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Default Bayesian hypothesis testing for the scale parameters in the half logistic distributions

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Abstract

This article deals with the problem of testing the equality of the scale parameters in the half logistic distributions. We propose Bayesian hypothesis testing procedures for the equality of the scale parameters under the noninformative priors. The noninformative prior is usually improper which yields a calibration problem that makes the Bayes factor to be defined up to a multiplicative constant. Thus we propose the default Bayesian hypothesis testing procedures based on the fractional Bayes factor and the intrinsic Bayes factors under the reference priors. Simulation study and an example are provided.

Keywords: Fractional Bayes factor, half logistic distribution, intrinsic Bayes factor, reference prior, scale parameter.

1. Introduction

The half logistic distribution as the distribution of an absolute logistic distribution is introduced as a probability model by Balakrishnan (1985). Since the support of half logistic distribution is non-negative and has increasing hazard rate, this distribution can be applied to model the data from quality control or reliability study. Application of the half-logistic distribution to life testing has been well demonstrated by Balakrishnan (1985) who derived some recurrence relations for the moments and product moments of order statistics.

Balakrishnan and Puthenpura (1986) obtained the coefficient of the best linear unbiased estimators for the location and scale parameters based on complete and censored samples. Balakrishnan and Wong (1991) obtained the approximate maximum likelihood estimators for the location and scale parameters. Adatia (1997) derived the approximate best linear unbiased estimators of the parameters. Kang and Park (2005) derived the approximate maximum likelihood estimators of the scale parameter based on multiply type-II censored samples. Kim and Han (2010) obtained the maximum likelihood estimator and Bayes estimator for the scale parameter of the half-logistic distribution based on a progressively type

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II censored sample assuming a natural conjugate prior. Recently, Rao and Kantam (2012) developed control charts of the mean and range for the half logistic distribution with fixed scale parameter. Kang *et al.* (2012) derived the noninformative priors for the ratio of scale parameters in the the half-logistic distributions. But, the problem of comparison for two scale parameters is not considered yet.

In Bayesian model selection or testing problem, the Bayes factor under proper priors or informative priors have been very successful. However, limited information and time constraints often require the use of noninformative priors. Since noninformative priors such as Jeffreys' prior or reference prior (Berger and Bernardo, 1989, 1992) are typically improper so that such priors are only defined up to arbitrary constants which affects the values of Bayes factors. Spiegelhalter and Smith (1982), O'Hagan (1995) and Berger and Pericchi (1996) have made efforts to compensate for that arbitrariness.

Spiegelhalter and Smith (1982) used the device of imaginary training sample in the context of linear model comparisons to choose the arbitrary constants. But the choice of imaginary training sample depends on the models under comparison, and so there is no guarantee that the Bayes factor of Spiegelhalter and Smith (1982) is coherent for multiple model comparisons. Berger and Pericchi (1996) introduced the intrinsic Bayes factor using a data-splitting idea, which would eliminate the arbitrariness of improper prior. O'Hagan (1995) proposed the fractional Bayes factor. For removing the arbitrariness he used to a portion of the likelihood with a so-called the fraction b. These approaches have shown to be quite useful in many statistical areas (Kang *et al.*, 2011, 2012). An excellent exposition of the objective Bayesian method to model selection is Berger and Pericchi (2001).

An objective Bayesian inference for the equality of the scale parameters in two independent half logistic distributions cab be performed by using the noninformative prior developed by Kang *et al.* (2012). We will use the above noninformative prior for testing the equality of the scale parameters. This equality problem is important when two independent groups of data are collected and one wants to know whether these two groups are identical or not. Therefore we develop hypothesis testing procedures for comparison of the scale parameters.

In this paper, we propose the objective Bayesian hypothesis testing procedures for the equality of the scale parameters in half logistic distributions based on the Bayes factors. The outline of the remaining sections is as follows. In Section 2, we introduce the Bayesian hypothesis testing procedures based on the Bayes factors. In Section 3, under the reference priors, we provide the Bayesian hypothesis testing procedures based on the fractional Bayes factor and the intrinsic Bayes factors. In Section 4, simulation study and an example are given.

