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Nonparametric test for unknown age class of life distributions

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Abstract. Based on the kernel function, a new test is presented, testing H_0: $\bar{F}$ is exponential against H_1: $\bar{F}$ is UBACT and not exponential is given in section 2. Monte Carlos null distribution critical points for sample sizes n = 5(5)100 is investigated in section 3. The Pitman asymptotic efficiency for common alternatives is obtained in section 4. In section 5 we propose a test statistic for censored data. Finally, a numerical examples in medical science for complete and censored data using real data is presented in section 6.

Key Words: *Asymptotic normality, efficiency, Hypothesis testing, Kernel method, UBACT Classes of life distributions*

1. INTRODUCTION

In reliability theory various concepts of aging have been proposed to study lifetimes of components or systems. Therefore statisticians and reliability analysts have shown a growing interest in modeling survival data using classification life distributions. As a criteria for comparing ages, for instance, electrical equipment, computers. Radio's or alike. The comparison of the additional residual life at different times has been used to produce several notations of aging. Let X be a non-negative continuous random variable representing equipment life with distribution function F and survival function $\bar{F}(t)$ = 1-F(t); such that F(0-) = 0, given a unit which has survived up to time t; with distribution function $F_t(x)$ and survival function

$$\bar{F}_t(x) = \frac{\bar{F}(x+t)}{\bar{F}(t)}, \quad x, t > 0,$$

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and assume that X has a finite mean

$$u = E(X) = \int_0^\infty \bar{F}(u)du.$$

Definition 1.

If X is non-negative random variable, its distribution function F(x) is said to be finitely and positively smooth if a number $\gamma \in (0, \infty)$ exists and,

$$\lim_{t\to\infty} \frac{\bar{F}(x+t)}{\bar{F}(t)} = e^{-\gamma x}, \quad (1)$$

where γ is called the asymptotic decay coefficient of X.

Definition 2.

The Kernel methods in reliability appears in early work of Rosnblat (1956) who view the idea of the kernel of function as

$$k(x) = \frac{1}{\alpha} k(\frac{x}{\alpha}). \quad (2)$$

Where the kernel function is a probability density function, so

$$\int K(x)dx = 1.$$

Definition 3.

The distribution function F is said to be Used better than aged (UBA) if it is finitely and positively smooth and satisfies

$$\bar{F}(x+t) \geq \bar{F}(t)e^{-\gamma x}. \quad (3)$$

Definition 4.

The distribution function F is said to be Used better than aged in convex ordering (UBAC) if it is finitely and positively smooth and satisfies,

$$v(x+t) \geq \gamma^{-1}\bar{F}(t)e^{-\gamma x}, \quad (4)$$

where

$$v(x+t) = \int_{x+t}^{\infty} F(z)dz.$$

Definition 5.

The distribution F(x) is called Used better than aged in convex tail ordering (UBACT) if,

$$\Gamma(x+t) \geq \gamma^{-2}\bar{F}(t)e^{-\gamma x} \text{ for } t \geq 0, \quad (5)$$

where,

$$V(z+t) = \int_{x+t}^{\infty} \bar{F}(y)dy, \text{ and } \Gamma(x+t) = \int_{x+t}^{\infty} V(u)du.$$

We can see the details for these definitions in Abu-Youssef and Bakr (2014). Its dual class is used worse than used in convex tail order, denoted by UWACT, which is defined by reversing the above inequality. Then, it is clear that

$$IHR \subset DMRL \subset UBA \subset UBAC \subset UBACT.$$

Note that F(x) has an exponential distribution with mean u When u equal to the coefficient of the asymptotic decay γ, and the exponential disttribution is the only which has the lack of memory property. Well known classes of life distributions include increasing failure rate (IFR), increasing failure rate in average (IFRA), new better than used (NBU), decreasing mean residual life (DMRL) and new better than used in expectation (NBUE). For definitions and properties of these criteria we refer Deshpande et al (1986), Barlow and Proschan (1981), Bryson and Siddique (1969).

