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A STUDY FOR DEVELOPMENT OF UNIVERSITY MATHEMATICS COURSE BASED ON REAL LIFE CONTEXT AND CLASSROOM DISCUSSION

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Abstract. Modern society demands leaders who are trained with competence to not only approach knowledge but also create new knowledge by comprehensively understanding and applying it, and a leader with character and commitment to share one's ideas with others and be able to accept criticisms. In response to these societal changes, universities are increasingly adopting 'small group discussion-based classes with an attempt to develop and strengthen communication skills through reading, writing and speaking. This paper seeks to introduce a case of a math lecture, where discussion-based class was applied to mathematical education, requiring practical problem-solving through an argumentative thought process.

1. Introduction

Recently, there has been an increased interest amongst universities in increasing the number of students studying science in response to the phenomenon of students evading science and engineering fields. Each university is analysing programs in various perspectives, generating interest in the field of scientific technology in education fields and running programs at different levels to spread the culture of science. There are

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number of reasons for this. One of them is the humanization education for a holistic education, related to character education. Our education has been criticized for being a memorization-intensive education focused at university entrance exams. As a result, it led to a shortcoming of character education such as logical thinking, creativity development and leadership qualities, showing a low academic achievement rate. In fact, there has been a rise in the percentage of students who dislike mathematics and the mathematical ability of our elementary, middle, secondary and post-secondary school students have declined. In the Achievement Test conducted by the Japan Science Foundation in 2004 for the high-school and university students in science fields for the four countries of South Korea, Japan, China and Singapore, Korean university students scored the lowest, with the exception of Singapore. According to the result of the Trends in International Mathematics and Science Study (TIMSS 2003) conducted by the International Association for the Evaluation of Educational Achievement for the second year middle school students in forty-six countries, Korean students ranked the second place in mathematics and third place in science. Compared to the results of 1995 and 1999, mathematics improved from 3rd place $\rightarrow$ 2nd place $\rightarrow$ 2nd place, and science improved from 4th place $\rightarrow$ 5th place $\rightarrow$ 3rd place. Even for the percentage of top highest range of students, we ranked in third place for both mathematics and science. On the Programme for International Student Assessment (PISA) conducted under the leadership of the Organization for Economic Cooperation and Development (OECD), Korea ranked the first place in problem-solving skills, second place in reading, third place in mathematics and fourth place in science (Press release data, Ministry of Education, Science and Technology, December 2004). However, despite the improved results in various indexes of the PISA, Korea ranked the 31st and 38th, respectively, on research results such as math-related interest or academic motivation, demonstrating that Korean students hold low interest or motivation in their studies. The memorization-based, cramming method of teaching could be blamed as the fundamental cause for this result. It is observed that that subjects are taught by focusing on knowledge, and proposals are being made to see what kind of efforts have been made for the improvement of our education and quality, and the need to question and bring change to the teaching style.

Another reason is the effect of undergraduate program officially introduced to universities since 1996. It is an education policy pursued with the goal of allowing students to engage in multiple disciplines, by lifting the pressure of fulfilling program requirements for specialized course of study and removing the barriers between departments, as the education curriculum is managed by each department. It brought forth a learner-based education from a supplier-based education, and served as an occasion to pursue practical studies rather than basic studies, there by supporting the values of the internet generation in the 21st century. For instance, if the mathematical education in the 80's was categorized as basic science in its acquisition and application and valued differential calculus and infinitesimal calculus, the mathematical education in the 90's is aimed at understanding the laws of nature and logical thinking, with importance given to general math subjects for all undergraduate students, such as ⌜understanding everything in the world⌟, or ⌜the world of mathematics⌟. Recently, beginning with the newly admitted students in 2009, universities are pursuing the establishment of free major, which trespasses the boundaries between departments. Free major refers to a program of study where students freely choose their courses in the first and second years without belonging to any particular college, and take a given number of credits for their intended field of specialization from the third year for graduation. This initiative is based on the appreciation of the ability of a global leader in the 21st century to freely analysis and criticize problems in various fields to apply them in one′s society and nature, as well as acquiring knowledge in a specific field of study. Moreover, amidst the chaos of world views due to rapid societal changes, a holistic character is demanded, by expanding the breadth of understanding and holding proper views on ethical and human problems.

As seen in the discussion above, universities today focus on character education and value education to cultivate global leaders who can freely analysis and criticize specialized knowledge from interdisciplinary studies. In this vein, an emphasis is given to discussion-based classes where students can develop analytical skills, widen their horizon through the group and establish a proper world view. The University which the author belongs to selects twelve books from social sciences and natural sciences and twelve books from literary fields for the first year students and ask them to read one book per week. After reading a book a week, students must take a class entitled ⌜Reading and

Expression」 for two semesters, where they debate in small groups for the discussion topics distributed earlier in the semester. This course was established as a mandatory elective course called 「The Reading Plan」 in 1969 and has continued for the past forty years. In this paper, cases of discussion-based lectures for mathematical education, with math-related books from the reading list, including 「A World Made of Mathematics」 and 「Chaos」 will be introduced. In the main section, the methodology for running a discussion-based class and the response from students will be outlined, and search for the future direction for application and improvement.

