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Resonance tunneling phenomena by periodic potential in type-II superconductor

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Abstract

We calculated the resonance tunneling energy band in the BCS gap for Type-II superconductor in which periodic potential is generated by external magnetic flux. In this model, penetrating magnetic flux was assumed to be in a fixed lattice state which is not moving by an external force. We observed the existence of two subbands when we used the same parameters as for the $Nd_{1.85}Ce_{0.15}CuO_x$ thin film experiment. The voltages at which the regions of negative differential resistivity (NDR) started after the resonant tunneling ended were in a good agreement with the experimental data in the field region of 1 T - 2.2 T, but not in the high field regions. Discrepancy occurred in the high field region is considered to be caused by that the potential barrier could not be maintained because the current induced by resonant tunneling exceeds the superconducting critical current. In order to have better agreement in the low field region, more concrete designing of the potential rather than a simple square well used in the calculation might be needed. Based on this result, we can predict an occurrence of the electromagnetic radiation of as much difference of energy caused by the 2nd order resonant tunneling in which electrons transit from the 2nd band to the 1st band in the potential wells.

Keywords: Resonance Tunneling, NDR, Flux-Flow, $Nd_{1.85}Ce_{0.15}CuO_X$,

1. INTRODUCTION

Negative differential resistivity (NDR) was reported to occur near the early part of resistivity on the field-dependent I-V characteristic curves in $Nd_{1.85}Ce_{0.15}CuO_x$ (NCCO) thin film measured at T = 1.92 K [1, 2]. One of reasons for an occurrence of the NDR phenomenon is the resonance tunneling effect (Fig. 1) [3, 4]. Periodic potential barriers are necessities of the resonance tunneling effect and in Type-II superconductors, periodic potential barriers can exist near the Fermi surface due to the penetrated magnetic flux [5].

In this paper, we assumed the occurrence of resonance tunneling effect of charge carrier by periodic potential barriers. By plotting the subband structure in the periodic potential barriers, we can predict the starting voltage of NDR as an indicator of resonance tunneling phenomena and the electromagnetic wave radiation. A better understanding about this phenomenon would help to study magnetic flux dynamics in type-II superconductor since the periodic potential barriers provide an ideal model right before magnetic flux flow occurs..

2. THEORY

Contrary to ordinary tunneling effect, resonance tunneling transmission shows much higher probability than general transmission probability in spite of potential barriers. Resonance tunneling effect needs two barriers and wells with the same condition inside barriers. For example is a quantum dot. Equation (1), calculated by the propagation matrix method at a single quantum dot, represents the transmission probability at square potential quantum dot.

$$T = \left(1+\left(\frac{\beta^2-k^2}{2\beta k}\right)^2 \sin^2(kL)\right)^{-1} \quad \left(\beta=\sqrt{\left[2m(U-E)\right]/\hbar^2}\right) \tag{1}$$

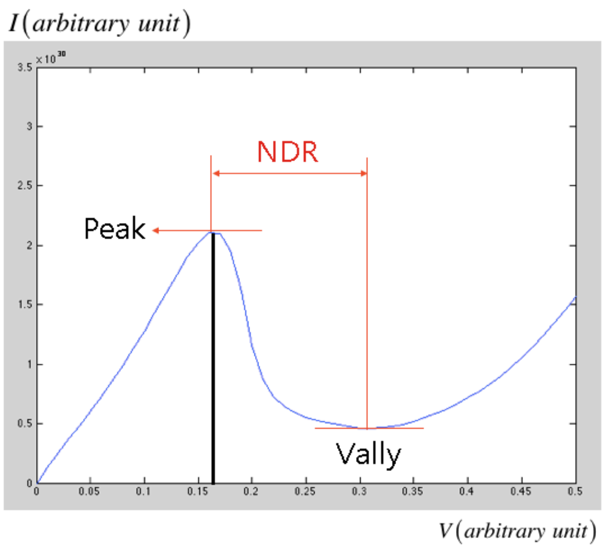

Fig. 1. I-V characteristics by resonance tunneling.