Testing exponentially against the classes of life distribution has seen a good deal of attention. For testing against IHR, we refer to Barlow and Proschan (1981) and Ahmad (1994), among others. While testing against DMRL see Ahmad (1992).and testing against UBA see Ahmad (2004).finally tasting against UBAC see Abu-Youssef (2009), and Mohie El-Din et.al (2013). Using Kernel methods in reliability appears in early work of Rosnblat (1956) who view the idea of the kernel of function as

$$k(x) = \frac{1}{\alpha} k(\frac{x}{\alpha}),$$

The Kernel method is used in some general goodness of fit problems for testing exponentiality versus the unknown age classes of life distributions successfully, Hendi (1999), Hendi et al (2000), Ahmed et al (1999, 2003), Hendi and Al-Ghufily (2005), Hendi et al (2007), finally Abu-Youssef (2007) for testing among many others.

2 TESTING FOR COMPLETE DATA

The test presented on a sample $X_1, X_2, \ldots, X_n$, from a population with distribution F(x). We wish to test the null hypothesis,

H_0 : $\bar{F}$ is exponential distribution with mean u, against,

H_1 : $\bar{F}$ is UBACT, and not exponential distribution.

Let the measure of departure from H0 in favor of H_1 is

$$\delta_K = E[f(x)(\Gamma(x+t) - \gamma^{-2}\bar{F}(t)e^{-\gamma x})]$$
$$= \int_0^\infty \int_0^\infty f(x)(\Gamma(x+t) - \frac{1}{\gamma^2}\bar{F}e^{-\gamma x})dF(x)dF(t), \qquad (6)$$

Remark that under $H_0 : \delta_K = 0$, while under $H1 : \delta_K > 0$. Then to estimate δ_K by $\hat{\delta}_K$, let $X_1, X_2, \ldots, X_n$ be a random sample from F and let $\widehat{\Gamma}_n(x) = \frac{1}{2n}\sum_{m=1}^{n}(X_m - x - t)^2 I(X_m > x + t)$ is the empirical distribution of $\Gamma(x)$, $d\hat{F}_n(x) = \frac{1}{n}$is the empirical distribution of dF(x), and pdf f(x) is estimated by $\hat{f}_n(x)=\frac{1}{n\alpha_n}\sum_{p=1}^{n} k(\frac{(X-X_p)}{\alpha_n})$ where k(.) be a known pdf. Then,

$$\hat{\delta}_K=\int_0^\infty \int_0^\infty \hat{f}_n(x)(\widehat{\Gamma}_n(x+t) - \frac{1}{\gamma^2}\bar{F}_n e^{-\hat{\gamma}x})d\hat{F}_n(x)\, d\hat{F}_n(t),$$

i.e,

$$\hat{\delta}_K=\frac{1}{2\alpha n^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{m=1}^{n}\sum_{p=1}^{n} k(\frac{(X_i-X_p)}{\alpha_n})(X^2{}_m + X^2{}_i + X^2{}_j + 2X_jX_i$$
$$-2X_mX_i - 2X_mX_j)I(X_m > X_i + X_j)\text{-}\frac{e^{-\hat{\gamma}X_i}}{\gamma^2}$$

where,

$$I(y>t) = \begin{cases} 1 & \text{if, } y > t \\ 0, & \text{if, o.w.,} \end{cases}$$

let us rewrite (6) as the following,

$$\hat{\delta}_K = \frac{1}{2\alpha n^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{m=1}^{n}\sum_{p=1}^{n} \emptyset(X_i, X_j, X_m, X_p),$$

where,

$$\emptyset(X_i, X_j, X_m, X_p) = k(\frac{(X_i - X_p)}{\alpha_n})[(X_m - X_i - X_j)^2 I(X_m > X_i + X_j) - \frac{1}{\gamma^2} e^{-\hat{\gamma} X_i}].$$

To make the test scale invariant, we take,

$$\hat{\Delta}_K = \frac{\hat{\delta}_K}{\bar{x}^2}, \quad (7)$$

Then $\hat{\Delta}_K$ in (7) is equivalent to the U-statistics and The following theorem summarizes the large sample properties of $\hat{\Delta}_K$.