2. Principles of Course Development

2.1. Mathematics found in real life. Mathematics has a more inclusive meaning beyond math education. Behind every day issues such as weather forecast, sports, music, arts, the nature and culture which we live in, such as computers, world of animals and plants, various career paths, love and marriage, mathematics is alive. However, the students' sentiments towards mathematics are recognized as a means of passively acquiring absolute knowledge. Keith Devlin, in his introduction, asserts: "This is not a typical math book that people often talk about. From the outset, it was not written with the goal of teaching techniques for solving math questions. Therefore, the readers will not be able to learn very much about that kind of math through this book or find mathematical formula or complicated practice questions anywhere in this book. Instead, you will find that mathematics is very different than how you thought of it before. And you will find out for sure that mathematics, often slightly deceptive its appearance, plays a very important role in almost all aspects of our daily lives. If there is anybody who thinks that math has nothing to with our daily lives, this book is for that kind of person." To encourage a discussion class with a math-related book, it is absolutely necessary to first remove pre-conceptions about math. As a result of education focused on university entrance exams, students automatically think of symbols, numbers, formula and problem solving when they hear the word "math." Therefore, for the first discussion topic, encourage students to discuss the definition of math provided by Keith Devlin in the World Made of Mathematics from their own perspectives. By introducing cases of mathematical cases found in the nature, such as

how did the leopard prints in leopards get created, or the mathematical symphony resounding in the fourth dimension, or Fight against virus with a knot, encourage students to naturally come up with their own definition.

The author explains that the mathematical principles and the curriculum taught at school are merely the tools necessary for doing the real math, and defines mathematics as a "scholarship about an effort to better understand ourselves and the world in which we live in."Moreover, he emphasizes that it is not that the world is mathematical, but that mathematics provides a powerful tool for us to examine the world. In his own words, "Mathematics provides a method of understanding the world, that is, a way of bringing the world into our minds." By comparing the author's view on mathematics and their own interpretations, it would be an effective way of teaching students that mathematics that they traditionally perceived as a difficult problem solving is actually something that can be easily discovered and which co-exists with us in our day to day interactions. Even though it is not visible to the eyes, our body is composed of over a 100 trillion cells. The cell is a basic functional and structural unit of all living creatures and is critical for our body's survival. Even though mathematics is not very visible to our eyes, from everything that happens in our world under the sun to the events that happen beyond the sun, mathematics exists everywhere. Therefore, the world cannot be understood without mathematics. Hence, mathematics can be defined as something akin to the cells. From this aspect, amongst the definitions provided by the author, the one that states "mathematics is transparent so it is not visible to the user's eyes,but the basis of everything we do is mathematics," best expresses a perspective for examining mathematics. In conclusion, by presenting their own definition of mathematics, students will be able to naturally express their preconceptions about mathematics. Moving away from a consciousness that mathematics is a difficult and boring subject that brings headaches when they think of symbols and calculations, students will be empowered to find interest and enthusiasm towards mathematics. The second topic is the question about mathematics that can be found in reality. Discussion classes may pose a burden for students that they must read one book for each week and give a daunting impression about the discussion culture which they are not familiar with. Therefore, one cannot help but ponder

upon the method of running the discussion to improve the class satisfaction rate. Mathematics found in real life asks students to research media releases, internet data and newspapers, so that they may share a wide breadth of information and perspective, as well as create a spontaneous opportunity to get involved in the discussion, in order to generate an active discussion. Examples of presentations include:
1) 1:of A4 sized paper: aesthetics of economic efficiency
2) Soju glass and Soju bottle 1:7.5: marketing strategy
3) Bar codes: a combination of black sticks and white blanks
4) The boundary between a long leg and short leg: the golden rate
5) Why a hive is a hexagon and a tile is a square
6) Hidden mathematical principle behind the seven musical scale
7) Probability in our life, lottery
8) Beautiful fractal found after repetition
9) Fibonacci sequence found in a pine cone
10) Efficiency of 21 teeth of a saw found in a bottle cap
As well, there are cases such as why the manhole covers are shapes of constant width, the mathematical principle behind breathalyzer, and cicada's wisdom for living in prime number cycles. Through mathematics found in real life, students will be able to establish the concept of mathematics. By applying them in our lives and understanding them, students will understand that mathematics is much more than just numbers and formulas, and that mathematics helps us to better understand the world of nature.

