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Optimum time-censored ramp soak-stress ALT plan for the Burr type XII distribution

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Abstract. Accelerated life tests (ALTs) are extensively used to determine the reliability of a product in a short period of time. Test units are subject to elevated stresses which yield quick failures. ALT can be carried out using constant-stress, step-stress, progressive-stress, cyclic-stress or random-stress loading and their various combinations. An ALT with linearly increasing stress is ramp-stress test. Much of the previous work on planning ALTs has focused on constant-stress, step-stress, ramp-stress schemes and their various combinations where the stress is generally increased. This paper presents an optimal design of ramp soak-stress ALT model which is based on the principle of Thermal cycling. Thermal cycling involves applying high and low temperatures repeatedly over time. The optimal plan consists in finding out relevant experimental variables, namely, stress rates and stress rate change points, by minimizing variance of reliability function with pre-specified mission time under normal operating conditions. The Burr type XII life distribution and time-censored data have been used for the purpose. Burr type XII life distribution has been found appropriate for accelerated life testing experiments. The method developed has been explained using a numerical example and sensitivity analysis carried out.

Key Words: *Accelerated life test, ramp soak-stress, thermal cycling, type-I censoring, variance of reliability function*

1. INTRODUCTION

The business environment of the twenty-first century is characterized by the intense global competition. As design and manufacturing technologies become more advanced it is difficult to obtain the reliability information of products such as mean time to failure within a short period of time under normal operating conditions. This problem is overcome by Accelerate Life Tests (ALTs) wherein the units are subjected to higher stress

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levels (for example, higher temperature, voltage, humidity and pressure levels) that yield quick failures. Failure data collected from ALT are then extrapolated by means of a regression model to estimate failure distribution under design condition.

Introduced by Chernoff (1962), and Bessler, Chernoff and Marshall (1962) acceleration of life test may be carried out in fully accelerated or partially accelerated environment. In fully accelerated life testing all the test units are run at accelerated condition, while in partially accelerated life testing they are run at both normal and accelerated conditions. The term fully accelerated life test has been coined by Bhattacharya and Soejoeti (1989), and the term partially accelerated life test is due to Degroot and Goel (1979). The fully accelerated life test is widely referred to as an accelerated life test in the literature, and therefore the two terms can be used interchangeably.

Stress under accelerated condition can be applied using constant-stress, step-stress, progressive-stress, cyclic-stress, random-stress, or combinations of such loadings. The choice of a stress loading depends on how the product or unit is used in service and other practical and theoretical limitations (Nelson (1980), Elsayed (1996)). A ramp-stress test results when stress is increased linearly. In particular, a ramp test with two linearly increasing stresses is a simple ramp test. Ramp tests are used for example in fatigue testing (Prot (1948)), capacitors (Endicott, Hatch and Schmer (1965), Starr and Endicott(1961)), insulation (Goba(1969), Solomon, Klein and Albert(1976)), and integrated circuits (Chan(1990)).

Optimum constant-stress, step-stress and ramp-stress ALT models as well as constant-stress and step-stress PALT models have been studied extensively in the literature. Most of the work on constant-stress ALT models have been summarized in Nelson (1990), Meeker and Escobar (1998). See also Nelson (2005a, 2005b). Gouno and Balakrishnan (2001) have provided a concise review of step-stress ALTs. Bai and Chung (1992) have considered the optimum design of the ramp tests with two stress rates for the Weibull distribution under type-I censoring. Bai and Chun (1993) have developed nonparametric inferences for ramp-stress tests under random censoring. Bai, Chun and Cha (1996) have obtained an optimum time-censored ramp test for items with the Weibull life distribution when there is stress upper bound. Bai and Chung (1992) have considered optimal designs for constant PALTs and step-stress PALTs under type-I censoring. Bai, Chun and Chung (1993) have used the maximum likelihood method to estimate the scale parameter and the acceleration factor for the log-normally distributed lifetime using constant-stress as well as step-stress PALTs and type-I censored data. See also Srivastava and Shukla((2008a, 2008b),(2009)), Srivastava and Mittal ((2010),(2012a, 2012b, 2012c),(2013a, 2013b, 2013c)) and the references therein. Park and Yum (1998) are the first to propose the optimum ALT plan under modified stress loading methods. Fei (2000), Wang (2001), Gao, Hu, Shi and Qin (2008) and Srivastava and Mittal (2013c) have also worked on modified stress loading methods.

In cyclic-stress testing, the stress level is changed according to a fixed pattern. For many products, the frequency and length of a cycle affect the lifetime of a product, so they are included in the model as stress variable. Common examples of such stress are thermal cycling, sinusoidal vibration, and triangular cyclic vibrations (Elsayed(2013)). Insulation under AC voltage exhibit sinusoidal stress. Also many metal components repeatedly undergo a mechanical stress cycle.

However, no work seems to have been done so far on the design of optimum ramp soak-stress ALT model. This model has its application in Thermal cycling (Yang (2007)). In this paper we have proposed optimum ramp soak ALT model using Burr Type XII life distribution and time-censored data. The Burr Type XII distribution has been found appropriate for accelerated life testing experiments (Soliman (2005)). The optimum plans consist in finding out optimum stress change point(s) and optimum stress rates by minimizing asymptotic variance of reliability function with pre-specified mission time.

Acronyms

ALT	accelerated life test
cdf	cumulative distribution function
pdf	probability density function
Asvar	Asymptotic variance

Notations

B_m	$\{t \mid t_m < t \le t_{m+1}\},\ m = 0,1,2,3,4,5$ where, $t_0 = 0, t_5 = \eta, t_6 = \infty$
$G_1(t)$	$\dfrac{e^{-\gamma_0} s_0^{-\gamma_1}((s_0+\beta_1 t)^{1+\gamma_1} - (s_0)^{1+\gamma_1})}{\beta_1(1+\gamma_1)}$
$G_2(t)$	$e^{-\gamma_0}\left(\dfrac{s_0}{s_1}\right)^{-\gamma_1}(t-t_1)$
$G_3(t)$	$\dfrac{e^{-\gamma_0} s_0^{-\gamma_1}(s_1^{1+\gamma_1} - (s_1-\beta_2(t-t_2))^{1+\gamma_1}}{\beta_2(1+\gamma_1)}$
$G_4(t)$	$e^{-\gamma_0}\left(\dfrac{s_0}{s_2}\right)^{-\gamma_1}(t-t_3)$
$G_5(t)$	$\dfrac{e^{-\gamma_0} s_0^{-\gamma_1}\left((s_2+\beta_3(t-t_4))^{1+\gamma_1} - s_2^{1+\gamma_1}\right)}{\beta_3(1+\gamma_1)}$
burr(c, k α)	Burr type XII distribution with parameters c, k, α
c, k	Shape parameters of Burr type XII distribution, $c > 0,\ k > 0$
$a_1(t)$	$G_1(t_1)+G_2(t)$
$a_2(t)$	$G_1(t_1)+G_2(t_2)+G_3(t)$
$a_3(t)$	$G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)$
$a_4(t)$	$G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)$
$\delta_j = \delta_j(t)$	Indicator function: $\delta_j(t) = \begin{cases} 1, \text{ if } t \in B_m,\ m = 0,1,2,3,4,5 \\ 0, \text{ otherwise} \end{cases}$
n	Total number of test units
Q_1	$kce^{-\gamma_0} s_0^{-\gamma_1}$