Theorem 1

i) When $an^4 \to 0$, $n \to \infty$, then $\sqrt{n}(\Delta_K - \hat{\delta}_K)$ is convergence asymptotically normal distribution with mean 0 and variance,

$$\sigma^2 = var(f(X)(\int_X^\infty \int_0^{v-X} (v - X - u)^2 f(u)f(v)dudv - e^{-\lambda X})$$
$$+ \int_X^\infty \int_0^{v-X} (v - u - X)^2 f^2(u)f(v)dudv - \int_0^\infty e^{-\lambda u} f^2(u)du$$
$$+ \int_0^X \int_0^{X-u} (X - u - v)^2 f^2(u)f(v)dvdu - \int_0^\infty e^{-\lambda u} f^2(u)du$$
$$+ f(X) \int_X^\infty \int_0^{v-X} (v - X - u)^2 f(u)f(v)dudv - e^{-\lambda X})).$$

ii) Under H0 : $\Delta_K = 0$, and $\sigma^2 = \frac{41}{30}$.

Proof. Note that

$$E(\hat{f}_n(x)) = f(x) + \frac{\alpha^2}{2} f^{\backslash\backslash}(x)\sigma_k^2,$$

which the second term ($f^{\backslash\backslash}(x)\,\sigma_k^2$) equal to zero under assumed on kernel. Hence

(i) To computeσ^2, we must compute

$$\emptyset_1(x_1) = E(\emptyset(X_1, X_2, X_3, X_4 | X_1)]$$
$$= E[k\left(\frac{(X_1 - X_4)}{\alpha_n}(X_3 - X_1 - X_2)^2 I(X_3 > X_1 - X_2) - \frac{1}{\gamma^2} e^{-\hat{\gamma} X_1}\right| X_1\Big]$$
$$= f(x_1) \int_{x_1}^\infty \int_0^{v-x_1} (v - X_1 - u)^2 f(u)f(v)dudv - e^{-2x_1} \quad (8)$$

$$\emptyset_2(x_2) = E(\emptyset(X_1, X_2, X_3, X_4 | X_2)]$$
$$= E[k\left(\frac{(X_1 - X_4)}{\alpha_n}(X_3 - X_1 - X_2)^2 I(X_3 > X_1 - X_2) - \frac{1}{\gamma^2} e^{-\hat{\gamma} X_1}\right| X_2\Big]$$
$$= \int_{x_2}^\infty \int_0^{v-x_2} (v - u - X_2)^2 f^2(u)f(v)dudv - \int_0^\infty f^2(u)\, e^{-u}du. \quad (9)$$

$$\emptyset_3(x_3) = E(\emptyset(X_1, X_2, X_3, X_4 | X_3)]$$
$$= E[k\left(\frac{(X_1 - X_4)}{\alpha_n}(X_3 - X_1 - X_2)^2 I(X_3 > X_1 - X_2) - \frac{1}{\gamma^2} e^{-\hat{\gamma} X_1}\right| X_3\Big]$$

$$= \int_0^{x_3}\int_0^{x_3-u}\left(X_3 - u - v\right)^2 f^2(u)f(v)dvdu - \int_0^{\infty} f^2(u)\, e^{-u}du. \quad (10)$$

Observe that E[∅(X1,X2,X3,X4|X4)] has the same representation as (8).

$$\emptyset_4(x_4) = E(\emptyset(X_1,X_2,X_3,X_4|X_4)]$$
$$= f(X_4)\int_{x_4}^{\infty}\int_0^{v-x_4}(v - X_4 - u)^2 f(u)f(v)dudv - e^{-2x_4} \quad (11)$$

Due to the fact that the variables X_1, X_2, X_3, X_4 independent and identical, it can write in short X instead of X_1, X_2, X_3, X_4,

set

$$\zeta(X) = \emptyset_1(X) + \emptyset_2(X) + \emptyset_3(X) + \emptyset_4(X).$$

Now, by substitution from equations (8), (9), (10), (11), we have

$$\sigma^2 = var(e^{-X}\int_X^{\infty}\int_0^{v-X}(v - X - u)^2 f(u)f(v)dudv - e^{-2X}$$
$$+ \int_X^{\infty}\int_0^{v-X}(v - u - X)^2 f^2(u)f(v)dudv - \int_0^{\infty} e^{-u}f^2(u)du$$
$$+\int_0^{x}\int_0^{X-u}(X - u - v)^2 f^2(u)f(v)dvdu - \int_0^{\infty} e^{-u}f^2(u)du$$
$$+e^{-X}\int_x^{\infty}\int_0^{v-X}(v - X - u)^2 f(u)f(v)dudv - e^{-2X})$$

