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최적 유한 임펄스 응답 평활기를 이용한 미지 입력 추정 기법

Unknown Input Estimation using the Optimal FIR Smoother

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Abstract: In this paper, an unknown input estimation method via the optimal FIR smoother is proposed for linear discrete-time systems. The unknown inputs are represented by random walk processes and treated as auxiliary states in augmented state space models. In order to estimate augmented states which include unknown inputs, the optimal FIR smoother is applied to the augmented state space model. Since the optimal FIR smoother is unbiased and independent of any a priori information of the augmented state, the estimates of each unknown input are independent of the initial state and of other unknown inputs. Moreover, the proposed method can be applied to stochastic singular systems, since the optimal FIR smoother is derived without the assumption that the system matrix is nonsingular. A numerical example is given to show the performance of the proposed estimation method.

Keywords: unknown input estimation, optimal FIR smoother, linear discrete-time system

I. INTRODUCTION

In the general state estimation problem, it is assumed that the system and noise information and inputs are known. However, in practice, there exist many situations in which some inputs of the system are inaccessible. Moreover, the estimation of unknown inputs arises in many areas such as fault detection and identification and target tracking systems. Hence, unknown input estimation methods have been investigated for decades. For deterministic noise-free systems, the unknown input observer approaches have been developed in order to estimate the unknown inputs [1-3]. But, since they do not consider the noise in their derivations, these approaches may have poor performance with system and measurement noises. To overcome the effect of noise, Kalman filtering approaches were suggested for the system with unknown inputs [4,5]. Kalman filter can give optimal solution, but the estimates of each unknown input depends on the initial information and other unknown inputs. Moreover, since Kalman filter use all of the information from initiation to the current time, it has some potential problems due to its structure [6,7]. Kalman filter may diverge for system with initial state uncertainty, modeling errors, and numerical errors. To overcome the shortcomings of the Kalman filtering approaches, FIR filtering approach was proposed in [8]. As shown in Fig. 1, since the optimal FIR filter estimate makes use of the recent finite measurements, it guarantees BIBO stability, robustness to temporary modeling uncertainties and fast convergency [9]. Moreover, since the optimal FIR filter is derived with unbiased constraint, which make exact estimates for unknown inputs, the estimates of each unknown input is completely independent of the initial state and other unknown inputs [8]. However, the optimal FIR filter used in [8] was derived assuming that the system matrix is nonsingular and the covariance of the initial state is infinity. Also, the optimal FIR filter was derived without known input. These assumptions and derivation are unclear and may prevent the optimal FIR filter from being applied to real problem.

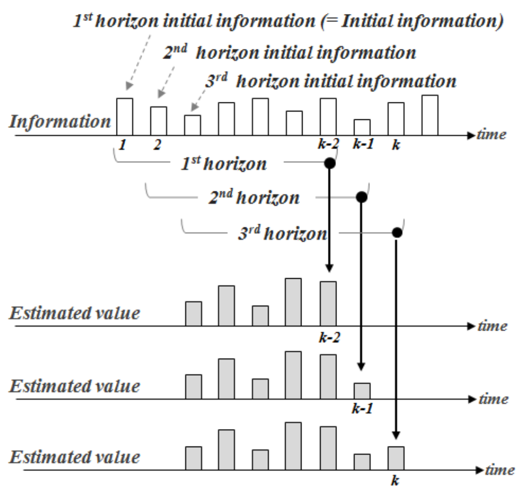

그림 1. FIR 구조의 개념.
Fig. 1. The concept of FIR structure.

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In this paper, the optimal FIR smoother without the assumptions described before is used to estimate unknown inputs. The optimal FIR smoother does not require any heuristic approaches for the initial state and is reduced to the optimal FIR filter when the fixed lag size is set to zero. Moreover, the optimal FIR smoother is derived for the system model with known input. Therefore, the proposed method cam give more general solution than the optimal FIR filtering approach. Due to the structure of the optimal FIR smoother which use additional measurements made after the estimation time, it can be expected that the optimal FIR smoother can give more exact estimate and faster convergence property than the optimal FIR filtering approach. The estimates of the each unknown input is completely independent of the any priori state information and other unknown inputs same as the optimal FIR filtering approach.

This paper is organized as follows. In Section 2, the unknown inputs estimation method by using the optimal FIR smoother will be introduced. In Section 3, numerical example is presented to show the performance of the proposed method. Finally, conclusions are drawn in Section 4.