2. Intrinsic and fractional Bayes factors

Consider X and Y are independently distributed random variables according to the half-logistic distribution $\mathcal{HL}(\sigma_1)$ with the scale parameter σ_1, and the half-logistic distribution $\mathcal{HL}(\sigma_2)$ with the scale parameter σ_2. Then the half logistic distributions of X and Y are given by

$$f(x|\sigma_1) = \frac{2}{\sigma_1} \frac{\exp\{-x/\sigma_1\}}{[1+\exp\{-x/\sigma_1\}]^2},\ x \geq 0,\ \sigma_1 > 0, \tag{2.1}$$

and

$$f(y|\sigma_2) = \frac{2}{\sigma_2}\frac{\exp\{-y/\sigma_2\}}{[1+\exp\{-y/\sigma_2\}]^2},\ y \geq 0,\ \sigma_2 > 0, \tag{2.2}$$

respectively. The present paper focuses on testing the equality of the scale parameters in the half logistic distributions.

Suppose that hypotheses $H_1,\ H_2,\ \cdots,\ H_q$ are under consideration, with the data $\mathbf{x} = (x_1, x_2,\ \cdots, x_n)$ having probability density function $f_i(\mathbf{x}|\theta_i)$ under hypothesis H_i. The parameter vector θ_i is unknown. Let $\pi_i(\theta_i)$ be the prior distributions of hypothesis H_i, and let p_i be the prior probability of hypothesis H_i,$i = 1, 2, \cdots, q$. Then the posterior probability that the hypothesis H_i is true is

$$P(H_i|\mathbf{x}) = \left(\sum_{j=1}^{q}\frac{p_j}{p_i}\cdot B_{ji}\right)^{-1}, \tag{2.3}$$

where B_{ji} is the Bayes factor of hypothesis H_j to hypothesis H_i defined by

$$B_{ji} = \frac{\int f_j(\mathbf{x}|\theta_j)\pi_j(\theta_j)d\theta_j}{\int f_i(\mathbf{x}|\theta_i)\pi_i(\theta_i)d\theta_i} = \frac{m_j(\mathbf{x})}{m_i(\mathbf{x})}. \tag{2.4}$$

The B_{ji} interpreted as the comparative support of the data for H_j versus H_i. The computation of B_{ji} needs specification of the prior distribution $\pi_i(\theta_i)$ and $\pi_j(\theta_j)$. When the noninformative priors $\pi_i^N(\theta_i)$ and $\pi_j^N(\theta_j)$ are used in (2.4), the undefined constants in prior distributions cause the indeterminacy problem in B_{ji}.

To solve this problem, Berger and Pericchi (1996) and O'Hagan (1995) proposed the intrinsic Bayes factor and the fractional Bayes factor, respectively.

The intrinsic Bayes factor of Berger and Pericchi (1996) uses the part of the data as a training sample to cancel undefined constants in (2.4). The fractional Bayes factor of O'Hagan (1995) uses a fraction of likelihood function for the same purpose of canceling undefined constants. For details, see Berger and Pericchi (1996), O'Hagan (1995), Kang *et al.* (2011, 2012).

3. Bayesian hypothesis testing procedures

Let $X_i, i = 1, \cdots, n_1$ denote observations from the half logistic distribution $\mathcal{HL}(\sigma_1)$, and $Y_i, i = 1, \cdots, n_2$ denote observations from the half logistic distribution $\mathcal{HL}(\sigma_2)$. Then likelihood function is given by