$R_0(t)$	$\dfrac{((s_0+t\beta_1)^{(1+\gamma_1)}\ln(s_0+t\beta_1)-s_0^{(1+\gamma_1)}\ln s_0)}{(s_0+t\beta_1)^{(1+\gamma_1)}-s_0^{(1+\gamma_1)}}$
$R_1(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}((s_0+t\beta_1)^{(1+\gamma_1)}\ln(s_0+t\beta_1)-s_0^{(1+\gamma_1)}\ln s_0)}{\beta_1(1+\gamma_1)}$
$R_2(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}(s_1^{(1+\gamma_1)}\ln s_1-(s_1-(t-t_2)\beta_2)^{(1+\gamma_1)}\ln(s_1-(t-t_2)\beta_2))}{\beta_2(1+\gamma_1)}$
$R_3(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}((s_2+(t-t_4)\beta_3)^{(1+\gamma_1)}\ln(s_2+(t-t_4)\beta_3)-s_2^{(1+\gamma_1)}\ln s_2))}{\beta_3(1+\gamma_1)}$
$Q_0(t)$	$\dfrac{((s_0+(t\beta_1)^{(1+\gamma_1)}(\ln(s_0+t\beta_1))^2-s_0^{(1+\gamma_1)}(\ln s_0)^2)}{(s_0+(t\beta_1)^{(1+\gamma_1)}-s_0^{(1+\gamma_1)}}$
$Q_1(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}((s_0+t\beta_1)^{(1+\gamma_1)}(\ln(s_0+t\beta_1))^2-s_0^{(1+\gamma_1)}(\ln s_0)^2)}{\beta_1(1+\gamma_1)}$
$Q_2(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}(s_1^{(1+\gamma_1)}(\ln s_1)^2-(s_1-(t-t_2)\beta_2)^{(1+\gamma_1)}(\ln(s_1-(t-t_2)\beta_2))^2)}{\beta_2(1+\gamma_1)}$
$Q_3(t)$	$\dfrac{e^{-\gamma_0}s_0^{-\gamma_1}((s_2+(t-t_4)\beta_3)^{(1+\gamma_1)}(\ln(s_2+(t-t_4)\beta_3))^2-s_2^{(1+\gamma_1)}(\ln s_2)^2)}{\beta_3(1+\gamma_1)}$
s_0	Design stress
s_1	High stress
s_2	Low stress
$s(t)$	Stress at time t
t_1^*	Optimal time at which stress reaches s_1
t_2^*	Optimal time at which stress leaves s_1
t_3^*	Optimal time at which stress reaches s_2
t_4^*	Optimal time at which stress leaves s_2
^	Maximum Likelihood estimate
α	Scale parameter of Burr type XII distribution, $\alpha > 0$
β_i	Stress rate, $\beta_i > 0, i = 1,2,3$
$\varepsilon(t)$	Cumulative Exposure at time t
γ_0, γ_1	Parameters of the inverse power law, $\gamma_1 > 0,\ -\infty < \gamma_0 < \infty$
η	Censoring time, $\eta > 0$
L_1	$\dfrac{2\ln s_0}{(1+\gamma_1)}+\dfrac{2}{(1+\gamma_1)^2}+(\ln s_0)^2$
L_2	$(-\ln s_0-\dfrac{1}{1+\gamma_1})$

2. The Model

The ramp soak-stress ALT model finds its application in thermal cycling that involves applying high and low temperatures repeatedly over time (Yang (2007)). The variables that define the profile of a thermal cycle include high temperature (T_{max}), low temperature (T_{min}), dwell time (soak time) at high temperature (t_{max}), dwell time at low temperature (t_{min}), and rate of temperature change (dT/dt), shown graphically in Figure 1. Thermal cycling is widely used in environmental stress testing. Automotive engine components experience this type of stress when the engine is ignited in cold weather or when the vehicle is driven through a flooding road. The engine components have to withstand rapidly increasing temperature in the former situation and a sharp temperature drop in the latter. Besides heavy metal parts; the products needing ramp and soak control are glass and ceramic. A slow ramp minimizes the risk of distortion or cracking due to differential thermal expansion within the work. It also helps to avoid temperature overshoot as ramp stops and becomes a fixed temperature for a specified dwell time, sometimes called the soak segment. At this stage the dwell time is set long enough to ensure that the parts attain a uniform temperature throughout and maintain it long enough to complete that stage of the processing. More important, thermal cycling is effective in precipitating fatigue failures in a test, especially for connections between two different materials, such as die attachments, wire bonds, and the plated wires of electronic products.

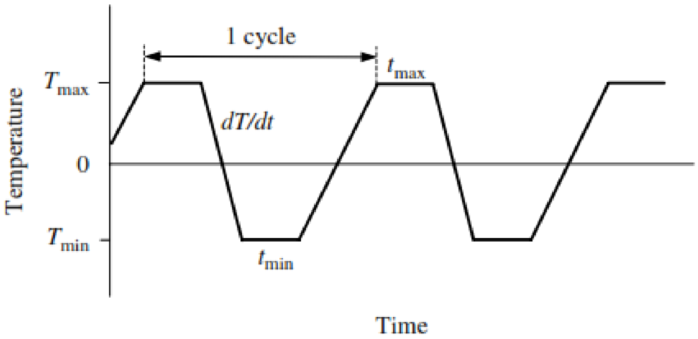

Figure 1. Thermal cycling Profile

2.1. Assumptions

a) The lifetimes of test units are independent and identically distributed.
b) The censoring time η is pre-specified.
c) At any constant stress s, the lifetime of a unit follows a Burr type XII model with scale parameter $\alpha(s)$ and shape parameters c & k, and the inverse power law holds for $\alpha(s)$:

$$\alpha(s) = e^{\gamma_0} (s_0 / s)^{\gamma_1}. \tag{1}$$

d) The shape parameter ‘c’ does not depend on the stress level, and the shape parameter ‘k’ is assumed to be known for the sake of mathematical convenience.

e) For the effect of changing stress levels, a cumulative exposure model holds ((Nelson (1980), (1990)), Nilsson (1985), Yin and Sheng (1987)).The cumulative exposure model is given by :

$$\varepsilon(t) = \int_0^t \frac{du}{\alpha(s(u))} \quad , \tag{2}$$

where $\alpha(\bullet)$ is defined in (1) , and s($\bullet$) is a function of time.

2.2. Test procedure

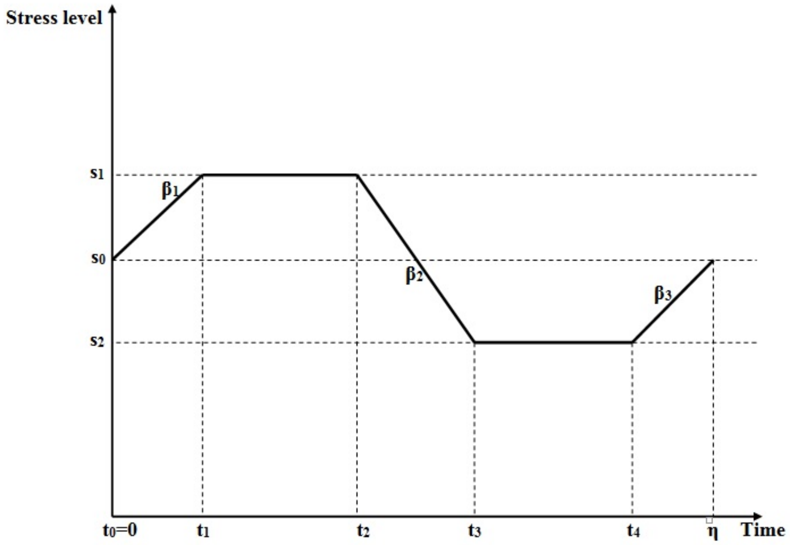

Figure 2. Ramp soak-stress ALT plan

The ramp soak-stress ALT proceeds as follows,

a) n test units are put to test.
b) The stress applied to a test unit is continuously increased with constant rate β_1 from s_0 till stress level s_1 at which the stress level is maintained for some time after which the stress applied is decreased with constant rate β_2 up to stress level s_2 at which it is maintained for some time, and then stress is again increased with constant rate β_3 till the censoring time η or till it fails, whichever occurs earlier (see Figure 2).
c) Test units are subject to type-I censoring with the censoring time η .
d) The stress is a function of time or it is directly proportional to time;
e) The test is continued until:
   i) all test items fail, or
   ii) a prescribed censoring time

whichever occurs earlier, and the test conditions remain the same.