II. THE OPTIMAL FIR SMOOTHER FOR SYSTEM WITH UNKNOWN INPUTS

Let's consider the system with unknown inputs

$$\tilde{x}_{k+1} = A\tilde{x}_k + Bu_k + Ep_k + G\tilde{w}_k, \tag{1}$$

$$y_k = Cx_k + Fp_k + v_k \tag{2}$$

where $\tilde{x}_k \in R^n$ and $y_k \in R^q$, are the state, the measurement and $u_k \in R^r$ and $p_k \in R^s$ are the known input and the unknown input vector, respectively. The system noise $\tilde{w}_k \in R^p$and the measurement noise $v_k \in R^q$ are zero-mean white Gaussian and mutually uncorrelated. The covariances of $\tilde{w}_k$ and v_k are Q and R, respectively. The pair (A, C) of system (1) and (2) is assumed to be observable. The unknown inputs can be represented by random walk processes as $p_{k+1} = p_k + \delta_k$, where $\delta_k \in R^s$ is a zero-mean Gaussian random process with covariance Q_d[8]. If we assume that the unknown inputs are uncorrelated with each other, the covariance matrix Q_d is to be diagonal matrix.

By augmenting the state and unknown input vector as $\begin{bmatrix}\tilde{x}_k^T & p_k^T\end{bmatrix}^T$, the system model (1) and (2) and the unknown input model represented by a random walk process can be represented in the following augmented system model:

$$\begin{bmatrix}\widetilde{x_{k+1}} \\ p_{k+1}\end{bmatrix} = \begin{bmatrix}A & E \\ 0 & I\end{bmatrix}\begin{bmatrix}\tilde{x}_k \\ p_k\end{bmatrix} + \begin{bmatrix}B \\ 0\end{bmatrix}u_k + \begin{bmatrix}G & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix}\tilde{w}_k \\ \delta_k\end{bmatrix}, \tag{3}$$

$$y_k = \begin{bmatrix}C & F\end{bmatrix}\begin{bmatrix}\tilde{x}_k \\ p_k\end{bmatrix} + v_k. \tag{4}$$

If we define the augmented state as $x_k = \begin{bmatrix}\tilde{x}_k^T & p_k^T\end{bmatrix}^T$, and the augmented system noise as $w_k = \begin{bmatrix}\tilde{w}_k^T & \delta_k^T\end{bmatrix}^T$, then the augmented system model can be rewritten as follows:

$$x_{k+1} = A_a x_k + B_a u_k + G_a w_k, \tag{5}$$

$$y_k = C_a x_k + v_k \tag{6}$$

where

$$A_a = \begin{bmatrix}A & E \\ 0 & I\end{bmatrix},\ B_a = \begin{bmatrix}B \\ 0\end{bmatrix},\ G_a = \begin{bmatrix}G & 0 \\ 0 & I\end{bmatrix},\ C_a = \begin{bmatrix}C & F\end{bmatrix}$$

Since we assume that the system noise $\tilde{w}_k$ and random walk process noise δ_k are zero-mean white Gaussian and they are mutually uncorrelated with each other and v_k, the augmented system noise w_k also is a zero-mean white Gaussian random process with covariance Q_a, which is defined as

$$Q_a = \begin{bmatrix}Q & 0 \\ 0 & Q_d\end{bmatrix}. \tag{7}$$

The pair (A_a, C_a) of the augmented system (5) and (6) is observable so that all modes are observed at the output and stabilized observers can be constructed [8].

Since the optimal FIR smoother is derived without the system input, we now derive the optimal FIR smoother for the general discrete-time systems with the system input. On the horizon $[k-N\ k]$, the optimal FIR smoother for estimation of the augmented state x_{k-h} at time $k-h$ can be represented by linear combination of the finite measurements and inputs as [9]

$$\hat{x}_{k-h} = H_a Y_{k-1} + L_a U_{k-1} \tag{8}$$

at the current time k, where h and N are the fixed-lag size and the size of the finite receding horizon, respectively and the finite measurement vector Y_{k-1} and the finite known input vector U_{k-1} are defined as

$$Y_{k-1} = \begin{bmatrix}y_{k-N}^T\, y_{k-N+1}^T \cdots\, y_{k-1}^T\end{bmatrix}^T \tag{9}$$

$$U_{k-1} = \begin{bmatrix}u_{k-N}^T\, u_{k-N+1}^T \cdots\, u_{k-1}^T\end{bmatrix}^T \tag{10}$$

respectively. H_a and L_a are the impulse responses with finite duration which are chosen to minimize the variance of the estimation error.