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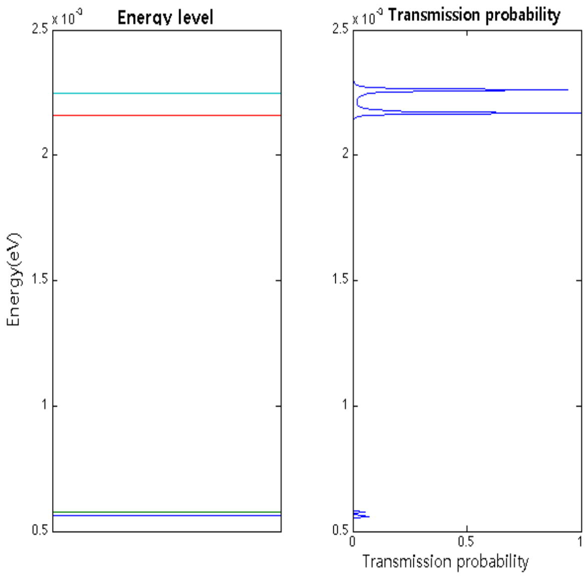

Fig. 2. Condition of resonance tunneling.

where k is a wave number in a potential well region, β is a wave number in a potential region, U is the potential energy, and L is a well width. When the sine function is 0, then the transmission probability T has a maximum value and this is a condition for the standing wave of particle wave function at one-dimensional square potential well. In other words, the transmission probability is a maximum at quantized energy levels in a quantum dot.

Fig. 2 compares the energy levels and the transmission probability calculated about all the energy levels for an arbitrary double quantum dot. It is confirmed that high transmission probability was obtained near the energy levels.

If the potential barriers are inclined linearly due to the potential difference between two ends, then the sub-bands inside the potential barriers are inclined with the same slope as shown in Fig. 3 [3].

$$V_c = \frac{\Delta E}{\lambda} X_c \quad (2)$$

where V_c in Eq. (2) is a starting voltage of NDR and when resonance tunneling can maximum slop. X_c is a length of the sample, ΔE is a width of energy band, λ is a length of period.

When an electron tunnels from (N+1)th quantum well to (N+2)th quantum well by resonance tunneling as shown in Fig. 4, an electron can transmit if the energy level of the corresponding quantum well is lower. In this case, electromangetic radiation corresponding to the energy gap could occur. If the energy gap between the subbands is large enough, detection of electromagnetic radiation is expected [6].

To explain the magnetic field and temperature dependence of the flux-flow resistance steps, Huebener *et al.* proposed that two relevant sub-bands exist between the Fermi energy and the gap energy in NCCO at H = 1.3 T [1].

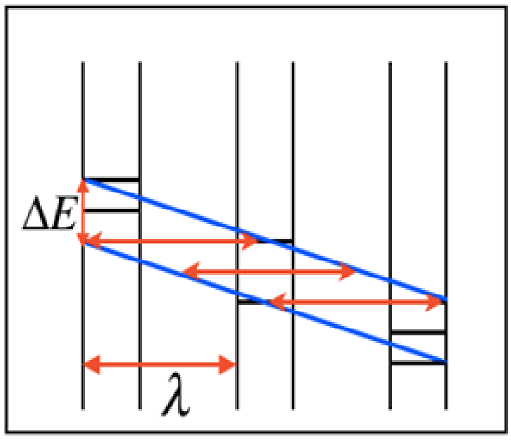

Fig. 3. Maximum possible slope for resonance tunneling.

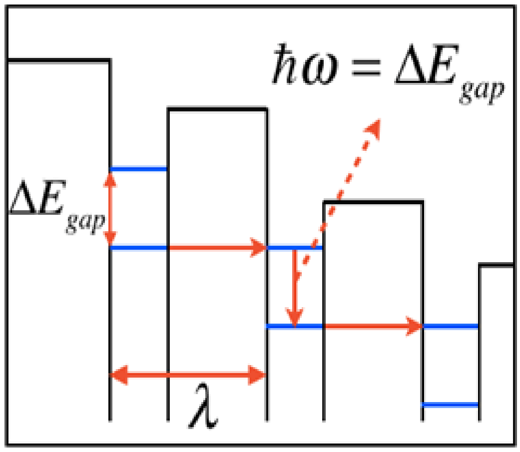

Fig. 4. Emission of the electromagnetic wave by resonance tunneling.

There may be two ways to verify the resonance tunneling phenomena occurred by magnetic flux penetration. First is a comparison of the calculated values of V_c with the experimental data, and second is a detection of electromagnetic radiation between two subbands due to the second-order resonance tunneling (Fig. 4).

3. SIMULATION METHOD

In order to calculate the subband structure, periodic potential function was constructed (Fig. 5) by using the NCCO parameters listed in Table. 1. Then we calculated the subband structure by using time-independent Schrödinger equation about single particle without charge (it is assumed to be a simple particle). And we obtained eigenvalues of the potential function.

If subbands are obtained by using the Kronig-penny model in k-space, it can be easily observed that the shape of subbands changes as the magnetic field increases from 1 T to 3 T. However, the energy levels for the tilted potential barriers with bias voltage could not be calculated.

TABLE I
SAMPLE PARAMETERS USED IN CALCULATION.