$$\begin{aligned} f(\mathbf{x},\mathbf{y}|\sigma_1,\sigma_2) &= 2^{n_1+n_2}\sigma_1^{-n_1}\sigma_2^{-n_2}\exp\left\{-\frac{\sum_{i=1}^{n_1}x_i}{\sigma_1} - \frac{\sum_{i=1}^{n_2}y_i}{\sigma_2}\right\} \\ &\times \prod_{i=1}^{n_1}\left(1+\exp\{-x_i/\sigma_1\}\right)^{-2}\prod_{i=1}^{n_2}\left(1+\exp\{-y_i/\sigma_2\}\right)^{-2}, \end{aligned} \tag{3.1}$$

where $\mathbf{x} = (x_1, \cdots, x_{n_1})$, $\mathbf{y} = (y_1, \cdots, y_{n_2})$, $\sigma_1 > 0$ and $\sigma_2 > 0$. We are interested in testing the hypotheses $H_1 : \sigma_1 = \sigma_2$ versus $H_2 : \sigma_1 \neq \sigma_2$ based on the fractional Bayes factor and the intrinsic Bayes factors.

3.1. Bayesian hypothesis testing procedure based on the fractional Bayes factor

From (3.1) the likelihood function under the hypothesis $H_1 : \sigma_1 = \sigma_2 \equiv \sigma$ is

$$\begin{aligned} L_1(\sigma|\mathbf{x},\mathbf{y}) &= 2^{n_1+n_2}\sigma^{-(n_1+n_2)} \exp\left\{-\frac{n_1\bar{x}+n_2\bar{y}}{\sigma}\right\} \\ &\times \prod_{i=1}^{n_1}(1+\exp\{-x_i/\sigma\})^{-2}\prod_{i=1}^{n_2}(1+\exp\{-y_i/\sigma\})^{-2}. \end{aligned} \tag{3.2}$$

And under the hypothesis H_1, the reference prior for σ is

$$\pi_1^N(\sigma) \propto \sigma^{-1}. \tag{3.3}$$

Then from the likelihood (3.2) and the reference prior (3.3), the element $m_1^b(\mathbf{x},\mathbf{y})$ of the fractional Bayes factor (FBF) under H_1 is given by

$$\begin{aligned} m_1^b(\mathbf{x},\mathbf{y}) &= \int_0^\infty L_1^b(\sigma|\mathbf{x},\mathbf{y})\pi_1^N(\sigma)d\sigma \\ &= 2^{b(n_1+n_2)}\int_0^\infty \sigma^{-b(n_1+n_2)-1}\exp\left\{-\frac{b(n_1\bar{x}+n_2\bar{y})}{\sigma}\right\} \\ &\times \prod_{i=1}^{n_1}(1+\exp\{-x_i/\sigma\})^{-2b}\prod_{i=1}^{n_2}(1+\exp\{-y_i/\sigma\})^{-2b}d\sigma, \end{aligned} \tag{3.4}$$

where $\bar{x} = \sum_{i=1}^{n_1} x_i/n_1$ and $\bar{y} = \sum_{i=1}^{n_2} y_i/n_2$. For the hypothesis $H_2 : \sigma_1 \neq \sigma_2$, the reference prior for (σ_1, σ_2) is

$$\pi^N(\sigma_1,\sigma_2) \propto \sigma_1^{-1}\sigma_2^{-1}. \tag{3.5}$$

Note that the derivation of the above reference priors (3.3) and (3.5), and the propriety of the posterior distribution are given in Appendix. The likelihood function under the hypothesis H_2 is

$$\begin{aligned} L_2(\sigma_1,\sigma_2|\mathbf{x},\mathbf{y}) &= 2^{n_1+n_2}\sigma_1^{-n_1}\sigma_2^{-n_2}\exp\left\{-\frac{n_1\bar{x}}{\sigma_1}-\frac{n_2\bar{y}}{\sigma_1}\right\} \\ &\times \prod_{i=1}^{n_1}(1+\exp\{-x_i/\sigma_1\})^{-2}\prod_{i=1}^{n_2}(1+\exp\{-y_i/\sigma_2\})^{-2}. \end{aligned} \tag{3.6}$$