Since the optimal FIR smoother should satisfy the unbiased constraint, i.e., $E[\hat{x}_{k-h}] = E[x_{k-h}]$, we have following constraints:

$$H_a \tilde{C}_{aN} = A_a^{N-h},\ L_a = N_{aN} - H_a \tilde{B}_{aN}, \tag{11}$$

where $\tilde{C}_{aN}$, $\tilde{B}_{aN}$ and N_{aN} are given as

$$\tilde{C}_{aN}=\begin{bmatrix} C_a \\ C_aA_a \\ \vdots \\ C_aA_a^{N-1}\end{bmatrix},\ \tilde{B}_{aN}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C_aB_a & 0 & \cdots & 0 & 0 \\ C_aA_aB_a & C_aB_a & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C_aA_a^{N-2}B_a & C_aA_a^{N-3}B_a & \cdots & C_aB_a & 0\end{bmatrix}$$

$$N_{aN}=\left[A_a^{N-h}B_a \ \ A_a^{N-h-1}B_a \cdots A_aB_a \ \ B_a \ 0 \ \cdots \ 0\right].$$

For the augmented system model (3) and (4), the optimal smoother gain matrix H_a can be obtained as [9]

$$H_a=\left[A_a^{N-h}-M_{aN}Q_N\tilde{G}_{aN}^T\Pi_N^{-1}\tilde{C}_{aN}\right]\left(\tilde{C}_{aN}^T \times\Pi_N^{-1}\tilde{C}_{aN}\right)^{-1}\tilde{C}_{aN}^T\Pi_N^{-1}+M_{aN}Q_N\tilde{G}_{aN}^T\Pi_N^{-1} \tag{12}$$

where

$$M_{aN}=\left[A_a^{N-h}G_a \ \ A_a^{N-h-1}G_a \cdots A_aG_a \ \ G_a \ 0 \ \cdots \ 0\right],$$

$$\Pi_N=\tilde{G}_{aN}Q_N\tilde{G}_{aN}^T+R_N,$$

$$Q_N=Q_a\oplus Q_a\oplus\cdots\oplus Q_a,\ R_N=R\oplus R\oplus\cdots\oplus R,$$

$$\tilde{G}_{aN}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C_aG_a & 0 & \cdots & 0 & 0 \\ C_aA_aG_a & C_aG_a & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C_aA_a^{N-2}G_a & C_aA_a^{N-3}G_a & \cdots & C_aG_a & 0\end{bmatrix}.$$

The optimal smoother gain L_a can be obtained from H_a by the unbiased constraint in (11).

Since the optimal smoother gain matrices H_a and L_a can be partitioned into two parts as $H_a=[H_x^TH_p^T]^T$ and $L_a=[L_x^TL_p^T]^T$, (8) can be rewritten as

$$\begin{bmatrix}\hat{x}_k \\ \hat{p}_k\end{bmatrix}=\begin{bmatrix}H_x \\ H_p\end{bmatrix}Y_{k-1}+\begin{bmatrix}L_x \\ L_p\end{bmatrix}U_{k-1} \tag{13}$$

Here, the matrices H_x and L_x represent the gain matrices for the state estimation part and the matrices H_p and L_p represent the gain matrices for the unknown inputs estimation part. Therefore, the estimates of unknown inputs can be represented as follows.

$$\hat{p}_{k-h}=H_pY_{k-1}+L_pU_{k-1} \tag{14}$$

From now on, we check the independency between the estimated each unknown input and the initial states information and other unknown inputs. Firstly, we will show that the estimated unknown input is independent of the initial state information as following theorem.

Theorem 1: The estimate of unknown input $\hat{p}_{k-h,i}$'s $(1\le i\le s)$ of (14) is independent of the horizon initial state information.

Proof: Since matrices H_p and L_p can be partitioned as $H_p=\left[H_{p,1}^T\ H_{p,2}^T\cdots H_{p,s}^T\right]^T$ and $L_p=\left[L_{p,1}^T\ L_{p,2}^T\cdots L_{p,s}^T\right]^T$, the estimate of the ith unknown input $\hat{p}_{k-h,i}$ is derived as

$$\hat{p}_{k-h,i}=H_{p,i}Y_{k-1}+L_{p,i}U_{k-1} \tag{15}$$

From (1) and (2), the finite measurements Y_{k-1} on the horizon $[k-N,k]$ can be expressed as