2.3. Burr type XII Life distribution

The Burr type XII distribution has a non-monotone hazard function, which can accommodate many shapes of hazard function. Zimmer, Keats and Wang (1998) have fitted Burr type XII distribution to the data from Nelson (1982) on the times to breakdown of an insulating fluid between electrodes at a voltage of 34 KV/min. They have also provided some other examples where Burr type XII distribution can be used as failure time distribution. This distribution has been found appropriate for accelerated life testing experiments (see Soliman (2005)).

The pdf and cdf, respectively of Burr type XII distribution are:

$$g(t;c,k,\alpha)=(kc/\alpha)\cdot(t/\alpha)^{c-1}\cdot(1+(t/\alpha)^{c})^{-(k+1)},\, t\geq 0, c>0, k>0, \alpha>0 \tag{3}$$

$$G(t;c,k,\alpha)=1-(1+(t/\alpha)^{c})^{-k},\, t\geq 0, c>0, k>0, \alpha>0 \tag{4}$$

where c and k are shape parameters and α is scale parameter. The Burr type XII distribution is unimodal, and its mode is $T_{mode}=\alpha\ [(c-1)/(ck+1)]^{1/c}$ if $c>1$; and the pdf is L-shaped if $c\leq 1$.

The reliability function and hazard function are given, respectively by

$$R(t)=(1+(t/\alpha)^{c})^{-k},\, t\geq 0, c>0, k>0, \alpha>0 \tag{5}$$

$$h(t)=(kc/\alpha)\cdot(t/\alpha)^{c-1}(1+(t/\alpha)^{c})^{-1},\, t\geq 0, c>0, k>0, \alpha>0\,. \tag{6}$$

The Weibull life distribution follows from the Burr type XII distribution as $k\to\infty$, such that $\alpha=k^{1/c}$. The exponential life distribution also follows from the Burr type XII distribution as $k\to\infty$, such that $\alpha=k^{1/c}$, and $c=1$, and the log-logistic distribution is a particular case of this distribution, as for k = 1, the distribution reduces to the log-logistic distribution. Tadikamalla (1980) has summarized relationship between Burr type XII distribution and various other distributions, namely, the Lomax, the Compound Weibull, the Weibull-Exponential, the Log-logistic, the Logistic, the Weibull and the Kappa family of distribution.

2.4. Life distribution under ramp soak stress

Based on the inverse power law (see assumption (c)), we calculate the cumulative exposure function $\varepsilon(t)$ at time t under stress level s as follows:

For $0<t\leq t_1$, we have

$$\varepsilon(t)=\int_0^t \frac{1}{\alpha(s(y))}dy=\int_0^t \frac{1}{e^{\gamma_0}\left(\frac{s_0}{s(y)}\right)^{\gamma_1}}dy=\int_0^t \frac{1}{e^{\gamma_0}\left(\frac{s_0}{s_0+y\beta_1}\right)^{\gamma_1}}dy$$

$$=\frac{e^{-\gamma_0}s_0^{-\gamma_1}((s_0+\beta_1 t)^{1+\gamma_1}-(s_0)^{1+\gamma_1})}{\beta_1(1+\gamma_1)}=G_1(t)\,. \tag{7}$$

For $t_1<t\leq t_2$, we have

$$\varepsilon(t)=\int_0^t \frac{1}{\alpha(s(y))}dy=\varepsilon(t_1)+\int_{t_1}^{t}\frac{1}{e^{\gamma_0}\left(\frac{s_0}{s_1}\right)^{\gamma_1}}dy=\varepsilon(t_1)+e^{-\gamma_0}\left(\frac{s_0}{s_1}\right)^{-\gamma_1}(t-t_1)$$
$$=\varepsilon(t_1)+G_2(t). \tag{8}$$

For $t_2 < t \le t_3$, we have

$$\varepsilon(t)=\int_0^t \frac{1}{\alpha(s(y))}dy=\varepsilon(t_2)+\int_{t_2}^{t}\frac{1}{e^{\gamma_0}\left(\frac{s_0}{s_1-\beta_2(y-t_2)}\right)^{\gamma_1}}dy$$
$$=\varepsilon(t_2)+\frac{e^{-\gamma_0}s_0^{-\gamma_1}(s_1^{1+\gamma_1}-(s_1-\beta_2(t-t_2))^{1+\gamma_1}}{\beta_2(1+\gamma_1)}$$
$$=\varepsilon(t_2)+G_3(t). \tag{9}$$

For $t_3 < t \le t_4$, we have

$$\varepsilon(t)=\int_0^t \frac{1}{\alpha(s(y))}dy=\varepsilon(t_3)+\int_{t_3}^{t}\frac{1}{e^{\gamma_0}\left(\frac{s_0}{s_2}\right)^{\gamma_1}}dy=\varepsilon(t_3)+e^{-\gamma_0}\left(\frac{s_0}{s_2}\right)^{-\gamma_1}(t-t_3)$$
$$=\varepsilon(t_3)+G_4(t). \tag{10}$$

For $t_4 < t \le \eta$, we have

$$\varepsilon(t)=\int_0^t \frac{1}{\alpha(s(y))}dy=\varepsilon(t_4)+\int_{t_4}^{t}\frac{e^{-\gamma_0}}{\left(\frac{s_0}{s(y)}\right)^{\gamma_1}}dy$$
$$=\varepsilon(t_4)+\int_{t_4}^{t}\frac{1}{e^{\gamma_0}\left(\frac{s_0}{\beta_3(y-t_4)+s_2}\right)^{\gamma_1}}dy$$
$$=\varepsilon(t_4)+\frac{e^{-\gamma_0}s_0^{-\gamma_1}\left((s_2+\beta_3(t-t_4))^{1+\gamma_1}-s_2^{1+\gamma_1}\right)}{\beta_3(1+\gamma_1)}$$
$$=\varepsilon(t_4)+G_5(t). \tag{11}$$

Then, cdf of the lifetime T of a unit tested under ramp soak-stress is $F(t)=G(\varepsilon(t))$, where $G(\bullet)$ is the assumed cdf (4) with the scale parameter α set equal to one and $\varepsilon(t)$ is the cumulative exposure (damage) model defined in (2).
Thus, the cdf is

$$F(t)=1-\left(1+\varepsilon(t)^{c}\right)^{-k}\ .$$

Therefore,

$$F(t)=\begin{cases}F_1(t), 0<t\le t_1\\ F_2(t), t_1<t\le t_2\\ F_3(t), t_2<t\le t_3\\ F_4(t), t_3<t\le t_4\\ F_5(t), t_4<t\le \eta\ ,\end{cases}$$

where

$$F_1(t)=1-\left(1+\left(G_1(t)\right)^{c}\right)^{-k},$$

$$F_2(t)=1-\left(1+\left(G_1(t_1)+G_2(t)\right)^{c}\right)^{-k},$$

$$F_3(t)=1-\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t)\right)^{c}\right)^{-k},$$

$$F_4(t)=1-\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)\right)^{c}\right)^{-k},$$

$$F_5(t)=1-\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)\right)^{c}\right)^{-k}.$$

The pdf is given by

$$f(t)=\begin{cases}f_1(t), 0<t\le t_1\\ f_2(t), t_1<t\le t_2\\ f_3(t), t_2<t\le t_3\\ f_4(t), t_3<t\le t_4\\ f_5(t), t_4<t\le \eta\ ,\end{cases}$$

where

$$f_1(t)=Q_1\left(s_0+\beta_1 t\right)^{\gamma_1}\left(G_1(t)\right)^{c-1}\left(1+\left(G_1(t)\right)^{c}\right)^{-k-1},$$

$$f_2(t)=Q_1\left(s_1\right)^{\gamma_1}\left(G_1(t_1)+G_2(t)\right)^{c-1}\left(1+\left(G_1(t_1)+G_2(t)\right)^{c}\right)^{-k-1},$$

$$f_3(t)=Q_1\left(s_1+\beta_2(t-t_2)\right)^{\gamma_1}\left(G_1(t_1)+G_2(t_2)+G_3(t)\right)^{c-1}\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t)\right)^{c}\right)^{-k-1},$$

$$f_4(t)=Q_1\left(s_2\right)^{\gamma_1}\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)\right)^{c-1}\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)\right)^{c}\right)^{-k-1},$$

$$f_5(t)=Q_1\left(s_2+\beta_3(t-t_4)\right)^{\gamma_1}\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)\right)^{c-1}$$
$$\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)\right)^{c}\right)^{-k-1}.$$