(ii) By direct calculation, under H0 : $f(u) = e^{-u}$, $f(v) = e^{-v}$ and $\hat{\gamma} = 1$, then

$$\emptyset_1(X_1) = e^{-X}\int_X^{\infty}\int_0^{v-X}(v - X - u)^2 e^{-u-v}dudv - e^{-2X} = e^{2(-X)} - e^{-2X} = 0$$
$$\emptyset_2(X_2) = \int_X^{\infty}\int_0^{v-X}(v - u - X)^2 e^{-2u-v}dudv - \int_0^{\infty} e^{-3u}du = \frac{2}{3}e^{-X} - \frac{1}{3}$$
$$\emptyset_3(X_3) = \int_0^{X}\int_0^{X-u}(X - u - v)^2 e^{-2u-v}dvdu - \int_0^{\infty} e^{-3u}du$$
$$= \frac{1}{4e^{2x}}(2X^2e^{2X} - 6Xe^{2X} + 7e^{2X} - 8e^{X} + 1) - \frac{1}{3}$$
$$\emptyset_4(X_4) = e^{-X}\int_X^{\infty}\int_0^{v-X}(v - X - u)^2 e^{-u-v}dudv - e^{-2X}) = 0$$

And

$$\zeta(X) = \emptyset_1(X) + \emptyset_2(X) + \emptyset_3(X) + \emptyset_4(X)$$
$$= \frac{2}{3}e^{-X} + \frac{1}{4e^{2X}}(2X^2e^{2X} - 6Xe^{2X} + 7e^{2X} - 8e^{X} + 1) - \frac{2}{3}$$

and the expectation of ζ (x) is

$$E\{\zeta(x)\} = \int_0^{\infty}(\frac{2}{3}e^{-X} + \frac{1}{4e^{2X}}(2X^2e^{2X} - 6Xe^{2X} + 7e^{2X} - 8e^{X} + 1) - \frac{2}{3})e^{-X}dX = 0.$$

The variance of ζ (x) is

$$\sigma^2(X) = \int_0^{\infty}(\frac{2}{3}e^{-X} + \frac{1}{4e^{2X}}(2X^2e^{2X} - 6Xe^{2X} + 7e^{2X} - 8e^{X} + 1) - \frac{2}{3})^2 e^{-X}dX = \frac{41}{30}.$$

$$\sigma^2 = \frac{41}{30}.$$

3. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. we have simulated the upper percentile points for 95%, 98%, 99%. Table 1 gives these percentile points of statistic $\hat{\Delta}_K$in (7) and the calculations are based on 5000 simulated samples of sizes n = 5(5)100. The percentiles values change slowly as n increase. To use the above test, calculate $\sqrt[2]{n}\hat{\delta}_K/\sigma^2$ and reject H0 if this exceeds the normal variate value $Z_{\alpha-1}$.

Table 1. Critical values of $\hat{\Delta}_K$

n	95%	98%	99%
5	1.026	1.22	1.351
10	0.725	0.862	0.955
15	0.592	0.704	0.78
20	0.513	0.61	0.675
25	0.459	0.545	0.604
30	0.419	0.498	0.551
35	0.388	0.461	0.51
40	0.363	0.431	0.477
45	0.342	0.38	0.45
50	0.324	0.386	0.427
55	0.321	0.379	0.419
60	0.296	0.352	0.39
65	0.284	0.338	0.374
70	0.267	0.319	0.354
75	0.257	0.307	0.341
80	0.256	0.305	0.338
85	0.254	0.304	0.337
90	0.251	0.296	0.327
95	0.235	0.279	0.31
100	0.23	0.278	0.308

It is clear from Table 1 that, the percentiles values decreases slowly as the sample size increases where is shown in Figure 1.

