delta^T(k)R_1\delta(k)-\frac{d_2}{d_2-d(k)}\sum_{l=k-d_2}^{k-d(k)-1}\delta^T(l)R_1\delta(l)\\
&\quad -\frac{d_2}{d(k)}\sum_{l=k-d(k)}^{k-1}\delta^T(l)R_1\delta(l)\\
&\le d_2^2\delta^T(k)R_1\delta(l)\\
&\quad -\begin{bmatrix}x(k)-x(k-d(k))\\ x(k-d(k))-x(k-d_2)\end{bmatrix}^T\begin{bmatrix}R_1 & T_1\\ \star & R_1\end{bmatrix}\begin{bmatrix}x(k)-x(k-d(k))\\ x(k-d(k))-x(k-d_2)\end{bmatrix}\\
&= d_2^2\delta^T(k)R_1\delta(k)-\begin{bmatrix}\zeta(k)\\ \zeta(k-d(k))\\ x(k-d_2)\end{bmatrix}^T\Pi_1\begin{bmatrix}\zeta(k)\\ \zeta(k-d(k))\\ x(k-d_2)\end{bmatrix},
\end{aligned} \tag{14}$$

where

$$\Pi_1=\begin{bmatrix} I_1^TR_1I_1 & -I_1^TR_1I_1+I_1^TT_1I_1 & -I_1^TT_1\\ \star & 2I_1^{\,T}R_1I_1-I_1^TT_1I_1-(I_1^TT_1I_1)^T & -I_1^{T^T}R_1+I_1^TT_1\\ \star & \star & R_1\end{bmatrix}$$

Note that if $d(k)=0$ or $d(k)=d_2$, we have $x(k)-x(k-d(k))=0$ or $x(k-d(k))-x(k-d_2)=0$, respectively. Thus Eq.(14) holds based on Lemma 1.

Similar to Eq. (11), the difference of $V_5(k)$ is

$$\begin{aligned}
\Delta V_5(k) &= (d_2-d_1)_2^2\delta^T(k)R_2\delta(k)-(d_2-d_1)_2\sum_{l=k-d_2}^{k-d_1-1}\delta^T(l)R_2\delta(l)\\
&\le (d_2-d_1)^2\delta^T(k)R_2\delta(k)-\begin{bmatrix}x(k-d_1)\\ \zeta(k-d(k))\\ x(k-d_2)\end{bmatrix}^T\Pi_1\begin{bmatrix}x(k-d_1)\\ \zeta(k-d(k))\\ x(k-d_2)\end{bmatrix},
\end{aligned} \tag{15}$$

where

$$\Pi_2=\begin{bmatrix} R_2 & -R_2I_1+T_2I_1 & -T_2\\ \star & 2I_1^{\,T}R_2I_1-I_1^TT_2I_1-(I_1^TT_2I_1)^T & -I_1^{T^T}R_2+I_1^TT_2\\ \star & \star & R_2\end{bmatrix}$$

Note that if $d(k)=d_1$ or $d(k)=d_2$, we have $x(k-d_1)-x(k-d(k))=0$ or $x(k-d(k))-x(k-d_2)=0$, respectively. Thus, Eq. (15) holds based on Lemma 1.

It follows readily from Eq. (2) and Eq. (3) that

$$\begin{bmatrix}\zeta(k)\\ f(x(k))\end{bmatrix}^T\begin{bmatrix}-I_1^T\overline{U_1}I_1 & -I_1^T\overline{U_2}\\ \star & -I\end{bmatrix}\begin{bmatrix}\zeta(k)\\ f(x(k))\end{bmatrix}\ge 0, \tag{16}$$

$$\begin{bmatrix}\zeta(k)\\ h(x(k))\end{bmatrix}^T\begin{bmatrix}-I_1^T\overline{V_1}I_1 & -I_1^T\overline{V_2}\\ \star & -I\end{bmatrix}\begin{bmatrix}\zeta(k)\\ h(x(k))\end{bmatrix}\ge 0, \tag{17}$$

$$\begin{bmatrix}\zeta(k-d(k))\\ h(x(k-d(k)))\end{bmatrix}^T\begin{bmatrix}-I_1^T\overline{W_1}I_1 & -I_1^T\overline{W_2}\\ \star & -I\end{bmatrix}\begin{bmatrix}\zeta(k-d(k))\\ h(x(k-d(k)))\end{bmatrix}\ge 0, \tag{18}$$