2.5. Likelihood function

The log-likelihood of an observation at time t under type-I censoring is derived. The log-likelihood of a single observation at time t is

$$
\begin{aligned}
L = L(\gamma_0,\gamma_1,c) &= \sum_{j=0}^{4} \delta_j \ln f_{j+1}(t) + \delta_5 \ln(1-F_5(\eta)) \\
&= \delta_0\left[\ln Q_1 + \gamma_1 \ln\left(s_0+\beta_1 t\right) + (c-1)\ln G_1(t) - (k+1)\ln\left(1+\left(G_1(t)\right)^c\right)\right] \\
&+\delta_1\left[\ln Q_1 + \gamma_1 \ln\left(s_1\right) + (c-1)\ln\left(G_1(t_1)+G_2(t)\right) - (k+1)\ln\left(1+\left(G_1(t_1)+G_2(t)\right)^c\right)\right] \\
&+\delta_2\left[\ln Q_1 + \gamma_1 \ln\left(s_1-\beta_2(t-t_2)\right)\right. \\
&\left.+(c-1)\ln\left(G_1(t_1)+G_2(t_2)+G_3(t)\right) - (k+1)\ln\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t)\right)^c\right)\right] \\
&+\delta_3\left[\ln Q_1 + \gamma_1 \ln\left(s_2\right) + (c-1)\ln\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)\right)\right. \\
&\left.-(k+1)\ln\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t)\right)^c\right)\right] \\
&+\delta_4\left[\ln Q_1 + \gamma_1 \ln\left(s_2+\beta_3(t-t_4)\right) + (c-1)\ln\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)\right)\right. \\
&\left.-(k+1)\ln\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(t)\right)^c\right)\right] \\
&+\delta_5\left[-k\ln\left(1+\left(G_1(t_1)+G_2(t_2)+G_3(t_3)+G_4(t_4)+G_5(\eta)\right)^c\right)\right].
\end{aligned}
\tag{12}
$$

Let the log-likelihood of unit j be L_j. The log-likelihood L_0 for n independent observations is

$$L_0 = L_1 + \ldots + L_n.$$

2.6. Parameter estimation

The likelihood equations are obtained by setting (15) – (17) (see Appendix A) to zero. The parameter values that solve "these equations summed over all test units" are the Maximum Likelihood estimates. As the system of likelihood equations has no closed form solution in γ_0, γ_1 and c, therefore the maximum likelihood estimates $\hat{\gamma}_0, \gamma_1$ and c are obtained by maximizing (11) using *NMaximize* option of *Mathematica 9*.
The first and second partial derivatives of (11) with respect to the model parameters for a single observation are given in the appendix A.

2.7. Fisher information matrix

The Fisher information is obtained by taking expectations of the negative of the second partial derivatives of the log (likelihood) function with respect to γ_0, γ_1, and c. The Fisher information matrix for an observation is,

$$F(\gamma_0,\gamma_1,c)=\begin{bmatrix} E\{-\partial^2 L/\partial\gamma_0^{\,2}\} & E\{-\partial^2 L/\partial\gamma_0\partial\gamma_1\} & E\{-\partial^2 L/\partial\gamma_0\partial c\} \\ E\{-\partial^2 L/\partial\gamma_0\partial\gamma_1\} & E\{-\partial^2 L/\partial\gamma_1^{\,2}\} & E\{-\partial^2 L/\partial\gamma_1\partial c\} \\ E\{-\partial^2 L/\partial\gamma_0\partial c\} & E\{-\partial^2 L/\partial\gamma_1\partial c\} & E\{-\partial^2 L/\partial c^2\} \end{bmatrix}. \tag{13}$$

Since for some set of parameters $\{\gamma_0, \gamma_1, s_0, s_1, s_2, \eta, \beta_1, \beta_2, \beta_3\ k, c, t_1, t_2, t_3, t_4\}$; $|F|$ or variance function may be negative, therefore, we choose only that parametric set for which $|F| > 0$, and variance function is positive. Because, 'n' units are tested, the Fisher information matrix, F, for the plan with a sample of n independent units is

$$F = nF_{\text{single}}\,.$$

The elements of $F(\gamma_0,\gamma_1,c)$ are given in Appendix A.

2.8. Asymtotic variance of reliability function at used stress s_0

For any plan, the asymptotic variance-covariance matrix of the model parameters is given by the inverse of the corresponding Fisher information matrix, that is,

$$F^{-1}=\begin{bmatrix} \mathrm{Var}(\hat\gamma_0) & \mathrm{Cov}(\hat\gamma_0,\hat\gamma_1) & \mathrm{Cov}(\hat\gamma_0,\hat c) \\ \mathrm{Cov}(\hat\gamma_0,\hat\gamma_1) & \mathrm{Var}(\hat\gamma_1) & \mathrm{Cov}(\hat\gamma_1,\hat c) \\ \mathrm{Cov}(\hat\gamma_0,\hat c) & \mathrm{Cov}(\hat\gamma_1,\hat c) & \mathrm{Var}(\hat c) \end{bmatrix}, \tag{14}$$

where F is the Fisher information matrix.

The asymptotic variance of the reliability function of the distribution at used stress s_0 and pre specified mission time t_0 is

$$\mathrm{Asvar}(\hat R(t_0)) = \mathrm{Asvar}\left(1+\left(\frac{t_0}{e^{\hat\gamma_0}}\right)^{\hat c}\right)^{-k},$$

where

$$\mathrm{Asvar}\left(1+\left(\frac{t_0}{e^{\hat\gamma_0}}\right)^{\hat c}\right)^{-k} = \underset{\sim}{\hat H}' F^{-1} \underset{\sim}{\hat H},$$

$$\underset{\sim}{\hat H}=\begin{bmatrix} \dfrac{\partial \hat h}{\partial\hat\gamma_0} \\ \dfrac{\partial \hat h}{\partial\hat\gamma_1} \\ \dfrac{\partial \hat h}{\partial\hat c} \end{bmatrix},$$

$$\hat{h}(\hat{\gamma}_0, \hat{\gamma}_1, \hat{c}) = (1 + (\frac{t_0}{e^{\hat{\gamma}_0}})^{\hat{c}})^{-k}. \tag{15}$$

2.9. Optimum plans

The optimal plan consists in finding optimum stress rates β_1, β_2 and β_3 and optimum stress rate change points t_1, t_2, t_3 and t_4 by minimizing variance of reliability function with pre-specified mission time at normal operating conditions.

3. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, a hypothetical ramp soak-stress ALT experiment is considered to illustrate the methods described in this paper with the following data set:
$\gamma_0 = 26.8,\ \gamma_1 = 22,\ s_0 = 20,\ s_1 = 30,\ s_2 = 10,\ \eta = 15,\ k = 10,\ c = 1,\ t_0 = 10{,}000,\ n = 35.$
The following relationship is assumed $\beta_2 = \frac{\beta_1}{2}, \beta_3 = 2\beta_1$. One can assume some other relationship depending on the objective of the experiment.

3.1. Optimal plans

Optimal β_1, t_1, t_2, t_3, and t_4 are obtained by minimizing the variance of the Reliability function of the distribution at used stress s_0 and pre-specified mission time t_0 using the *NMinimize* option of *Mathematica 9.0*. They are obtained as $t_1^* = 1.7116$, $t_2^* = 4.10926$, $t_3^* = 7.365, t_4^* = 11.18$ and $\beta_1^* = 7.94625$.
Thus, optimum value $\beta_2^* = 3.973125$ and $\beta_3^* = 15.8925$.