Thus from the likelihood (3.6) and the reference prior (3.5), the element $m_2^b(\mathbf{x},\mathbf{y})$ of FBF under H_2 is given as follows.

$$\begin{aligned} m_2^b(\mathbf{x},\mathbf{y}) &= \int_0^\infty\int_0^\infty L_2^b(\sigma_1,\sigma_2|\mathbf{x},\mathbf{y})\pi_2^N(\sigma_1,\sigma_2)d\sigma_1 d\sigma_2 \\ &= 2^{b(n_1+n_2)}\int_0^\infty\int_0^\infty \sigma_1^{-bn_1-1}\sigma_2^{-bn_2-1}\exp\left\{-\frac{bn_1\bar{x}}{\sigma_1}-\frac{bn_2\bar{y}}{\sigma_1}\right\} \\ &\times \prod_{i=1}^{n_1}(1+\exp\{-x_i/\sigma_1\})^{-2b}\prod_{i=1}^{n_2}(1+\exp\{-y_i/\sigma_2\})^{-2b}d\sigma_1 d\sigma_2. \end{aligned} \tag{3.7}$$

Using equation (3.4) and (3.7), the FBF of H_2 versus H_1 is given by

$$B_{21}^F = \frac{m_2^1(\mathbf{x},\mathbf{y})}{m_1^1(\mathbf{x},\mathbf{y})} \cdot \frac{m_1^b(\mathbf{x},\mathbf{y})}{m_2^b(\mathbf{x},\mathbf{y})},$$

usually $b = 1/m$, and m is the number of minimal training sample of Berger and Pericchi (1996).

Note that the calculations of the FBF of H_2 versus H_1 requires tow dimensional integration. This can be evaluated by the numerical method such as Gaussian quadrature.

3.2. Bayesian hypothesis testing procedure based on the intrinsic Bayes factor

The element B_{21}^N of the intrinsic Bayes factor is computed in the fractional Bayes factor. So under minimal training sample, we only calculate the marginal densities for the hypotheses H_1 and H_2, respectively. The marginal density of X_j and Y_k is finite for all $1 \le j \le n_1$ and $1 \le k \le n_2$ under each hypothesis. Thus we conclude that any training sample of size 2 is a minimal training sample.

The marginal density $m_1^N(x_j, y_k)$ under H_1 is given by

$$\begin{aligned} m_1^N(x_j, y_k) &= \int_0^\infty f(x_j, y_k|\sigma)\pi_1^N(\sigma)d\sigma \\ &= \int_0^\infty 4\sigma^{-3}\left(1+\exp\{-x_j/\sigma\}\right)^{-2}\left(1+\exp\{-y_k/\sigma\}\right)^{-2}\exp\left\{-\frac{x_j+y_k}{\sigma}\right\}d\sigma. \end{aligned}$$

And the marginal density $m_2^N(x_j, y_k)$ under H_2 is given by

$$\begin{aligned} m_2^N(x_j, y_k) &= \int_0^\infty\int_0^\infty f(x_j, y_k|\sigma_1,\sigma_2)\pi_2^N(\sigma_1,\sigma_2)d\sigma_1 d\sigma_2 \\ &= \frac{1}{x_j y_k}. \end{aligned}$$

Therefore the arithmetic intrinsic Bayes factor (AIBF) of H_2 versus H_1 is given by

$$B_{21}^{AI} = \frac{m_2^1(\mathbf{x},\mathbf{y})}{m_1^1(\mathbf{x},\mathbf{y})}\left[\frac{1}{L}\sum_{j=1}^{n_1}\sum_{k=1}^{n_2}\frac{T_1(x_j,y_k)}{T_2(x_j,y_k)}\right],$$

where $L = n_1 n_2$,

$$T_1(x_j, y_k) = \int_0^\infty \sigma^{-3}\left(1+\exp\{-x_j/\sigma\}\right)^{-2}\left(1+\exp\{-y_k/\sigma\}\right)^{-2}\exp\left\{-\frac{x_j+y_k}{\sigma}\right\}d\sigma$$