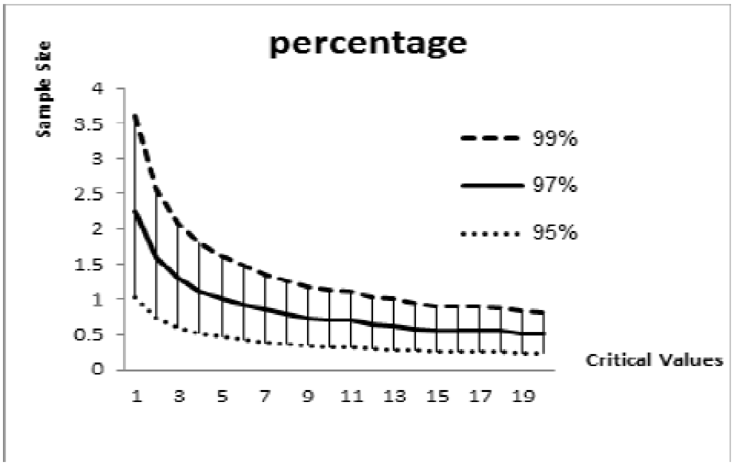

Figure 1. The Relation between sample size and critical values

4. ASYMPTOTIC RELATIVE EFFICIENCY (ARE)

Since the above test statistic $\hat{\Delta}_K = \frac{\hat{\delta}_K}{x^2}$ is new and no other tests are known for these class UBACT. We may compare this to those of the other classes classes. Here we choose The test U_n presented by Kanjo for NBUE class of life distribution, $\Delta_{\widehat{UK}}$ presented M. M. Mohie et el for (UBAC) class of life distribution. The comparisons are achived by using Pitman asymptotic relative efficiency (PARE), which is defined as follows:
Let T_{1n} and T_{2n} be two statistics for testing $H_0 : F_{\theta_x} \in \{F_x\}, \theta_n = \theta + \frac{C}{\sqrt[2]{n}}$ with C an arbitrary constant, then PARE of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n}, T_{2n}) = \frac{\grave{\mu}_1(\theta_0)}{\sigma_1(\theta_0)} / \frac{\grave{\mu}_2(\theta_0)}{\sigma_2(\theta_0)} \tag{12}$$

where $\grave{\mu}_i = \lim_{n\to\infty} \frac{\partial}{\partial\theta} E(T_{in})_{\theta\to\theta_0}$ and $\sigma_i^2(\theta_0) = \lim_{n\to\infty} var(T_{in}), i = 1, 2.$
Two of the most commonly used alternatives (cf. Hollander and Proschan (1972)) they are:
(i) Linear failure rate family

$$\bar{F}_1(x) = e^{-x-\frac{x^2}{2}\theta}, \quad x, \theta \geqslant 0 \tag{13}$$

(ii) Makeham family:

$$\bar{F}_2(x) = e^{-x-\theta(x+e^{-x}-1)}, \quad x, \theta \geqslant 0 \tag{14}$$

Note that Ho (the exponential disribution) is attained at θ = 0 in (i) and (ii). The Pitman's asymptotic efficiency (PAE) of $\hat{\Delta}$ is equal to

$$\text{eff}_F = \frac{\left|\frac{\partial}{\partial\theta}\Delta \mid_{\theta=\theta_0}\right|}{\sigma_0} \tag{15}$$

by substituting of Δ we get

$$\delta`(\theta) = \int_0^\infty \int_0^\infty \frac{\partial}{\partial\theta}[\Gamma(x+t) - \gamma^{-2}\bar{F}(t)e^{-\gamma x}]f_\theta(x)dF_\theta(x)dF_\theta(t) \mid_{\theta=\theta_0}$$

where

$$\delta`(\theta) = \frac{\partial}{\partial\theta}\Delta \mid_{\theta=\theta_0},$$

and by differentiation,

$$\delta`(\theta) = \int_0^\infty \int_0^\infty \left(\gamma^2 \Gamma`_{\theta_0}(x+t) - \bar{F}`_{\theta_0}(t)e^{-\gamma x}\right) e^{-t}e^{-2x}dxdt \mid_{\theta=\theta_0} \tag{16}$$