3.2. Sensitivity analysis

To use an optimum test plan, one needs estimates of the design parameters γ_0, γ_1 and c. These estimates sometimes may significantly affect the values of the resulting decision variables; therefore, their incorrect choice may give a poor estimate of the variance of reliability function at design constant stress. Hence, it is important to conduct sensitivity analysis to evaluate the robustness of the resulting ALT plan.
The percentage deviations of the optimal settings are measured by $PD = (|Z^{**} - Z^*| / Z^*) \times 100$, where Z^* is the setting obtained with the given design parameters, and Z^{**} is the one obtained when the parameter is mis-specified. Table 1 shows the optimal test plans for various deviations from the design parameter estimates. The results show that the optimal setting of Z is robust to the small deviations from baseline parameter estimates.

Table 1. Sensitivity analysis with $t_1^* = 1.7116$, $t_2^* = 4.10926$, $t_3^* = 7.365$, $t_4^* = 11.18$, $\beta_1^* = 7.94625$ and Z^*=5.91x10^{-14}

Parameter	% change	t_1	t_2	t_3	t_4	Z^{**}	PD %
$\hat{\gamma}_0$	+0.1%	1.7116	4.10926	7.365	11.18	5.49x10^{-14}	7.11
$\hat{\gamma}_0$	–0.1%	1.7116	4.10926	7.365	11.18	6.38x10^{-14}	7.95
$\hat{\gamma}_1$	+0.1%	1.7116	4.10926	7.365	11.18	6.05x10^{-14}	2.37
$\hat{\gamma}_1$	–0.1%	1.7116	4.10926	7.365	11.18	5.79x10^{-14}	2.03
$\hat{c}$	+0.1%	1.7116	4.10926	7.365	11.18	5.71x10^{-14}	3.38
$\hat{c}$	–0.1%	1.7116	4.10926	7.365	11.18	6.13x10^{-14}	3.72

4. CONCLUDING REMARKS

Most of the electronic gadgets are affected by both extremely high and extremely low stress conditions. Ramp soak-stress ALT model incorporates Thermal cycling which makes a product fail quickly as it is encountered with different stresses in a short span of time. Thermal cycling involves applying high and low temperatures repeatedly over time. This paper focuses on formulation of optimal ramp soak-stress ALT plans. The optimal plan consist in finding out relevant experimental variables, namely, stress rates and stress rate change points, by minimizing variance of reliability function with pre-specified mission time under normal operating conditions. The Burr type XII distribution and time-censored data have been used for the purpose. The Burr type XII distribution has been found appropriate for modeling failures that occur with less frequency and also when there is high occurrence of early failures. This distribution has been found appropriate for accelerated life testing experiments. The method develop has been explained using a numerical example and sensitivity analysis carried out. The results of sensitivity analysis show that optimum plan is robust for small deviations in the true values of the model parameters.

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APPENDIX A

The first partial derivatives are :

$$\frac{\partial L}{\partial \gamma_0} = -c\sum_{j=0}^{4}\delta_j + (k+1)c\left(\frac{\delta_0(G_1(t))^c}{(1+(G_1(t))^c)} + \frac{\delta_1(a_1(t))^c}{(1+(a_1(t))^c)} + \frac{\delta_2(a_2(t))^c}{(1+(a_2(t))^c)} + \frac{\delta_3(a_3(t))^c}{(1+(a_3(t))^c)} + \frac{\delta_4(a_4(t))^c}{(1+(a_4(t))^c)}\right)$$
$$+ \frac{\delta_5 kc(a_4(\eta))^c}{(1+(a_4(\eta))^c)}, \tag{16}$$

$$\frac{\partial L}{\partial \gamma_1} = \delta_0\left[-\ln s_0 + \ln(s_0+\beta_1 t) + (c-1)\left(L_2 + R_0(t)\right) - \frac{(k+1)c(G_1(t))^c}{(1+(G_1(t))^c)}\left(L_2 + R_0(t)\right)\right]$$
$$+\delta_1[-\ln s_0 + \ln(s_1) + (c-1)\left(\frac{L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t) + R_1(t)}{a_1(t)}\right)$$
$$-\frac{(k+1)c(a_1(t))^c}{(1+(a_1(t))^c)}\left(L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t) + R_1(t)\right)]$$
$$+\delta_2[-\ln s_0 + \ln(s_1 - (t-t_2)\beta_2) + (c-1)\left(\frac{(L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t_2) + L_2G_3(t) + R_1(t_1) + R_2(t))}{a_2(t)}\right)$$
$$-\frac{(k+1)c(a_2(t))^{c-1}}{(1+(a_2(t))^c)}(L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t_2) + L_2G_3(t) + R_1(t_1) + R_2(t))]$$
$$+\delta_3\{-\ln s_0 + \ln(s_2)$$
$$+(c-1)\left(\frac{L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t_2) + L_2G_3(t_3) - \ln\left(\frac{s_0}{s_2}\right)G_4(t) + R_1(t_1) + R_2(t_3)}{a_3(t)}\right)$$
$$-\frac{(k+1)c(a_3(t))^c}{(1+(a_3(t))^c)}\left(L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t_2) + L_2G_3(t_3) - \ln\left(\frac{s_0}{s_2}\right)G_4(t) + R_1(t_1) + R_2(t_3)\right)\}$$
$$+\delta_4\{-\ln s_0 + \ln(s_2 + (t-t_4)\beta_3)$$
$$+(c-1)\left(\frac{(L_2G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right)G_2(t_2) + L_2G_3(t_3) - \ln\left(\frac{s_0}{s_2}\right)G_4(t_4) + L_2G_5(t) + R_1(t_1) + R_2(t_3) + R_3(t))}{a_4(t)}\right)$$

$$-\frac{(k+1)c(a_4(t))^c}{(1+(a_4(t))^c)}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+L_2G_5(t)+R_1(t_1)+R_2(t_3)+R_3(t))\}$$

$$+\delta_5\{\frac{-ck(a_4(\eta))^{c-1}}{(1+(a_4(\eta))^c)}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+L_2G_5(\eta)+R_1(t_1)+R_2(t_3)+R_3(\eta))\}, \tag{17}$$

$$\frac{\partial L}{\partial c}=\sum_{j=0}^{4}\frac{\delta_j}{c}+\delta_0\ln G_1(t)+\delta_1\ln(a_1(t))+\delta_2\ln(a_2(t)+\delta_3\ln(a_3(t)+\delta_4\ln(a_4(t))$$

$$-(k+1)(\frac{\delta_0\ln G_1(t)(G_1(t))^c}{(1+(G_1(t))^c)}+\frac{\delta_1\ln(a_1(t))(a_1(t))^c}{(1+(a_1(t))^c)}+\frac{\delta_2\ln(a_2(t))(a_2(t))^c}{(1+(a_2(t))^c)}$$

$$+\frac{\delta_3\ln(a_3(t))(a_3(t))^c}{(1+(a_3(t))^c)}+\frac{\delta_4\ln(a_4(t))(a_4(t))^c}{(1+(a_4(t))^c)})-\frac{\delta_5k\ln(a_4(\eta))(a_4(\eta))^c}{(1+(a_4(\eta))^c)}, \tag{18}$$