and

$$T_2(x_j, y_k) = \frac{1}{4x_j y_k}.$$

Also the median intrinsic Bayes factor (MIBF) of H_2 versus H_1 is given by

$$B_{21}^{MI} = \frac{m_2^1(\mathbf{x},\mathbf{y})}{m_1^1(\mathbf{x},\mathbf{y})} ME\left[\frac{T_1(x_j,y_k)}{T_2(x_j,y_k)}\right],$$

where ME indicates the median. Note that the calculations of the AIBF and the MIBF of H_2 versus H_1 require only one dimensional integration.

4. Numerical studies

In order to assess the Bayesian hypothesis testing procedures, we evaluate the posterior probability for several configurations of (σ_1, σ_2) and (n_1, n_2). In particular, for fixed (σ_1, σ_2), we take 1,000 independent random samples of X_i and Y_i with sample size n_1 and n_2 from the models (2.1) and (2.2), respectively. We want to test the hypotheses $H_1 : \sigma_1 = \sigma_2$ versus $H_2 : \sigma_1 \neq \sigma_2$. The posterior probabilities of H_1 being true are computed assuming equal prior probabilities. Table 4.1 shows the results of the averages and the standard deviations in parentheses of posterior probabilities. In Table 4.1, $P^F(\cdot)$, $P^{AI}(\cdot)$ and $P^{MI}(\cdot)$ are the posterior probabilities of the hypothesis H_1 being true based on FBF, AIBF and MIBF, respectively. From the results of Table 4.1, the FBF, the AIBF and the MIBF give fairly reasonable answers for all configurations. Also the FBF, the AIBF and the MIBF give a similar behavior for all sample sizes. However the AIBF and the MIBF slightly favor the hypothesis H_1 than the FBF.

Table 4.1 The averages and the standard deviations (in parentheses) of posterior probabilities

σ_1	σ_2	(n_1, n_2)	$P^F(H_1\|\mathbf{x},\mathbf{y})$	$P^{AI}(H_1\|\mathbf{x},\mathbf{y})$	$P^{MI}(H_1\|\mathbf{x},\mathbf{y})$
1.0	1.0	5,5	0.630 (0.124)	0.694 (0.137)	0.681 (0.135)
		5,10	0.671 (0.139)	0.720 (0.145)	0.706 (0.142)
		10,10	0.690 (0.141)	0.751 (0.144)	0.736 (0.144)
		10,20	0.734 (0.135)	0.780 (0.133)	0.765 (0.133)
	2.0	5,5	0.502 (0.197)	0.556 (0.219)	0.550 (0.213)
		5,10	0.504 (0.220)	0.547 (0.235)	0.539 (0.229)
		10,10	0.437 (0.256)	0.490 (0.276)	0.478 (0.271)
		10,20	0.408 (0.266)	0.451 (0.281)	0.439 (0.276)
	3.0	5,5	0.344 (0.225)	0.378 (0.254)	0.377 (0.247)
		5,10	0.303 (0.220)	0.333 (0.241)	0.333 (0.235)
		10,10	0.180 (0.210)	0.208 (0.237)	0.204 (0.231)
		10,20	0.116 (0.160)	0.134 (0.181)	0.130 (0.175)
	5.0	5,5	0.167 (0.185)	0.181 (0.210)	0.187 (0.208)
		5,10	0.116 (0.145)	0.129 (0.163)	0.133 (0.162)
		10,10	0.030 (0.080)	0.035 (0.093)	0.035 (0.090)
		10,20	0.011 (0.041)	0.013 (0.049)	0.013 (0.047)
	10.0	5,5	0.037 (0.079)	0.038 (0.090)	0.044 (0.095)
		5,10	0.015 (0.037)	0.016 (0.042)	0.019 (0.047)
		10,10	0.001 (0.003)	0.001 (0.004)	0.001 (0.004)
		10,20	0.000 (0.000)	0.000 (0.001)	0.000 (0.001)

Example 4.1 This example taken from Balakrishnan and Puthenpura (1986). The data is failure times, in minutes, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress. For this data, Balakrishnan and Puthenpura (1986) concluded that the half logistic distribution fits the data better than an exponential distribution. To test the equality of the scale parameters, we randomly divided this data into two groups. The data sets are given by

Group 1: 21.8, 70.7, 151.9, 75.3, 12.3, 28.6.
Group 2: 24.4, 138.6, 95.5, 98.1, 43.2, 46.9.