First: For Linear failure rate family:
By direct calculation from equation (13), (16),

$$\delta`(\theta) = \int_0^\infty \int_0^\infty \left(\frac{1}{2}e^{-t-x}(t^2 + 2tx + 4t + x^2 + 4x + 6) + \frac{t^2}{2}e^{-t-x}\right) e^{-2x}e^{-t}dxdt = \frac{49}{54}$$

and

$$eff_F = \frac{\frac{49}{54}}{2\sqrt{\frac{41}{30}}} = 0.77620 .$$

SECOND: For Makeham family:
By direct calculation from equation (14), (16),

$$\delta`(\theta) = \int_0^\infty \int_0^\infty \left(-e^{-t-x} - \frac{1}{4}e^{-2t-2x} - te^{-t-x} - xe^{-t-x} - e^{-t-x}(1-t-e^{-t})\right) e^{-2x}e^{-t}dxdt$$
$$= -0.31389$$

and

$$eff_F = \frac{0.31389}{2\sqrt{\frac{41}{30}}} = 0.25543 .$$

The null hypothesis is at θ =0 for linear failure rate and Makeham distributions respectively. Direct calculations of PAE of $\hat{\delta}_2$, $\hat{\Delta}_{UK}$ and $\hat{\Delta}_K$ are summarized in table (12), the efficiencies in Table 2 shows clearly our U-statistic $\hat{\Delta}_K$ perform well for F_1 and F_2.

Table 2. PAE of $\hat{\delta}_2$ & $\hat{\Delta}_{UK}$and $\hat{\Delta}_K$

Distribution	$\hat{\delta}_2$	$\hat{\Delta}_{UK}$	$\hat{\Delta}_K$
F_1 Linear failure rate	0.630	0.565	0.776
F_2 Makeham	0.385	0.245	0.255

In Table 3, we give PARE.s of $\hat{\Delta}_K$ with respect to $\hat{\delta}_2$ and $\hat{\Delta}_{UK}$ whose PAE are mentioned in Table 2.

Table 3. PARE of $\hat{\Delta}_K$ with respect to $\hat{\delta}_2$ and $\hat{\Delta}_{UK}$

Distribution	$eff_i(\hat{\Delta}_K, \hat{\delta}_2)$	$eff_i(\hat{\Delta}_K, \hat{\Delta}_{UK}$
F_1 Linear failure rate	1.2	1.4
F_2 Makeham	0.7	1.0

It is clear from Table 3 that the statistic $\hat{\Delta}_K$ perform well for $\bar{F}_1$ and $\bar{F}_2$ and it is more efficient than both $\hat{\delta}_2$ and $\hat{\Delta}_{UK}$ for all cases mentioned above. Hence our test, which deals the much larger UBAC is better and also simpler.

5. TESTING FOR CENSORED DATA

In this section, a test statistic is proposed to test H_0 ($\bar{F}$ is exponential distribution with mean μ) versus, H_1 ($\bar{F}$ is UBACT and not exponential distribution); with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows:
Suppose n objects are put on test, and X_1, X_2, …, X_n denote their true life time. We assume that $X_1, X_2, \ldots, X_n$ be independent, identically distributed (i.i.d.) according to a

continuous life distribution F. Let $Y_1, Y_2, \ldots, Y_n$ be (i.i.d.) according to a continuous life distribution G and assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_i, δ_i), $i = 1, \ldots, n$, where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1 & if \quad Z_i = X_i \ (i \text{ th observation is uncensored}) \\ 0 & if \quad Z_i = Y_i \ (i \text{ th observation is censored}). \end{cases}$$

Let $Z_{(0)} < Z_{(1)} < \ldots < Z_{(n)}$ denoted the ordered of Z's and δ_i is the δ corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier estimator in the case of censored data (Z_i, δ_i), $i = 1, \ldots, n$, then the proposed test statistic is given by (7) can be written using right censored data as

$$\hat{\delta}_K^c = \sum_{i=1}^n \sum_{j=1}^n \hat{f}(x)\left[\hat{\Gamma}_n(x+t) - \bar{F}_n(t)e^{-\gamma Z_{(j)}}\right]\left[\prod_{p=1}^{i-2} C_i^{\delta_i} - \prod_{p=1}^{i-1} C_i^{\delta_i}\right]\left[\prod_{q=1}^{j-2} C_i^{\delta_i} - \prod_{q=1}^{j-1} C_i^{\delta_i}\right] \quad (17)$$