2.2. Mathematics is an analysis and categorization of all possible patterns. "Mathematics is an analysis and categorization of all possible patterns." The term pattern should be interpreted in a broad sense. In another words, pattern can be defined as something inclusive, from the regularity of forms that are visibly identified to the regularity of invisible things that are only recognized through our hearts." In the natural world, there is some degree of regularity. For instance, when the temperatures go down in autumn, the trees create a separate layer in the leafstalk to bring down the leaves. Also, to survive in a dry climate and coldness, they move the water and nutrients saved in branches and stalks to the roots. The phenomenon of how the trees that barely preserved themselves as if they were dead pumps up water in the spring; or the phenomenon of how different parts of our body functions actively to the environment in spring, when the daytime for the biorhythm of

our body extends and the temperature difference between day and night varies greatly; or the phenomenon of how the birds recognize the black and yellow ties surrounding the hornets, are cases of how all living things with life recognize patterns. Furthermore, the nature and the society are filled with confusing patterns. Yet, today's physics has proven the fact that even the most complicated patterns can be written in an equation with just a few variables, and that it is possible to predict some parts. To recognize that mathematics is a science of patterns, define and discuss examples of a fractal in our body and nature as the third topic of the book, ⌜Chaos⌟. Some examples for presentation include:
1) Crystallized shape of snow: Koch's snowflake curve
2) Lightening which appears to be irregular
3) Stock Market, regularity within irregularity
4) A field of study using fractal - Statistics
5) Similarity of the blood vessel structure stemming from the heart
6) Repetitive quality of brain wrinkles/ surface area of the lung
7) Fractal found in plants (leaves of brackens, broccoli and water stratum of leaves)
8) Computer graphic art created by applying the fractal formula
9) The Universe & I
10) The spirit and chaos of oriental philosophy
Rather than delivering objective knowledge in a discussion class, an experience to encounter mathematics that appears to have very little in common with reality, as a play, naturally changes the image. Fractal cards were made using the origami papers and knives. Simple patterns using self-resemblance looks most beautiful and you may experience how the simple patterns of geometry is the most efficient structure accepted by the nature. The patterns and relations studied by mathematicians are seen in all parts of the nature. Some examples include the symmetrical patterns of flowers, various types of complicated patterns found in knots, patterns of dots on the external skin of leopards, voting patterns of certain population group, patterns found in a dice play or roulette game, relationships between the words that make up a sentence, the pattern of sounds we recognize in a music, and many more. As such, it is evident that we live within the nature surrounded by patterns and develop close relations with mathematics in our lives, before we even realize it.

2.3. We are able to see the invisible through mathematics. Mathematics enables us to see things that are not visible to the eye. If you let go of something in your hand, it falls down. This is because of the force of gravity. However, gravity is just a name given to the cause. The name itself does not help very much in understanding the real cause. And that cause is not even visible to the eye. The correct answer is Newton's equation of movement from the 17th century. Newton's mathematics made it possible to see the force that was not visible to the eye. That force is the reason why the earth can continue to revolve around the sun and an apple falls down from a tree. Furthermore, the equation that Daniel Bernoulli discovered in the early 18th century made visible the invisible power of making an airplane fly in the air. Two thousand years before a spacecraft took a photograph of the earth, Greek mathematician Eratosthenes demonstrated the fact that the earth is circular using mathematics. We use mathematics to see the invisible things. Ranging from the origins of the universe to the bottom of the sea, from the pattern of coincidental events to the application into the inner mind of humans, we would not be able to understand our world without mathematics. Logical thinking is an act which distinguishes humans as advanced living creatures. Through the training of critical and rational thinking, human beings are able to arrive at a more logical mental world. For the training of invisible logical thinking, an advertisement called the beauty of 30 seconds was used. To emphasize the strengths of the product, the advertisement skips some parts of the preposition. If the advertisements are carefully examined from a critical perspective, it is possible to discover how the first impression from the advertisement and the outcome of the advertisement are unclear and explained in a distorted way. Look at the following advertisements and think about how the first impression from each advertisement and the meaning of the advertisement are different.

1) An automobile advertisement where a driver is speeding up in a curved road of a highway with no other cars in traffic

2) An advertisement of a long-distance phone company that says "You can save more money by talking longer."

3) "Get a 50% discount if you live in the same house." only when the years of membership for the family members totals up to more than 40 years

4) A beverage company which advertises the recognition gained from the

Association of Korean Oriental Medicine the period for recognition is only one year
5) Apartment advertisement which runs, "This is the first time I'm bringing him to my home," or "This is the first time I'm going to meet her parents."
6) Female-priority loan company's advertisement which screams "No interest for 30 days!"
Training of critical thinking through these advertisements is not simply about understanding the meaning of advertisements and assessing whether they are reasonable. It is a training to comprehensively restructure one's experiences to understand the advertisement, and subsequently, think critically and creatively. In another words, it is a process of changing one self. After assessing what is true or false, you will no longer be the same person that you were before.