The likelihood equations are obtained by setting (16) – (18) to zero. The parameter values that solve "these equations summed over all test units" are the Maximum Likelihood estimates. As the system of likelihood equations has no closed form solution in γ_0, γ_1 and c, therefore the maximum likelihood estimates $\hat{\gamma}_0, \gamma_1$ and c are obtained by maximizing (12) using *NMaximize* option of *Mathematica 9*.
The second partial derivatives are:

$$\frac{\partial^2 L}{\partial\gamma_0^2}=-\left\{(k+1)c^2\left(\frac{\delta_0(G_1(t))^c}{(1+(G_1(t))^c)^2}+\frac{\delta_1(a_1(t))^c}{(1+(a_1(t)^c)^2}+\frac{\delta_2(a_2(t))^c}{(1+(a_2(t)^c)^2}+\frac{\delta_3(a_3(t))^c}{(1+(a_3(t)^c)^2}+\frac{\delta_4(a_4(t))^c}{(1+(a_4(t)^c)^2}\right)\right.$$

$$\left.-kc^2\frac{\delta_5(a_4(\eta))^c}{(1+(a_4(\eta)^c)^2}\right\}, \tag{19}$$

$$\frac{\partial^2 L}{\partial\gamma_1\partial\gamma_0}=(k+1)c^2\{\frac{\delta_0(G_1(t))^c}{(1+(G_1(t))^c)^2}[-\ln s_0-\frac{1}{1+\gamma_1}+R_0(t)]$$

$$+\frac{\delta_1(a_1(t))^{c-1}}{(1+(a_1(t))^c)^2}[L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1)]$$

$$+\frac{\delta_2(a_2(t))^{c-1}}{(1+(a_2(t))^c)^2}[L_2G_1(t_1)+L_2G_3(t)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t)]$$

$$+\frac{\delta_3(a_3(t))^{c-1}}{(1+(a_3(t))^c)^2}[L_2G_1(t_1)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3)]$$

$$+\frac{\delta_4(a_4(t))^{c-1}}{(1+(a_4(t))^c)^2}[L_2G_1(t_1)+L_2G_3(t_3)+L_2G_5(t)$$

$$-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t)]\}$$

$$+\frac{\delta_5 kc^2(a_4(\eta))^{c-1}}{(1+(a_4(\eta)^c)^2}[L_2G_1(t_1)+L_2G_3(t_3)+L_2G_5(\eta)$$

$$-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta)]\ , \tag{20}$$

$$\frac{\partial^2 L}{\partial c\partial\gamma_0}=\frac{1}{c}\frac{\partial L}{\partial\gamma_0}+(k+1)c(\frac{\delta_0\ln G_1(t)(G_1(t))^c}{(1+(G_1(t))^c)^2}+\frac{\delta_1\ln(a_1(t))(a_1(t))^c}{(1+(a_1(t))^c)^2}+\frac{\delta_2\ln(a_2(t))(a_2(t))^c}{(1+(a_2(t))^c)^2}$$

$$+\frac{\delta_3\ln(a_3(t))(a_3(t))^c}{(1+(a_3(t))^c)^2}+\frac{\delta_4\ln(a_4(t))(a_4(t))^c}{(1+(a_4(t))^c)^2})+\frac{\delta_5 kc\ln(a_4(\eta))(a_4(\eta))^c}{(1+(a_4(\eta))^c)^2}\ , \tag{21}$$

$$\frac{\partial^2 L}{\partial\gamma_1^2}=\delta_0\{(c-1)((-(L_2+R_0(t))^2+L_1+2L_2R_0(t)+Q_0(t))-(k+1)(-c^2\frac{(G_1(t))^{2c}}{(1+(G_1(t))^c)^2}(L_2+R_0(t))^2$$

$$+c(c-1)\frac{(G_1(t))^c}{(1+(G_1(t))^c)}(L_2+R_0(t))^2+c\frac{(G_1(t))^{c-1}}{(1+(G_1(t))^c)}(L_1+2L_2R_0(t)+Q_0(t)))\}$$

$$+\delta_1\{(c-1)[\frac{-(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2}{(a_1(t))^2}+\frac{L_1G_1(t_1)+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t)+2L_2R_1(t_1)+Q_1(t_1)}{a_1(t)}]$$

$$-(k+1)[-c^2\frac{(a_1(t))^{2c-2}}{(1+(a_1(t))^c)^2}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2$$

$$+c(c-1)\frac{(a_1(t))^{c-2}}{(1+(a_1(t))^c)}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2$$

$$+c\frac{(a_1(t))^{c-1}}{(1+(a_1(t))^c)}(L_1G_1(t_1)+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t)+2L_2R_1(t_1)+Q_1(t_1))]\}$$

$$+\delta_2\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2}{(a_2(t))^2}$$

$$+\frac{L_1(G_1(t_1)+G_3(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+2L_2(R_1(t_1)+R_2(t))+Q_1(t_1)+Q_2(t)}{a_2(t)}]$$

$$-(k+1)[-c^2\frac{(a_2(t))^{2c-2}}{(1+(a_2(t))^c)^2}(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2$$

$$+c(c-1)\frac{(a_2(t))^{c-2}}{(1+(a_2(t))^c)}(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2$$

$$+c\frac{(a_2(t))^{c-1}}{(1+(a_2(t))^c)}(L_1(G_1(t_1)+G_3(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+2L_2(R_1(t_1)+R_2(t))+Q_1(t_1)+Q_2(t))]\}$$

$$+\delta_3\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2}{(a_3(t))^2}+$$

$$\frac{L_1(G_1(t_1)+G_3(t_3))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t)+2L_2(R_1(t_1)+R_2(t_3))+Q_1(t_1)+Q_2(t_3)}{a_3(t)}]$$

$$-(k+1)[-c^2\frac{(a_3(t))^{2c-2}}{(1+(a_2(t))^c)^2}(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2$$

$$+c(c-1)\frac{(a_3(t))^{c-2}}{(1+(a_3(t))^c)}(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2$$

$$+c\frac{(a_3(t))^{c-1}}{(1+(a_3(t))^c)}(L_1(G_1(t_1)+G_3(t_3))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t)+2L_2(R_1(t_1)+R_2(t_3))+Q_1(t_1)+Q_2(t_3))]\}$$

$$+\delta_4\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2}{(a_4(t))^2}+$$

$$\frac{L_1(G_1(t_1)+G_3(t_3)+G_5(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(t))+Q_1(t_1)+Q_2(t_3)+Q_3(t)}{a_4(t)}]$$

$$-(k+1)[-c^2\frac{(a_4(t))^{2c-2}}{(1+(a_4(t))^c)^2}(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2$$

$$+c(c-1)\frac{(a_4(t))^{c-2}}{(1+(a_4(t))^c)}(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2$$

$$+c\frac{(a_4(t))^{c-1}}{(1+(a_4(t))^c)}(L_1(G_1(t_1)+G_3(t_3)+G_5(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(t))+Q_1(t_1)+Q_2(t_3)+Q_3(t))]\}$$

$$+\delta_5\{-k[-c^2\frac{(a_4(\eta))^{2c-2}}{(1+(a_4(\eta))^c)^2}(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))^2$$

$$+c(c-1)\frac{(a_4(\eta))^{c-2}}{(1+(a_4(\eta))^c)}(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))^2$$

$$+c\frac{(a_4(\eta))^{c-1}}{(1+(a_4(\eta))^c)}(L_1(G_1(t_1)+G_3(t_3)+G_5(\eta))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(\eta))+Q_1(t_1)+Q_2(t_3)+Q_3(\eta))]\}\,, \tag{22}$$

$$\frac{\partial^2 L}{\partial\gamma_1\partial c}=\delta_0\left[\left((L_2+R_0(t))-\left(\frac{(k+1)\ln(G_1(t))(G_1(t))^c}{(1+(G_1(t))^c)}(1+\frac{c}{(1+(G_1(t))^c)})(L_2+R_0(t))\right)\right]\right.$$

$$+\delta_1\Big(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))}{a_1(t)}-\Big(\frac{(k+1)c\ln(a_1(t))(a_1(t))^{c-1}}{(1+(a_1(t))^c)^2}(1+\frac{c}{(1+(a_1(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))$$

$$+\delta_2\Big(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t)+R_1(t_1)+R_2(t))}{a_2(t)}$$

$$-\Big(\frac{(k+1)\ln(a_2(t))(a_2(t))^{c-1}}{(1+(a_2(t))^c)}(1+\frac{c}{(1+(a_2(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t)+R_1(t_1)+R_2(t)))$$