$$Y_{k-1}=\tilde{C}_Nx_{k-N}+\tilde{E}_NP_{k-1}+\tilde{B}_NU_{k-1}+\tilde{G}_N\tilde{W}_{k-1}+V_{k-1} \tag{16}$$

where

$$\tilde{E}_N=\begin{bmatrix} F & 0 & \cdots & 0 & 0 \\ CE & F & \cdots & 0 & 0 \\ CAE & CE & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N-2}E & CA^{N-3}E & \cdots & CE & F\end{bmatrix},$$

$$\tilde{W}_{k-1}=\left[\tilde{w}_{k-N}^T\,\tilde{w}_{k-N+1}^T\cdots\tilde{w}_{k-1}^T\right]^T,$$

$$V_{k-1}=\left[v_{k-N}^T\,v_{k-N+1}^T\cdots v_{k-1}^T\right]^T,$$

and $\tilde{C}_N$, $\tilde{B}_N$, and $\tilde{G}_N$ are obtained by replacing $A_a\rightarrow A$ $B_a\rightarrow B$, $C_a\rightarrow C$, $G_a\rightarrow G$ in the definitions of $\tilde{C}_{aN}$, $\tilde{B}_{aN}$, and $\tilde{G}_{aN}$, respectively. By using the equation (16), the estimate of the ith unknown input (15) can be rewritten as follows:

$$\hat{p}_{k-h,i}=H_{p,i}\left[\tilde{C}_Nx_{k-N}+\tilde{E}_NP_{k-1}+\tilde{G}_NW_{k-1}+V_{k-1}\right]+\left[H_{p,i}\tilde{B}_N+L_{p,i}\right]U_{k-1},\quad 1\le i\le s. \tag{17}$$

From the unbiased constraint (11), we have

$$H_a\tilde{C}_{aN}=\begin{bmatrix}H_x \\ H_{p,1} \\ \vdots \\ H_{p,s}\end{bmatrix}\left[\tilde{C}_N\ \Lambda_N\right]=\begin{bmatrix}A^{N-h} & \sum_{l=0}^{N-h}A^lE \\ 0 & I\end{bmatrix}, \tag{18}$$

where

$$\Lambda_N=\begin{bmatrix}F \\ CE+F \\ \vdots \\ \sum_{i=0}^{N-1}CA^lE+F\end{bmatrix}.$$

Since the equation (18) gives the following matrix equality:

$$H_{p,i}\tilde{C}_N=0,\ (1\le i\le s), \tag{19}$$

$\hat{p}_{k-h,i}$ is independent of the horizon initial state x_{k-N}.

This completes the proof. ■

Secondly, let's check that the estimated unknown input is independent of other unknown inputs as following theorem.

Theorem 2: The estimate of each unknown input $\hat{p}_{k-h,i}$ $(1\le i\le s)$ of (16) is independent of other unknown inputs.

Proof: The estimate of the ith unknown input $\hat{p}_{k,i}$ (18) can be rewritten as

$$\begin{aligned}\hat{p}_{k-h,i} = H_{p,i}[&\tilde{C}_N x_{k-N} + \tilde{E}_{N,i}P_{k-1,i} + \sum_{j\neq i}\Lambda_{N,j}\, p_{k-N,j} \\ &+ \sum_{j\neq i}\tilde{E}_{N,j}\Delta_{N,j} + \tilde{G}_N W_{k-1} + V_{k-1}] \\ &+ [H_{p,i}\tilde{B}_N + L_{p,i}]\, U_{k-1}, \quad 1 \le i \le s.\end{aligned} \tag{20}$$

where

$$\Delta_{N,j} = \begin{bmatrix} 0 \\ \delta_{k-N,j} \\ \vdots \\ \sum_{i=0}^{N-1}\delta_{k-N+i,j} \end{bmatrix},$$

where $\delta_{k,j}$ is the jth component of δ_k. $\Lambda_{N,j}$ is the jth columns of the matrix Λ_N and $\tilde{E}_{N,j}$ is defined by replacing $E \to E_j$ and $F \to F_j$ of $\tilde{E}_N$ where E_j and F_j are the jth columns of the matrices E and F, respectively.

From the unbiased constraint (18), we can have additional matrix equality as $H_p\Lambda_N = I$. Thus, the following identities are obtained:

$$H_{p,i}\Lambda_{N,i} = 1, \; H_{p,i}\Lambda_{N,j} = 0 (1 \le j \le s, i \neq j). \tag{21}$$

The second equation of (21) implies that the estimate of each unknown input $\hat{p}_{k-h,i}$ is independent of other unknown inputs. This completes the proof. ■

Since the inverse of the augmented system matrix A_a does not appear in the optimal smoother gain H_a, the proposed approach can avoid the singularity problem. Also, the optimal gain matrix H_a [9] is derived without infinite covariance of initial state assumption whereas the optimal FIR filter used in [8] is derived by assuming that the covariance of initial state is infinity. This makes better adaptability to real problems.