PARAMETER NAME	QUANTITY
SAMPLE	$Nd_{1.85}Ce_{0.15}CuO_X$ THIN FILM
T_C	24 K
TEMPERATURE	1.92 K
GAP PARAMETER	4 MEV
COHERENCE LENGTH	8 NM
EFFECTIVE MASS RATIO (M*/M)	0.9
LENGTH	100 μM
WIDTH	40 μM
THICKNESS	0.09 μM

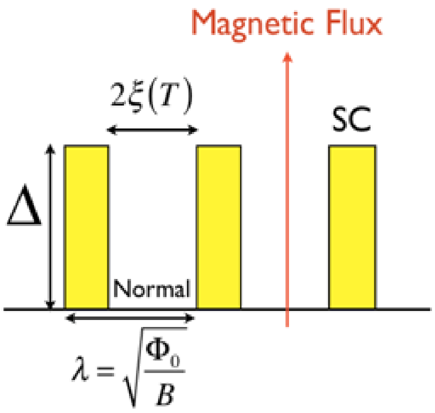

Fig. 5. Modeling of potential function.

Therefore, we solved the Schrödinger equation by using the Matlab to obtain a change in energy levels for a virtual sample (sample length is about 500 nm) when the potential is tilted by applying voltage. Simulation is focused to confirm that this model can be applied to explain the experimental results, so we used a simple square potential model and a simple particle problem. The Hamiltonian is

$$\hat{H}\psi(x)=\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U(x)\right]\psi(x)=E\psi(x) \qquad (3)$$

where $U(x)$ is a potential function as shown in Fig. 5. Minimum length step is 1nm and total length is designed to be 500 nm. Since the minimum length step was set to be 1 nm, the differential term in the Hamiltonian equation was modified as the following.

$$\frac{d^2}{dx^2}\psi(x_j)\rightarrow\frac{\psi(x_{j+1})-2\psi(x_j)+\psi(x_{j-1})}{h^2} \qquad (4)$$

where h is not a Planck constant but a minimum length step to find a slope in finite region, and was set to be 1 nm in the calculation. If the differential term is replaced by (4), the eigenvalue problem with the Hamiltonian becomes

$$\hat{H}\psi(x_j)=-L_{j+1}\psi(x_{j+1})+D_j\psi(x_j)-L_j\psi(x_{j-1})=E\psi(x_j) \qquad (5)$$

where $L=\frac{\hbar^2}{2mh^2}$, $D=\frac{\hbar^2}{mh^2}+U(x_j)$.

The matrix was constructed as in (6) and then the eigenvalues were obtained by using the Matlab [7].

$$(\hat{H}-EI)\psi=\begin{bmatrix}(D_1-E) & -L_2 & & & \cdots \\ -L_2 & (D_2-E) & -L_3 & & \cdots \\ 0 & -L_3 & (D_3-E) & -L_4 & \cdots \\ \vdots & \vdots & \vdots & \ddots & -L_N \\ & & & -L_{N-1} & (D_N-E)\end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \vdots\\ \psi_{N-1}\end{bmatrix}=0 \qquad (6)$$

Since the slope of the potential barriers depends on a length of the sample, we first calculated the Schrödinger equation for a finite potential function, then corrected it to the length of a real sample according to the following formula.

$$\frac{V_c}{X_c}X_s=V_s \qquad (7)$$

where V_c is the calculated voltage, X_c is the length of a virtual sample, X_s is the length of a real sample, and V_s is the voltage for a real sample corrected from the result of virtual sample.

4. RESULTS

First of all, we confirmed the existence of two subbands in agreement with Huebener's prediction. We calculated the starting voltage of NDR for each magnetic field by using the width of the first subband and compared the values of the voltage with the experimental data. As shown in Fig. 6, the calculated values matched well with the experimental data only for H = 2.2 T and showed discrepancies in high and low field regions. The discrepancy in high magnetic fields could be explained by that the superconducting state was broken by the critical current even before the NDR region started. The deviation in low field region is considered to come from using a very simple square well potential. In addition, the Hamiltonian used for a simple particle need to be reconstructed for a charged particle since the periodic potential well is the area where the magnetic flux is penetrated. In spite of using very simple model, the starting voltage of NDR was found to show a similar trend with the experimental data in mV scale.

The 2nd order resonance tunneling may be possible due to the two-subband structure. The change in the energy levels due to inclined potential barriers by applying voltage was calculated and shown in Fig. 7. When applied voltage is 0, most of energy levels were degenerated in subbands and they were undegenerated as the potential barriers were tilted by applying voltage. The spread of the energy levels were found to be linear to the voltages applied. As shown in Fig. 7, the energy level of the 1st band in (ground +1) well matched with the energy level of the 2nd band in ground well and the 2nd order resonance tunneling could occur.