To establish the H_∞ performance for the filtering error system (5), if the difference of $V(k)$ is negative, then $z(k)$ goes to zero as $k\to\infty$. Next, assuming zero initial conditions for the filtering error system, the performance index is

$$\begin{aligned}
J &= \sum_{k=0}^{\infty}\{\mathrm{e(k)^Te(k)}-\gamma^2\mathrm{w(k)^Tw(k)}\}\\
&= \sum_{k=0}^{\infty}\{\mathrm{e(k)^Te(k)}-\gamma^2\mathrm{w(k)^Tw(k)}+\Delta\mathrm{V(k)}\}+\mathrm{V(0)}-\mathrm{V}(\infty)
\end{aligned}$$

If the inequality $e(k)^Te(k)-\gamma^2w(k)^Tw(k)+\Delta V(k)<0$ holds, then $V(k)$ goes to zero as $k\to\infty$.

Combining with Eq. (10)-(18), one can obtain

$$\begin{aligned}
&\Delta V(k)+e(k)^Te(k)-\gamma^2w(k)^Tw(k)\\
&\quad\le \eta^T(k)(\Omega+\Phi^TP\Phi+\Psi^T\Psi)\eta(\mathrm{k}).
\end{aligned} \tag{19}$$

By Schur complement, inequality (19) is equivalent to

$$\begin{bmatrix}\Omega & \Gamma^TR & \Phi^T & \Psi^T\\ \star & -R & 0 & 0\\ \star & \star & -P^{-1} & 0\\ \star & \star & \star & -I\end{bmatrix}<0 \tag{20}$$

In the inequality (20), the positive-definite matrix P and the filter parameters A_f, B_f, C_f, D_f, which included in the matrix $\overline{A},\overline{B},\overline{C}$ are unknown. Hence it should be converted to LMI via proper variable substitution method.

Let us define V as

$$V=\begin{bmatrix}Z & M\\ \star & M\end{bmatrix},$$

For positive definite matrix P^{-1} and nonzero matrix V, it follows that

$$(V-P)P^{-1}(V^T-P)=-V-V^T+VP^{-1}V^T+P>0. \tag{21}$$

Combined with the Eq. (21), pre and post multiplying the matrix inequality (20) by the matrix $diag\{\mathrm{I,I,V,I}\}$ and $diag\{\mathrm{I,I,V^T,I}\}$, then one can get the following inequality

$$\begin{bmatrix}\Omega & \Gamma^TR & \Phi^TV^T & \Psi^T\\ \star & -R & 0 & 0\\ \star & \star & P-V-V^T & 0\\ \star & \star & \star & -I\end{bmatrix}<0 \tag{22}$$

By simple matrix calculation, it is straightforward to verify that

$$V\overline{A}=\begin{bmatrix}ZA+MB_FC & MA_F\\ M^TA+MB_FC & MA_F\end{bmatrix},\ V\overline{A_d}=\begin{bmatrix}ZA_d+MB_FC_d & 0\\ M^TA & 0\end{bmatrix},$$

$$V\overline{F}=\begin{bmatrix}ZF\\ M^TF\end{bmatrix},\ V\overline{H_1}=\begin{bmatrix}B_FMH_1\\ B_FMH_1\end{bmatrix},$$

$$V\overline{H_2}=\begin{bmatrix}B_FMH_2\\ B_FMH_2\end{bmatrix},\ V\overline{B_w}=\begin{bmatrix}ZB_w+MB_FD_w\\ M^TB_w+MB_FD_w\end{bmatrix}$$