Moreover, the unknown input estimation method by using the optimal smoother is more general than the optimal Filtering approach, since the optimal FIR smoother is reduced to the current augmented state x_k when the fixed-lag size h is set to zero. And, since the optimal fixed-lag FIR smoother use more additional information made after the estimation time, it is expected that the proposed method can give more exact estimate and have faster convergence property than the optimal FIR filtering approach.

III. NUMERICAL EXAMPLE

To demonstrate the performance of the proposed estimation method, a numerical example on the following discretized DC motor system [8] is simulated. The corresponding dynamic model is represented as

$$\begin{aligned}\tilde{x}_{k+1} = &\begin{bmatrix} -0.0005 & -0.0084 \\ 0.0517 & 0.8069 \end{bmatrix}\tilde{x}_k + \begin{bmatrix} 0.1815 \\ 1.7902 \end{bmatrix} u_k \\ &+ \begin{bmatrix} 0.0129 & 0 \\ -1.2504 & 0 \end{bmatrix} p_k + \begin{bmatrix} 0.0006 \\ 0.0057 \end{bmatrix}\tilde{w}_k,\end{aligned} \tag{22}$$

$$y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} p_k + v_k. \tag{23}$$

The covariance of system noise, the measurement noise and the random walk process noise are taken as

$$Q = 0.01^2, R = \begin{bmatrix} 0.001^2 & 0 \\ 0 & 0.01^2 \end{bmatrix}, Q_d = \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.1^2 \end{bmatrix},$$

respectively.

In order to design the optimal FIR smoother, the receding horizon size and fixed-lag size are taken as $N=12$ and $h=9$, respectively. Unknown inputs $p_k = [p_{k,1}\; p_{k,2}]$ applied to the DC motor system (22) and (23) are set as

$$p_{k,1} = \begin{cases} 0.5, (100 \le k \le 300) \\ 0, \; (otherwise) \end{cases}, p_{k,2} = \begin{cases} 0.3, (200 \le k \le 400) \\ 0, \; (otherwise) \end{cases}.$$

The estimates and estimation errors of the optimal FIR smoother based approach and the optimal filter based approach are compared in Figs. 2, 3, and 4. It is shown that the estimate of each unknown input does not affect other estimates of unknown inputs in Fig. 2.

As shown in Figs. 2 and 3, the estimate of the proposed method is converged to the real values much faster than that of the optimal FIR filter based approach at time of unknown input occurrence. Moreover, Fig. 4 shows that the estimation error of the optimal FIR smoother based approach is much smaller than the optimal FIR filter based approach.

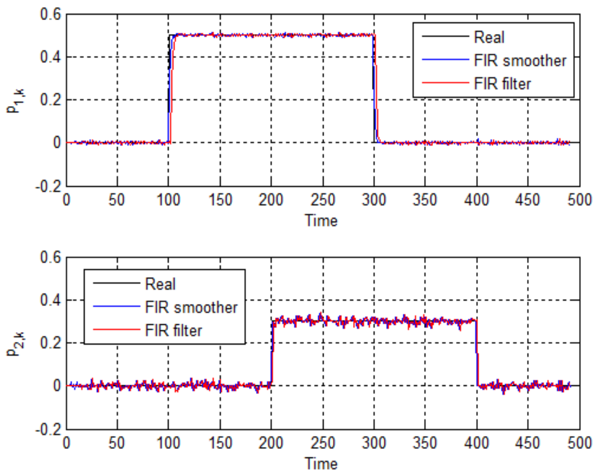

그림 2. DC 모터에 인가된 미지 입력과 추정치들.
Fig. 2. Unknown inputs applied to the DC motor and estimates.