2.4. Look at the future using mathematics. One of the biggest factors affecting the national competitiveness in the present information-based society is likely the advancement of scientific technology. Through creative research in a new field, with basic science as the foundation, scientists have understood the principles and phenomenon of the nature and created new knowledge. Technology derived through such basic science not only improves the quality of life for the citizens of a particular country, but the added value from technology is often directly related to the national competitiveness. Thus, it can be said that basic science determines the future of a nation. Mathematics is sometimes hidden in the nature. However, it creates technology as the foundation of basic science. Furthermore, mathematics changes human life. Ubiquitous technology,which is persistently under spotlight, is expected to transform the mankind's society and culture, beyond simply improving environment. For this reason, we visited the Ubiquitous Dream Exhibition to experience the ubiquitous lifestyle which will be realized in real life and find out about the level of Korea's ubiquitous IT technology and had an educational session about experiencing the future. This exhibition was opened by the Korea Communications Commission (previously called the Information Communication Commission) at Kwanghwamun in 2004. It allows the visitors to experience the future home environment, U-Caf, U-Street and offices and various domestic advanced technologies such as digital TV, artificial intelligence service robot and next generation's mobile communication,so that everything from the current

IT technology to the future technology could be seen at a glance. The following are the students' responses about this field trip.

■ I could not taste the charm of mathematics as a memory-intensive subject filled with symbols and formulas, but this opportunity brought a change to my opinion that we cannot expect a complete improvement in our lives without mathematics.

■ It served as an occasion for me to realize the importance of enjoyable and meaningful mathematics, which I used to simply consider on a practical level.

■ It was an opportunity to directly see and experience ubiquitous networking and informatization in our lives, a future living environment which I only heard of before.

3. Conclusion

With the 7th Education Curriculum applied to all grades in elementary, middle and high-schools since 2004, the curriculum for mathematics set the goal of extending Mathematical Power which can create self-directed intellectual value, along with problem-solving skills and all sorts of advanced thinking capability. It seeks to promote creative thinking ability, critical thinking ability, problem-solving skill, reasoning ability, communication ability, confidence about mathematics, positive attitude, and relevance of mathematics to other related field of study and realization of the utility of mathematics. Towards that goal, appropriate study contents, increasing classes focusing on the learner's activity and education for each step and level. However, the mathematics curriculum has not yet escaped from a teacher-focused, cramming method of teaching, and passive and standardized type of problem solving persists. This type of learning is showing limitation in giving a real mathematical perspective to students or cultivating creative and productive problem solving skills. This paper introduced a case as a solution to this problem, by suggesting thoughtful mathematics and discussion-based mathematical education. Further research is in order to select appropriate texts for discussion-based mathematical education, development of discussion topics and for the use and assessment of instructors and teaching aids. Lastly, some excerpts from the students' evaluations at the end of the

term are directly quoted here. Student #1 "I used to think of mathematics as a difficult subject which is only defined and learned through a textbook. It is really interesting to find out how mathematics is related to objects, buildings and nature that we can easily find around us. I also felt that mathematics, with such multiple purposes, is an absolutely important subject in education. I came to re-consider what is meant by mathematical thinking and why it is necessary. If the educational curriculum provides more examples of mathematics used in real life and allows students to encounter a wide variety of mathematical cases, as opposed to just teaching mathematics that deals with numbers, I think more students will find mathematics interesting and make contribution to the discipline of mathematics." Student #2 "Through this discussion, I realized that mathematics is a really important part of my life, as opposed to a source of headache. I wanted to have more discussion about the importance of basis science but regret that we could not do so due to the time constraint." Student #3 "Mathematics and music appear to be different. At least, that is what I thought before reading this book. But they are the same in that they use an abstract way of statement to express an abstract pattern. They read various musical or mathematical patterns through the patterns found in a musical score and a mathematical symbol. Within mathematics, which I used to consider as the most conservative and old-fashioned discipline, I found a soaring creativity that was stronger than any other disciplines. That the world looks so much different to me now is likely because of this book." Student #4 " I came to realize that my thoughts about the math I have been learning to this date have been biased and terribly misunderstood. As written in the cover of this book, this book clearly washed away all my misconceptions and prejudices about mathematics. I was surprised by the new examples of mathematics and had fun with them. I became embarrassed of myself for complaining why do I have to learn math that will have nothing to do with my life, to my friends in high school." Student #5 "It was an opportunity to think about basic science. Even though I thought of applying to natural sciences because of my love for chemistry, I had changed my course to engineering because of poor research environment and weak support. The reason I could not apply, even though I knew the importance of basic science, was because of practical reasons such as job opportunity. I am very sorry about this unfortunate reality and thought that more support is definitely necessary at the policy level."

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