where

$$\hat{\Gamma}_n(x+t) = \int_x^\infty \int_z^\infty \bar{F}_n(u+t)dudt = \int_x^\infty \int_{z+t}^\infty \bar{F}_n(u)dudt \quad (18)$$

$$= \int_x^\infty \left[\hat{\mu} - \sum_{k=1}^{l} \prod_{m=1}^{k-1} C_m^{\delta_m}(Z_k - Z_{k-1})\right]$$

$$l = i + j \quad if \ Z_i + Z_j < Z_n$$

$$l = n \quad if \ Z_i + Z_j > Z_n$$

$$\hat{\mu} = \sum_{j=1}^{l} \prod_{k=1}^{j-1} C_k^{\delta_k}\left(Z_{(j)} - Z_{(j-1)}\right), \quad (19)$$

$$dF_n(Z_i) = \prod_{q=1}^{j-2} C_i^{\delta_i} - \prod_{q=1}^{j-1} C_i^{\delta_i}, \quad (20)$$

$$\hat{f}(x) = \sum_{k=1}^{l} \delta_k K(X - Z_{(k)}) \quad (21)$$

$$\bar{F}_n(t) = \prod_{m < z_{(m)} < t} C_m^{\delta_m}, \quad (22)$$

$$C_m = \frac{n-m}{n-m+1}, \quad t \in \left[0, z_{(m)}\right]. \quad (23)$$

Table 4 shows the critical values percentiles $\hat{\delta}_{uk}^c$ for sample size n=(2)(1)(20) (51)(10)(101) and Figure 2 shows the relation between the sample size and critical values in the case of censored data.

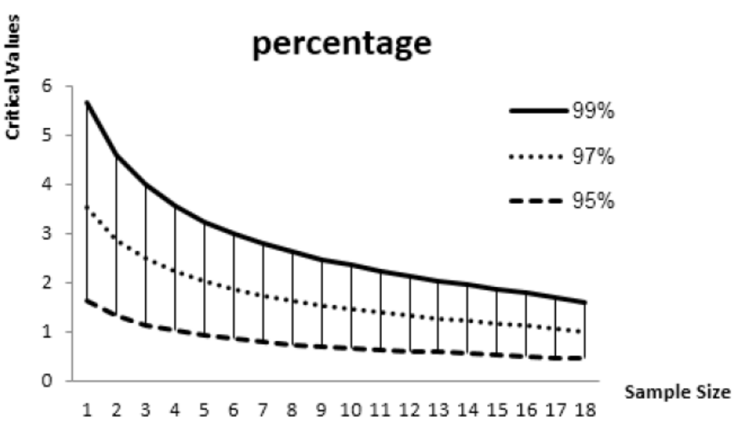

Figure 2. The relation between sample size and critical values

Table 4. Critical values of $\hat{\Delta}_K^c$

n	95^{th}	98^{th}	99^{th}
2	1.61	1.92	2.12
3	1.31	1.56	1.73
4	1.13	1.35	1.50
5	1.01	1.20	1.33
6	0.92	1.1	1.21
7	0.85	1.01	1.12
8	0.79	0.94	1.05
9	0.74	0.89	0.99
10	0.70	0.84	0.93
11	0.67	0.8	0.88
12	0.63	0.76	0.84
13	0.60	0.72	0.80
14	0.58	0.69	0.77
15	0.55	0.66	0.74
16	0.53	0.63	0.71
17	0.50	0.61	0.68
18	0.47	0.58	0.65
19	0.45	0.54	0.61
20	0.41	0.51	0.57
51	0.33	0.24	0.28
61	0.39	0.32	0.36
71	0.38	0.31	0.35
81	0.36	0.3	0.33
91	0.34	0.28	0.31
101	0.32	0.27	0.30

6. NUMERICAL EXAMPLES

6.1. Applications for complete data

Example 1.