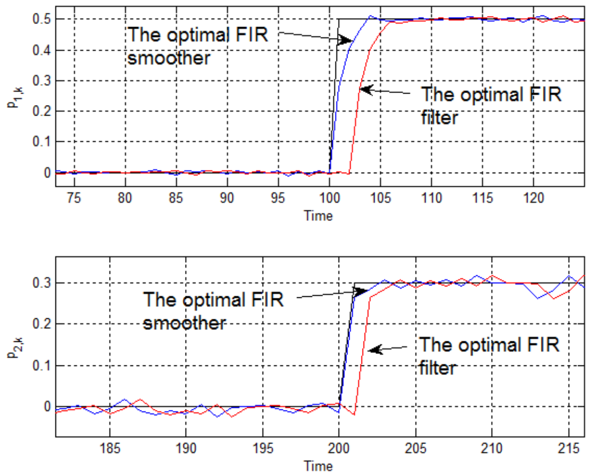

그림 3. 실제 미지 입력과 추정된 미지입력 ($k=100$, $k=200$).
Fig. 3. Real unknown inputs and estimates ($k=100$, $k=200$).

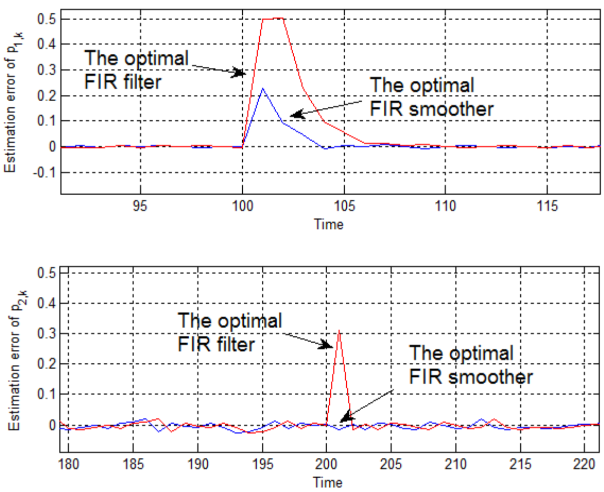

그림 4. 추정 오차 ($k=100$, $k=200$).
Fig. 4. The estimation errors ($k=100$, $k=200$).

IV. CONCLUSION

In this paper, the unknown input estimation method by using the optimal FIR smoother was proposed for linear discrete-time systems. The optimal FIR smoother applied to estimate the unknown inputs does not require any heuristic approaches and assumptions such as infinite covariance of initial state and nonsingular system matrix. Since the optimal FIR smoother can be reduced to the optimal FIR filter by setting the fixed-lag size as zero, the proposed method gives more general solution than the previous optimal FIR filtering approach. It was also shown that the estimate of each unknown input was completely independent of the initial state and other unknown inputs. Moreover, since the optimal FIR smoother use additional measurements made after the estimation time, the proposed approach can give more exact estimate and fast convergence property than the optimal FIR filtering approach. To demonstrate the performance of the proposed method, a numerical example was given for the discretized DC motor system. From the simulation result, it was shown that the proposed method has much better tracking and estimating ability than the optimal filtering based approach.

REFERENCES

[1] J. White and J. Speyer, "Detection filter design: Spectral theory and algorithms," *IEEE Transactions on Automatic Control*, vol. 7, no. 32, pp. 593-603, 1987.

[2] P. Frank, "Fault diagnosis in dynamic systems using analytic and knowledge-based redundancy-a survey and some new results," *Automatica*, vol. 3, no. 26, pp. 459-474, 1990.

[3] J. Park and G. Rizzoni, "An eigenstructure assignment algorithm for the design of fault detection filters," *IEEE Transactions on Automatic Control*, vol. 7, no. 39, pp. 1521-1524, 1994.

[4] B. Friedland, "Treatment of bias in recursive filtering," *IEEE Transactions on Automatic Control*, vol. 14, no. 4, pp. 359-367, 1969.

[5] M. B. Ignagni, "Separate-bias Kalman estimator with bias state noise," *IEEE Transactions on Automatic Control*, vol. 3, no. 33, pp. 338-341, 1990.

[6] H. Heffes, "The effect of erroneous models on the Kalman filter response models," *IEEE Transactions on Automatic Control*, vol. 11, no. 3, pp. 541-543, 1966.

[7] R. J. Fitzgerald, "Divergence of the Kalman filter," *IEEE Transactions on Automatic Control*, vol. 16, pp. 736-747, 1971.

[8] S. H. Park, P. S. Kim, O. Kwon, and W. H. Kwon, "Estimation and detection of unknown inputs using optimal FIR filter," *Automatica*, vol. 36, no. 10, pp. 1481-1488, 2000.

[9] B. K. Kwon, S. Han, O. K. Kwon, and W. H. Kwon, "Minimum variance FIR smoothers for discrete-time state space models," *IEEE Signal Processing Letters*, vol. 14, no. 8, pp. 557-560, 2007.

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