For this data sets, the maximum likelihood estimates of σ_1 in group 1 is 43.285 and for group 2, the maximum likelihood estimates of σ_2 is 51.188.

We want to test the hypotheses $H_1 : \sigma_1 = \sigma_2$ versus $H_2 : \sigma_1 \neq \sigma_2$. The values of the Bayes factors and the posterior probabilities of H_1 are given in Table 4.2. From the results of Table 4.2, the posterior probabilities based on various Bayes factors give the same answer, and select the hypothesis H_1. The FBF has the smallest posterior probability of H_1 than any other posterior probabilities based on the AIBF and the MIBF, but the values of three posterior Bayes factors are almost the same results.

Table 4.2 Bayes factors and posterior probabilities of $H_1 : \sigma_1 = \sigma_2$

B_{21}^F	$P^F(H_1\|\mathbf{x}, \mathbf{y})$	B_{21}^{AI}	$P^{AI}(H_1\|\mathbf{x}, \mathbf{y})$	B_{21}^{MI}	$P^{MI}(H_1\|\mathbf{x}, \mathbf{y})$
0.4069	0.7108	0.2797	0.7814	0.3243	0.7551

5. Concluding remarks

In this paper, we developed the objective Bayesian hypothesis testing procedures based on the fractional Bayes factor and the intrinsic Bayes factors for the equality of the scale parameters in half logistic distributions under the reference priors. From our numerical results, the developed hypothesis testing procedures give fairly reasonable answers for all parameter configurations. However the AIBF and the MIBF slightly favor the hypothesis H_1 than the FBF. From our simulation and example, we recommend the use of the FBF than the AIBF and MIBF for practical application in view of its simplicity and ease of implementation.

To test hypotheses about this problem, a classical test needs exact or asymptotic distribution of test function. But the proposed Bayesian testing procedures can be used without resorting the distribution of test function.

When the sample size and the difference between parameters are small, the proposed procedures fail to choose correct model. This problem is hard to overcome.

For the further study, developing the intrinsic prior for this problem is an important and interesting work which is worthy to consider. The intrinsic prior is a proper prior which can make the computation of Bayes factor without splitting the data into two parts. And based on the intrinsic prior, the value of Bayes factor is asymptotically equivalent to AIBF.

Appendix

The likelihood function of parameters σ_1 for the model (2.1) is given by

$$L(\sigma_1) \propto \sigma_1^{-1} \frac{\exp\{-x/\sigma_1\}}{[1+\exp\{-x/\sigma_1\}]^2}.$$

From the above likelihood function, the Fisher information is $\frac{3+\pi^2}{9\sigma_1^2}$. Thus the reference prior is given by

$$\pi(\sigma_1) \propto \sigma_1^{-1}.$$

And the posterior distribution for σ_1 given $\mathbf{x}$ is

$$\pi(\sigma_1|\mathbf{x}) \propto \sigma_1^{-n_1-1} \frac{\exp\{-n_1\bar{x}/\sigma_1\}}{\prod_{i=1}^{n_1}[1+\exp\{-x_i/\sigma_1\}]^2}.$$

Thus

$$\int_0^\infty \pi(\sigma_1|\mathbf{x}) \quad \leq \quad \int_0^\infty \sigma_1^{-n_1-1} \exp\{-n_1\bar{x}/\sigma_1\} < \infty,$$

if $n_1 \geq 1$.

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