Now, define a new set of variables as follows

$\overline{A}=MA_F, \overline{B}=MB_F, \overline{C}=C_F, \overline{D}=D_F.$

Note that the inequality (8) implies that $J<0$ for any nonzero $w(k)\in L_2$, i.e., the filtering error system has a guaranteed γ level of disturbance attenuation. This completes the proof. ■

Remark 2. If $F=H_1=H_2=0$, the system (1) is reduced to the following linear system:

$$\begin{aligned} x(k+1) &= Ax(k)+A_d x(k-d(k))+B_w w(k), \\ y(k) &= Cx(k)+C_d x(k-d(k))+D_w w(k), \\ z(k) &= Lx(k)+L_d x(k-d(k))+G_w w(k), \end{aligned} \tag{23}$$

Combining the same filter system with Eq. (4), the corresponding error system is obtained as following

$$\begin{aligned} \zeta(k+1) &= \overline{A}\zeta(k)+\overline{A_d}\zeta(k-d(k))+\overline{B_w}w(k), \\ e(k) &= \overline{C}\zeta(k)+\overline{C_d}\zeta(k-d(k))+\overline{D_w}w(k) \end{aligned} \tag{24}$$

In convenience, we define

$$\widehat{\eta}^T(k)=[\zeta^T(k)\ \ x^T(k-d_1)\ \ \zeta^T(k-d(k))\ \ x^T(k-d_2)\ \ \ w^T(k)],$$

$$\hat{Y}=\left[I_1^T(A-I)^T 0\ \ I_1^T A_d^T\ \ 0\ \ B_w^T\right]^T,$$

$$\hat{\Omega}=\begin{bmatrix} \widehat{\Omega_{11}} & 0 & I_1^T R_1 I_1 - I_1^T T_1 I_1 & I_1^T T_1 & 0 \\ \star & -R_2 & R_2 I_1 - T_2 I_1 & T_2 & 0 \\ \star & \star & \Omega_{33} & \Omega_{34} & 0 \\ \star & \star & \star & -R_1-R_2-Q_2 & 0 \\ \star & \star & \star & \star & -\gamma^2 I \end{bmatrix}$$

$$\widehat{\Omega_{11}}=-P-I_1^T R_1 I_1+Q_1+I_1^T Q_2 I_1,$$

Based on Theorem 1, the following Corollary provides a sufficient condition, which ensures the system (24) to be asymptotically stable with H_∞ performance γ.

Corollary 1. *For given $d_2>d_1>0$, $\gamma>0$, the linear filtering error system (24) is asymptotically stable with H_∞ performance γ, if there exist positive definite symmetric matrix P, Q_1, Q_2, R_1, R_2 and appropriate dimension matrix $\overline{A}, \overline{B}, \overline{C}, \overline{D}, T_1, T_2$, satisfying the following LMIs*

$$\begin{bmatrix} \hat{\Omega} & \widehat{\Gamma}^T R & \widehat{\Xi}^T & \widehat{\Psi}^T \\ \star & -R & 0 & 0 \\ \star & \star & P-V-V^T & 0 \\ \star & \star & \star & -I \end{bmatrix}<0 \tag{25}$$

$$\begin{bmatrix} R_1 & T_1 \\ \star & R_1 \end{bmatrix}\geq 0, \begin{bmatrix} R_2 & T_2 \\ \star & R_2 \end{bmatrix}\geq 0, \tag{26}$$

Remark 3. For any solutions of the LMIs (7}-(8) and LMIs (25)-(26) in Theorem 1 and Corollary 1, respectively, a corresponding filter of the form (4} can be reconstructed from the relations

$$A_F=M^{-1}\overline{A}, B_F=M^{-1}B, C_F=\overline{C}, D_F=\overline{D}.$$

Remark 4. In [13], the time-varying delay term $x(k-d(k))$ was not considered for estimating the bound of $-(d_2-d_1)\sum_{l=k-d_2}^{k-d_1-1}\delta^T(l)R_2\delta(l)$. In order to obtain a less conservative result, Lemma 2 is applied by using the time-varying delay term $x(k-d(k))$ in the Eq. (14) and (15). The following examples will be given to demonstrate the effectiveness of this method.