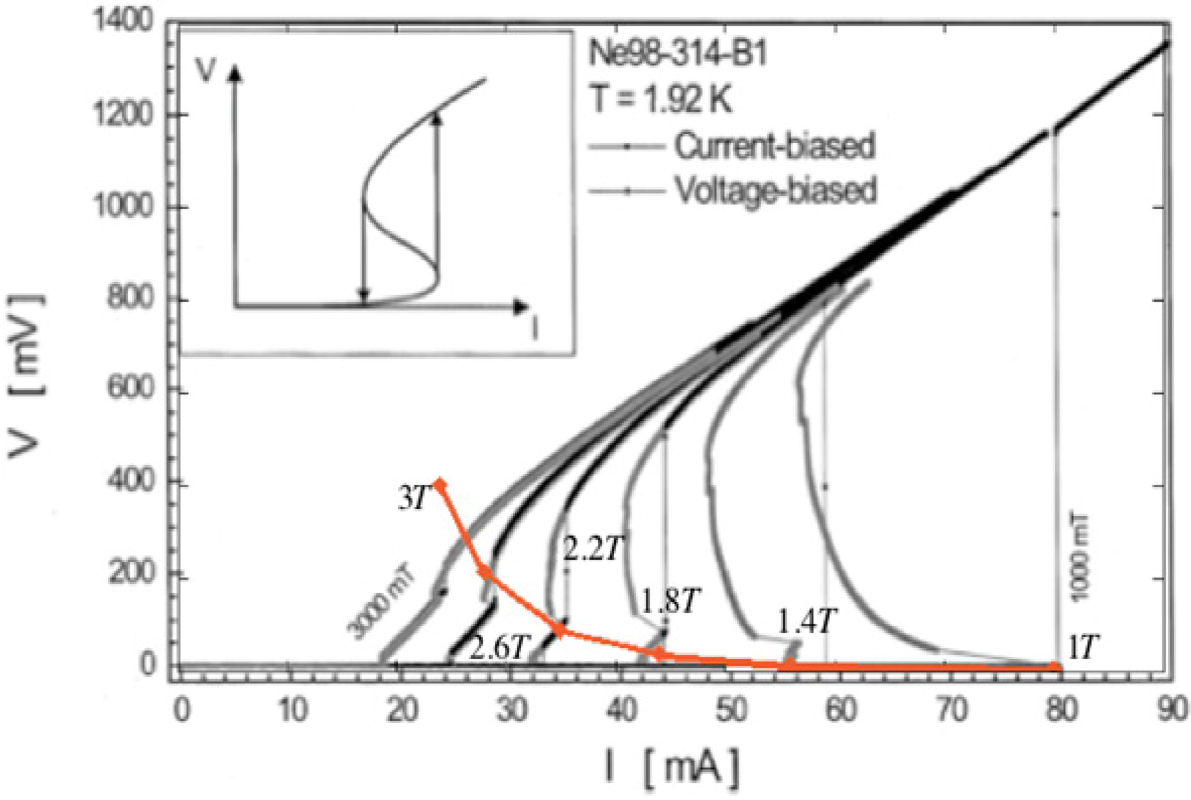

Fig. 6. Comparison of the calculation of the starting voltage of NDR and the experiment result of NCCO.