In an experiment at Florida state university to study the effect of methyl mercury poisoning on the life lengths of fish goldfish were subjected to various dosages of methyl mercury (Kochar (1985)). At one dosage level the ordered times to death in week are :

6 6.143 7.286 8.714 9.429 9.857 10.143 11.571 11.714 11.714

It is found that the test statistics for the set data by using equation (7) is $\hat{\Delta}_K = 0.377$, which is smaller than the crossposting critical value of the Table 1(0.523). Then we accept H_0 which states that the set of data have exponential property under significant level α = 0.05. Therefore the data has exponential Property.

6.2. Applications for censored data

Example 1.
On the basis of right-censored data for lung cancer patients from Pena (2002). These data consists of 86 survival times (in month) with 22 right censored. The whole life times

i) Non-censored data

0.99	1.28	1.77	1.97	2.17	2.63	2.66	2.76	2.79	2.86
2.99	3.06	3.15	3.45	3.71	3.75	3.81	4.11	4.27	4.34
4.40	4.63	4.73	4.93	4.93	5.03	5.16	5.17	5.49	5.68
5.72	5.85	5.98	8.15	8.62	8.48	8.61	9.46	9.53	10.05
10.15	10.94	10.94	11.24	11.63	12.26	12.65	12.78	13.18	13.47
13.96	14.88	15.05	15.31	16.13	16.46	17.45	17.61	18.20	18.37
19.06	20.70	22.54	23.36						

ii) Censored data

11.04	13.53	14.23	14.65	14.91	15.47	15.47	17.05
17.28	17.88	17.97	18.83	19.55	19.55	19.75	19.78
19.95	20.04	20.24	20.73	21.55	21.98		

It is found that the test statistics for the set of data $\hat{\Delta}_K^c = 0.29$. Then we accept H0 which states that the set of data have exponentail distribution under significant level α = 0.05.

REFERENCES

Abu-Youssef, S. E. (2007). Testing decreasing (increasing) variance residual class of life distributions using kernel method, *Applied Mathematical Science*, **1**, 1915-1926.

Abu-Youssef, S.E. (2009). Nonparametric Test for Used Better Than Aged in Convex Ordering Class (UBAC) of Life Distributions with Hypothesis Testing, *Intentional Journal of Reliability and Applications*, **10**, 81-88.

Abu-Youssef, S. E. and Bakr, M. E. (2014). Some properties of UBACT class of life distribution, *Journal of Advanced Research in Applied Mathematics*, **6**, 1-9.

Abu-Youssef, S. E. and Bakr, M. E. (2014). A goodness of fit approach to class of life distribution with unknown age, *International Journal of Computer Applications*, **104**, 30-35.

Abu-Youssef, S. E. and Bakr , M. E. (2014). Testing for used better than aged in convex tail ordering class (UBACT) of life distribution based on U-statistics, submitted.

Ahmad, I. A. and Q. Li (1997). Testing symmetry of unknown density function by kernel method, *Journal of Nonparametric Statistics*, **7**, 279-293.

Ahmed, I. A. (2004). Some Properties of Classes of Life Distributions with Unknown Age, *Statistics and Probability Letters*, **69**, 333-342.

Barlow ,R. E. and Doksum, K. A. Isoomic (1972). Tests for Covex Ordering, *Proceeding of 6th Berkeley Symposium*, 93-323.

Barlow, R. E. and Proschan, F. (1981). *Statistical Theory of Reliability and Life Testing Probability Models*, To Begin With, Silver-Spring, MD.

Deshpand, J. V., Kocher, S. C. and Singh, H. (1986). Aspects of Positive aging, *Journal of Applied Probability*, **23**, 1472-1483.

Kanjo, A. J. (1993). Testing for new is better than used, *Communications in Statistics-Theory and Methods*, **22**, 311-321.

Kochar, S. C. (1985). Testing exponentiality against monotone failure rate average, *Communications in Statistics-Theory and Methods*, **14**, 381-392.

Mohie El-Din, M. M., Abu-Youssef, S. E. and Hassan, M. K H. (2014). On testing of unknown age classes of life distributions, submitted.

Willmot, G. E. and Cai, J. (2000). On Classes of Lifetime Distributions with unknown age, *Probability in the Engineering and Informational Sciences,* **14**, 473-484.