4. Numerical Examples

In this section, two examples are given to show the effectiveness of our method on the design of the robust H_∞ filter.

Example 1 Consider the following simplified longitudinal flight system [10]:

$$x_1(k+1)=0.9944x_1(k)-0.1203x_2(k)-0.4302x_3(k), \tag{27}$$

$$\begin{aligned} x_2(k+2) &= 0.0017x_1(k)+0.9902x_2(k) \\ &\quad -(0.0747+0.01\sin(x_1(k)))x_3(k), \end{aligned} \tag{28}$$

$$x_3(k+1)=0.8187x_2(k)+0.1w(k), \tag{29}$$

The measurement signal and signal to be estimated are

$$\begin{aligned} y(k) &= 0.2x_1(k)+0.1x_2(k)+(0.1+0.01\sin(x_1(k))x_3(k) \\ &\quad +0.1x_1(k-d(k))+(0.1+0.01\sin(x_1(k)))x_2(k-d(k)) \\ &\quad +0.02\cos(x_1(k))x_2(k-d(k))+0.04\sin(x_2(k))x_3(k)+0.1w(k), \end{aligned} \tag{30}$$

$$z(k)=0.1x_2(k)+0.2x_3(k)+0.1x_1(k-d(k))+0.2x_2(k-d(k)). \tag{31}$$

It is easy to see that the system described by Eqs. (27)-(31) which is satisfied Eqs. (2) and (3) and has the form (1) with

$$A=\begin{bmatrix} 0.9944 & -0.1203 & -0.4302 \\ 0.0017 & 0.9902 & -0.0747 \\ 0 & 0.8187 & 0 \end{bmatrix}, A_d=0, F=1, B_w=\begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix},$$

$$C=[0.2\ \ 0.1\ \ 0.1], C_d=[0.1\ \ 0.1\ \ 0], H_1=H_2=[1\ \ 1\ \ 1], D_w=0.1$$

$$L=[0\ \ 0.1\ \ 0.2], L_d=[0.1\ \ 0.2\ \ 0], G_w=0,$$

$$U_1=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -0.01 \\ 0 & 0 & 0 \end{bmatrix}, U_2=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.01 \\ 0 & 0 & 0 \end{bmatrix}, V_1=\begin{bmatrix} 0 & 0 & -0.01 \\ 0 & 0 & 0 \\ 0 & 0 & -0.04 \end{bmatrix},$$

$$V_2=\begin{bmatrix} 0 & 0 & 0.01 \\ 0 & 0 & 0 \\ 0 & 0 & 0.04 \end{bmatrix}, W_1=\begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.01 & 0 \\ 0 & -0.02 & 0 \end{bmatrix}, W_2=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0.02 & 0 \end{bmatrix}.$$

When $d_1=1, d_2=5, \gamma=0.0670$, applying Theorem 1 to above system by utilizing MATLAB(with YALMIP 3.0 an SeDuMi 1.3), the corresponding parameter of the filter gains (Remark 3) are obtained by

$$A_F=\begin{bmatrix} 0.0363 & -0.0348 & -1.6053 \\ 0.1602 & 0.7897 & 0.1834 \\ 0.0558 & 0.2915 & 0.1052 \end{bmatrix}, B_F=\begin{bmatrix} -3.3934 \\ 0.5590 \\ 0.2407 \end{bmatrix},$$

$$C_F=[-0.0640\ \ -0.3102\ \ -0.1543], D_F=0.0956.$$

Fig. 1 shows that the output $z(k)$ and its estimated output $\hat{z}(k)$ under the initial condition $z(k)=[1.5\ -1\ 1.5]^T$, $\hat{z}(k)=[-0.5\ 0\ 0.5]^T$ respectively. Also, the estimated error $e(k)$ is described in Fig. 2, where the external disturbance $w(k)=\sin(k)e^{-10k}$. From these simulation results, we can see that the disturbance is effectively attenuated by designed H_∞ filter for the discrete-time system (1) with nonlinearities and time-varying delay.