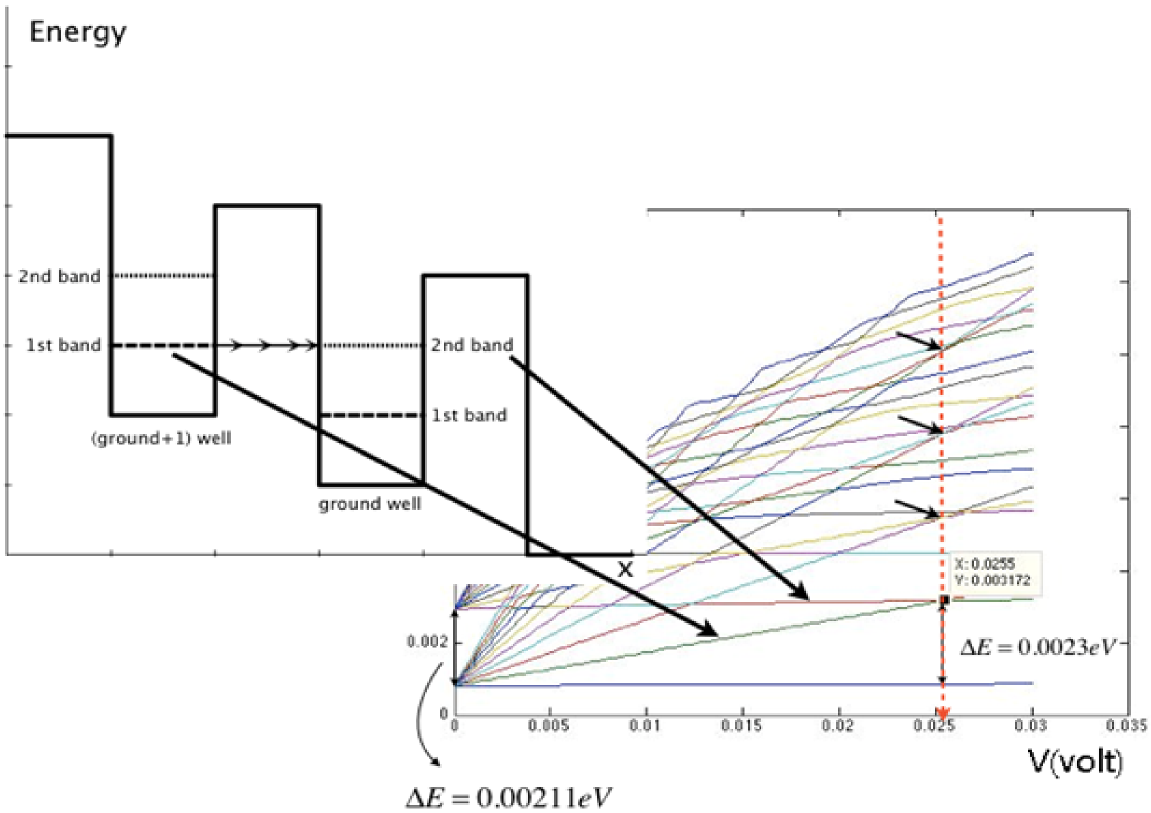

Fig. 7. Calculation result of voltages for the 2nd order resonance tunneling. The red dotted line indicates the starting voltages of 2nd order resonance tunneling.

We calculated the voltages where the 2nd order resonance tunneling occurs for each magnetic field and the results were represented in Fig. 8. For example, the voltages are about 4.5 V and 3.5 V for H = 1 T and 0.5 T, respectively. According to the experimental data of NCCO, the superconducting state changed to the normal state at 1.2 V, so no information on the 2nd order resonance tunneling could be obtained in this experiment. Possibility of the 2nd order resonance tunneling may be obtained either by using the fact that the voltage for the 2nd order resonance tunneling decreases and the critical current increases as magnetic field increases, or by changing the materials and/or the experimental conditions.

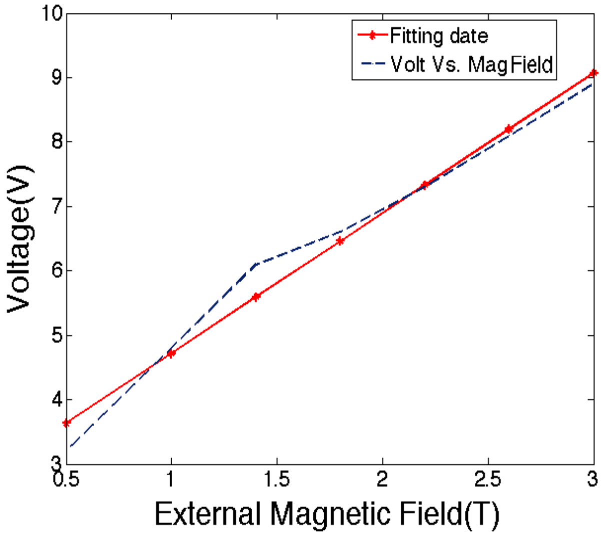

Fig. 8. Calculation of the 2nd-order resonance tunneling voltage as a function of magnetic field.

5. CONCLUSION

We proposed that periodic potential was generated near the Fermi energy by penetrated magnetic flux in Type-II superconductors and resonance tunneling phenomena of charge carriers through these potential barriers could occur. Indication of the NDR to be result of the resonance tunneling by periodic potential is that the calculation shows a similar tendency with the starting voltages of NDR.

According to the calculation, the 2nd order resonance tunneling and furthermore, electromagnetic radiation in IR region were expected to occur due to the existence of two subbands. However, the voltage where the 2nd order resonance tunneling occurs was high enough for the superconducting state to be changed to the normal state. In order to clarify the possibility of resonance tunneling, experiments need to be conducted in different conditions.

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