$$+\delta_3\Big(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))}{a_3(t)}$$

$$-\Big(\frac{(k+1)\ln(a_3(t))(a_3(t))^{c-1}}{(1+(a_3(t))^c)}(1+\frac{c}{(1+(a_3(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3)))$$

$$+\delta_4\Big(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+L_2G_5(t)+R_1(t_1)+R_2(t_3)+R_3(t))}{a_4(t)}$$

$$-\Big(\frac{(k+1)\ln(a_4(t))(a_4(t))^{c-1}}{(1+(a_4(t))^c)}(1+\frac{c}{(1+(a_4(t))^c)})$$

$$(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t)))$$

$$+\delta_5\Big(\frac{(-k\ln(a_4(\eta))(a_4(\eta))^{c-1}}{(1+(a_4(\eta))^c)}(1+\frac{c}{(1+(a_4(\eta))^c)})$$

$$(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))), \tag{23}$$

and

$$\frac{\partial^2 L}{\partial c^2} = -\sum_{j=0}^{4} \frac{\delta_j}{c^2} - (k+1)\Big(\frac{\delta_0 (\ln G_1(t))^2 (G_1(t))^c}{(1+(G_1(t))^c)^2} + \frac{\delta_1 (\ln(a_1(t)))^2 (a_1(t))^c}{(1+(a_1(t))^c)^2} + \frac{\delta_2 (\ln(a_2(t)))^2 (a_2(t))^c}{(1+(a_2(t))^c)^2}$$
$$+ \frac{\delta_3 (\ln(a_3(t)))^2 (a_3(t))^c}{(1+(a_3(t))^c)^2} + \frac{\delta_4 (\ln(a_4(t)))^2 (a_4(t))^c}{(1+(a_4(t))^c)^2}\Big) - \frac{\delta_5 k (\ln(a_4(\eta))^2 (a_4(\eta))^c}{(1+(a_4(\eta))^c)^2} \ . \tag{24}$$

The elements of the Fisher information matrix for an observation are the negative expectations of these second partial derivatives:

$$E\left[-\frac{\partial^2 L}{\partial \gamma_0^2}\right] = (k+1)c^2 \Big(\int_0^{t_1} \frac{(G_1(t))^c f_1(t)dt}{(1+(G_1(t))^c)^2} + \int_{t_1}^{t_2} \frac{(a_1(t))^c f_2(t)dt}{(1+(a_1(t)^c)^2} + \int_{t_2}^{t_3} \frac{(a_2(t))^c f_3(t)dt}{(1+(a_2(t)^c)^2} + \int_{t_3}^{t_4} \frac{(a_3(t))^c f_4(t)dt}{(1+(a_3(t)^c)^2}$$
$$+ \int_{t_4}^{\eta} \frac{(a_4(t))^c f_5(t)dt}{(1+(a_4(t)^c)^2}\Big) + \frac{kc^2 (a_4(\eta))^c}{(1+(a_4(\eta)^c)^{2+k}} \ , \tag{25}$$

$$E\left[-\frac{\partial^2 L}{\partial \gamma_1 \partial \gamma_0}\right] = (k+1)c^2 \Big\{\int_0^{t_1} -\frac{(G_1(t))^c}{(1+(G_1(t))^c)^2}[L_2 + R_0(t)]f_1(t)dt$$
$$+ \int_{t_1}^{t_2} -\frac{(a_1(t))^{c-1}}{(1+(a_1(t))^c)^2}[L_2 G_1(t_1) - \ln\left(\frac{s_0}{s_1}\right) G_2(t) + R_1(t_1)]f_2(t)dt$$
$$+ \int_{t_2}^{t_3} -\frac{(a_2(t))^{c-1}}{(1+(a_2(t))^c)^2}[L_2 G_1(t_1) + L_2 G_3(t) - \ln\left(\frac{s_0}{s_1}\right) G_2(t_2) + R_1(t_1) + R_2(t)]f_3(t)dt$$
$$+ \int_{t_3}^{t_4} -\frac{(a_3(t))^{c-1}}{(1+(a_3(t))^c)^2}[L_2 G_1(t_1) + L_2 G_3(t_3) - \ln\left(\frac{s_0}{s_1}\right) G_2(t_2) - \ln\left(\frac{s_0}{s_2}\right) G_4(t) + R_1(t_1) + R_2(t_3)]f_4(t)dt$$
$$+ \int_{t_4}^{\eta} -\frac{(a_4(t))^{c-1}}{(1+(a_4(t))^c)^2}[L_2 G_1(t_1) + L_2 G_3(t_3) + L_2 G_5(t)$$
$$- \ln\left(\frac{s_0}{s_1}\right) G_2(t_2) - \ln\left(\frac{s_0}{s_2}\right) G_4(t_4) + R_1(t_1) + R_2(t_3) + R_3(t)]f_5(t)dt\Big\}$$
$$- \frac{kc^2 (a_4(\eta))^{c-1}}{(1+(a_4(\eta)^c)^{2+k}}[L_2 G_1(t_1) + L_2 G_3(t_3) + L_2 G_5(\eta)$$
$$- \ln\left(\frac{s_0}{s_1}\right) G_2(t_2) - \ln\left(\frac{s_0}{s_2}\right) G_4(t_4) + R_1(t_1) + R_2(t_3) + R_3(\eta)] \ , \tag{26}$$

$$E\left[-\frac{\partial^2 L}{\partial c \partial \gamma_0}\right] = -(k+1)c\Big(\int_0^{t_1} \frac{\ln G_1(t)(G_1(t))^c f_1(t)dt}{(1+(G_1(t))^c)^2} + \int_{t_1}^{t_2} \frac{\ln(a_1(t))(a_1(t))^c f_2(t)dt}{(1+(a_1(t))^c)^2} + \int_{t_2}^{t_3} \frac{\ln(a_2(t))(a_2(t))^c f_3(t)dt}{(1+(a_2(t))^c)^2}$$
$$+ \int_{t_3}^{t_4} \frac{\ln(a_3(t))(a_3(t))^c f_4(t)dt}{(1+(a_3(t))^c)^2} + \int_{t_4}^{\eta} \frac{\ln(a_4(t))(a_4(t))^c f_5(t)dt}{(1+(a_4(t))^c)^2}\Big) - \frac{kc \ln(a_4(\eta))(a_4(\eta))^c}{(1+(a_4(\eta))^c)^{2+k}} \ , \tag{27}$$

$$E\left[-\frac{\partial^2 L}{\partial \gamma_1^2}\right]=\int_0^{t_1} -\{(c-1)((-(L_2+R_0(t))^2+L_1+2L_2R_0(t)+Q_0(t))$$

$$-(k+1)(-c^2\frac{(G_1(t))^{2c}}{(1+(G_1(t))^c)^2}(L_2+R_0(t))^2$$

$$+c(c-1)\frac{(G_1(t))^c}{(1+(G_1(t))^c)}(L_2+R_0(t))^2$$

$$+c\frac{(G_1(t))^{c-1}}{(1+(G_1(t))^c)}(L_1+2L_2R_0(t)+Q_0(t)))\}f_1(t)dt$$

$$+\int_{t_1}^{t_2} -\{(c-1)[\frac{-(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2}{(a_1(t))^2}$$

$$+\frac{L_1G_1(t_1)+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t)+2L_2R_1(t_1)+Q_1(t_1)}{a_1(t)}]$$

$$-(k+1)[-c^2\frac{(a_1(t))^{2c-2}}{(1+(a_1(t))^c)^2}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2$$

$$+c(c-1)\frac{(a_1(t))^{c-2}}{(1+(a_1(t))^c)}(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))^2$$

$$+c\frac{(a_1(t))^{c-1}}{(1+(a_1(t))^c)}(L_1G_1(t_1)+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t)+2L_2R_1(t_1)+Q_1(t_1))]\}f_2(t)dt$$