Furthermore, when $d_1=d_2=2$, the time delay become constant delay, we can obtain the minimal H_∞ performance lever $\gamma=0.0456$, which is less conservative than 1.5 derived in [10].

Example 2. Consider the system (23) with the following parameters:

$$A=\begin{bmatrix}0.85 & 0.1\\ -0.1 & 0.7\end{bmatrix}, A_d=\begin{bmatrix}0.2 & 0\\ -0.2 & 0.1\end{bmatrix}, B_w=\begin{bmatrix}0.1\\ 0.4\end{bmatrix},$$
$$C=[0.2\ \ 2.5], C_d=[-0.5\ \ 0.5], D_w=1,$$
$$L=[0\ \ 2.2], L_d=[1.5\ \ -0.4], G_w=-0.1.$$

The allowable H_∞ performance lever γ obtained by different methods is depicted in Table 1. When $d_1=1, d_2=4$, we can see that the minimal H_∞ performance lever γ is 3.8447, which is much small than 4.9431. It means that the obtained result in this paper is less conservative than the one derived in [13]. Solving the LMIs (25) and (26) by using the MATLAB(with YALMIP 3.0 an SeDuMi 1.3), the corresponding parameter of the filter gains (Remark 3) are given by

$$A_F=\begin{bmatrix}0.3727 & 2.0729\\ 0.0121 & 0.0416\end{bmatrix}, B_F=\begin{bmatrix}0.9248\\ -0.3347\end{bmatrix},$$
$$C_F=[-0.7806\ \ -4.1024], D_F=0.7745.$$

Table 1 Comparison of H_∞ performance γ

$d(k)$	$d_1=1$, $d_2=4$	$d_1=1$, $d_2=5$	$d_1=2$, $d_2=5$	$d_1=2$, $d_2=6$
[3]	5.0782	6.4910	5.5151	7,0533
[13]	4.9431	6.1604	5.3551	6.7581
Corollary 1	3.8447	3.9562	3.9562	3.9919

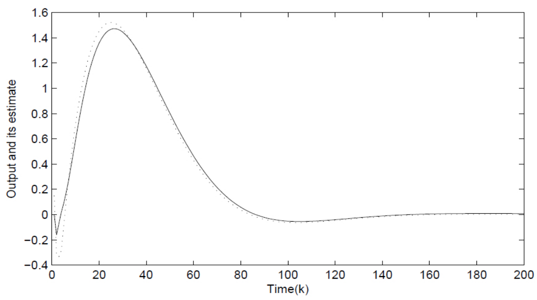

Fig. 1 $z(k)$ (dashed) and its estimate $\tilde{z}(k)$ (solid) in Example 1.

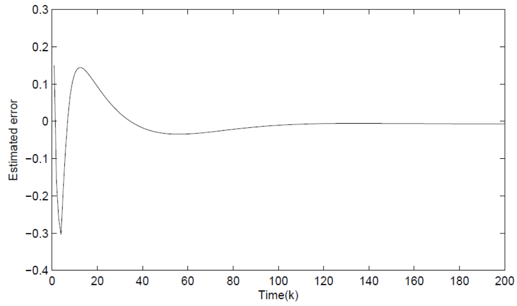

Fig. 2 Estimated error $e(k)$ in Example 1.

5. Conclusions

In this paper, a robust H_∞ filtering problem for a class of discrete-time systems with nonlinear sensor and interval time-varying delay has been proposed. Based on the Lyapunov-Krasovskii functional approach, sufficient conditions have been provided for the stability of the filtering error system with a prescribed H_∞ performance level. Finally, numerical examples have been given to show the usefulness and effectiveness of the proposed filter design method.

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