$$+\int_{t_2}^{t_3} -\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2}{(a_2(t))^2}$$

$$+\frac{L_1(G_1(t_1)+G_3(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+2L_2(R_1(t_1)+R_2(t))+Q_1(t_1)+Q_2(t)}{a_2(t)}]$$

$$-(k+1)[-c^2\frac{(a_2(t))^{2c-2}}{(1+(a_2(t))^c)^2}(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2$$

$$+c(c-1)\frac{(a_2(t))^{c-2}}{(1+(a_2(t))^c)}(L_2(G_1(t_1)+G_3(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+R_1(t_1)+R_2(t))^2$$

$$+c\frac{(a_2(t))^{c-1}}{(1+(a_2(t))^c)}(L_1(G_1(t_1)+G_3(t))$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+2L_2(R_1(t_1)+R_2(t))+Q_1(t_1)+Q_2(t))]\}f_3(t)dt$$

$$+\int_{t_3}^{t_4} -\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2}{(a_3(t))^2}+$$

$$\frac{L_1(G_1(t_1)+G_3(t_3))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t)+2L_2(R_1(t_1)+R_2(t_3))+Q_1(t_1)+Q_2(t_3)}{a_3(t)}]$$

$$-(k+1)[-c^2\frac{(a_3(t))^{2c-2}}{(1+(a_2(t))^c)^2}(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2$$

$$+c(c-1)\frac{(a_3(t))^{c-2}}{(1+(a_3(t))^c)}(L_2(G_1(t_1)+G_3(t_3))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))^2$$

$$+c\frac{(a_3(t))^{c-1}}{(1+(a_3(t))^c)}(L_1(G_1(t_1)+G_3(t_3))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t)+2L_2(R_1(t_1)+R_2(t_3))+Q_1(t_1)+Q_2(t_3))]\}f_4(t)dt$$

$$+\int_{t_4}^{\eta} -\{(c-1)[\frac{-(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2}{(a_4(t))^2}+$$

$$\frac{L_1(G_1(t_1)+G_3(t_3)+G_5(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(t))+Q_1(t_1)+Q_2(t_3)+Q_3(t)}{a_4(t)}]$$

$$-(k+1)[-c^2\frac{(a_4(t))^{2c-2}}{(1+(a_4(t))^c)^2}(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2$$

$$+c(c-1)\frac{(a_4(t))^{c-2}}{(1+(a_4(t))^c)}(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t))^2$$

$$+c\frac{(a_4(t))^{c-1}}{(1+(a_4(t))^c)}(L_1(G_1(t_1)+G_3(t_3)+G_5(t))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(t))+Q_1(t_1)+Q_2(t_3)+Q_3(t))]\}f_5(t)dt$$

$$+\{k[-c^2\frac{(a_4(\eta))^{2c-2}}{(1+(a_4(\eta))^c)^{2+k}}(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))^2$$

$$+c(c-1)\frac{(a_4(\eta))^{c-2}}{(1+(a_4(\eta))^{c+k})}(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)$$

$$-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))^2$$

$$+c\frac{(a_4(\eta))^{c-1}}{(1+(a_4(\eta))^{c+k})}(L_1(G_1(t_1)+G_3(t_3)+G_5(\eta))+(\ln\left(\frac{s_0}{s_1}\right))^2G_2(t_2)$$

$$+(\ln\left(\frac{s_0}{s_1}\right))^2G_4(t_4)+2L_2(R_1(t_1)+R_2(t_3)+R_3(\eta))+Q_1(t_1)+Q_2(t_3)+Q_3(\eta))]\} \quad , \qquad (28)$$

$$E\left[-\frac{\partial^2 L}{\partial\gamma_1\partial c}\right]=\int_0^{t_1}-\left[\left((L_2+R_0(t))-(\frac{(k+1)\ln(G_1(t))(G_1(t))^c}{(1+(G_1(t))^c)}(1+\frac{c}{(1+(G_1(t))^c)})(L_2+R_0(t))\right)\right]f_1(t)dt$$

$$+\int_{t_1}^{t_2}-(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))}{a_1(t)}-(\frac{(k+1)c\ln(a_1(t))(a_1(t))^{c-1}}{(1+(a_1(t))^c)^2}(1+\frac{c}{(1+(a_1(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t)+R_1(t_1))f_2(t)dt$$

$$+\int_{t_2}^{t_3}-(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t)+R_1(t_1)+R_2(t))}{a_2(t)}$$

$$-(\frac{(k+1)\ln(a_2(t))(a_2(t))^{c-1}}{(1+(a_2(t))^c)}(1+\frac{c}{(1+(a_2(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t)+R_1(t_1)+R_2(t)))f_3(t)dt$$

$$+\int_{t_3}^{t_4}-(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3))}{a_3(t)}$$

$$-(\frac{(k+1)\ln(a_3(t))(a_3(t))^{c-1}}{(1+(a_3(t))^c)}(1+\frac{c}{(1+(a_3(t))^c)})$$

$$(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t)+R_1(t_1)+R_2(t_3)))f_4(t)dt$$

$$+\int_{t_4}^{\eta}-(\frac{(L_2G_1(t_1)-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)+L_2G_3(t_3)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+L_2G_5(t)+R_1(t_1)+R_2(t_3)+R_3(t))}{a_4(t)}$$

$$-(\frac{(k+1)\ln(a_4(t))(a_4(t))^{c-1}}{(1+(a_4(t))^c)}(1+\frac{c}{(1+(a_4(t))^c)})$$

$$(L_2(G_1(t_1)+G_3(t_3)+G_5(t))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(t)))f_5(t)dt$$

$$+(\frac{(k\ln(a_4(\eta))(a_4(\eta))^{c-1}}{(1+(a_4(\eta))^c)^{k+1}}(1+\frac{c}{(1+(a_4(\eta))^c)})$$

$$(L_2(G_1(t_1)+G_3(t_3)+G_5(\eta))-\ln\left(\frac{s_0}{s_1}\right)G_2(t_2)-\ln\left(\frac{s_0}{s_2}\right)G_4(t_4)+R_1(t_1)+R_2(t_3)+R_3(\eta))), \tag{29}$$

and

$$E\left[-\frac{\partial^2 L}{\partial c^2}\right]=\int_0^{t_1}\frac{f_1(t)dt}{c^2}+\int_{t_1}^{t_2}\frac{f_2(t)dt}{c^2}+\int_{t_2}^{t_3}\frac{f_3(t)dt}{c^2}+\int_{t_3}^{t_4}\frac{f_4(t)dt}{c^2}+\int_{t_4}^{\eta}\frac{f_5(t)dt}{c^2}+$$

$$(k+1)(\int_0^{t_1}\frac{(\ln G_1(t))^2(G_1(t))^c f_1(t)dt}{(1+(G_1(t))^c)^2}+\int_{t_1}^{t_2}\frac{(\ln(a_1(t)))^2(a_1(t))^c f_2(t)dt}{(1+(a_1(t))^c)^2}$$

$$+\int_{t_2}^{t_3}\frac{(\ln(a_2(t)))^2(a_2(t))^c f_3(t)dt}{(1+(a_2(t))^c)^2}+\int_{t_3}^{t_4}\frac{(\ln(a_3(t)))^2(a_3(t))^c f_4(t)dt}{(1+(a_3(t))^c)^2}+$$

$$+\int_{t_4}^{\eta}\frac{(\ln(a_4(t)))^2(a_4(t))^c f_5(t)dt}{(1+(a_4(t))^c)^2})+\frac{k(\ln(a_4(\eta))^2(a_4(\eta))^c}{(1+(a_4(\eta))^c)^{2+k}} \tag{30}$$

These expectations (25-30) are calculated with the aid of

$$E\{\delta_5(t)\}=1-F_5(\eta),\ E\left\{\frac{\partial L}{\partial\gamma_i}\right\}=0,\ \text{for } i=0,1,\ E\left\{\frac{\partial L}{\partial c}\right\}=0.$$

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