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Pedagogical *Mathematica* Platform Visualizing the Coriolis Effects in 3-Cell Atmospheric Circulation Model

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The atmospheric flow in the 3-Cell model of global atmosphere circulation is described by the Lagrange's equation of the non-inertial frame where pressure force, frictional force and fictitious force are mixed in complex form. The Coriolis force is an important factor which requires calculation of fictitious force effects on atmospheric flow viewed from the rotating Earth. We make new Mathematica platform to solve Lagrange's equation by numerical analysis in order to analyze dynamics of atmospheric general circulation in the non-inertial frame. It can simulate atmospheric circulation process anywhere on the earth. It is expected that this pedagogical platform can be utilized to help students studying the atmospheric flow understand the mechanisms of atmospheric global circulation.

Keywords: Mathematica, fictitious forces, Coriolis force, non-inertial frame.

1. INTRODUCTION

Motion observed on the rotating Earth is generally explained by invoking inertial forces described in the non-inertial frame of reference (Symon 1971, Landau & Lifshitz 1976). Because viewing the motion from accelerating or rotating frame of reference introduces fictitious forces added to actual forces. On account of the inequality in energy absorbed at a spherical surface of the rotating Earth, seven zones of atmospheric pressure are formed over the Earth surface: an intertropical convergence zone (ITCZ) near the equator, two subtropical highs in both Hemispheres at the latitude of 30°, two subpolar lows on both Hemispheres at the latitude of 60°, two polar highs on both poles. Accordingly, global scale circulation of the atmosphere is described by the Lagrange's equation in the non-inertial frame of reference according to the 3-Cell general circulation of the atmosphere (GCA) model rather than Hadley Cell model (Ahrens 2001). Long-term recording data from the satellites approve of the 3-Cell GCA model ("AMNH-Weather and Climate Events" 2014, "Global Climate Animation" 2014). On the rotating Earth frame, the Coriolis force acts as a most important force to change the direction of surface airflows on the Earth. The deflection is not only instrumental in large-scale atmospheric circulations, the development of tropical cyclones, hurricanes and typhoons, also it can affect missile launching, satellite operation, and GPS position sensors (Bikonis & Demkovicz 2013) in the modern sciences. Effects upon the weather, ocean currents, rivers and projectile motions are well documented (Graney 2011, Mclntyre 2000), but the motions over very long distance are required for discernible effects. Common pedagogical tools are helpful to explain the Coriolis effects. For example, Merry-Go-Round table ("Merry-Go-Round" 2014) or Bath-Tub Vortex (Trefethen et al. 1965) is helpful for explaining the Coriolis effects, but it cannot be distinguished whether its effect is from Coriolis force or centrifugal force unless we calculate the forces with their vector components. While this approach simplifies some problems, there is often little physical insight into the motion, in particular, into the fictitious force of the vectorial characteristic.

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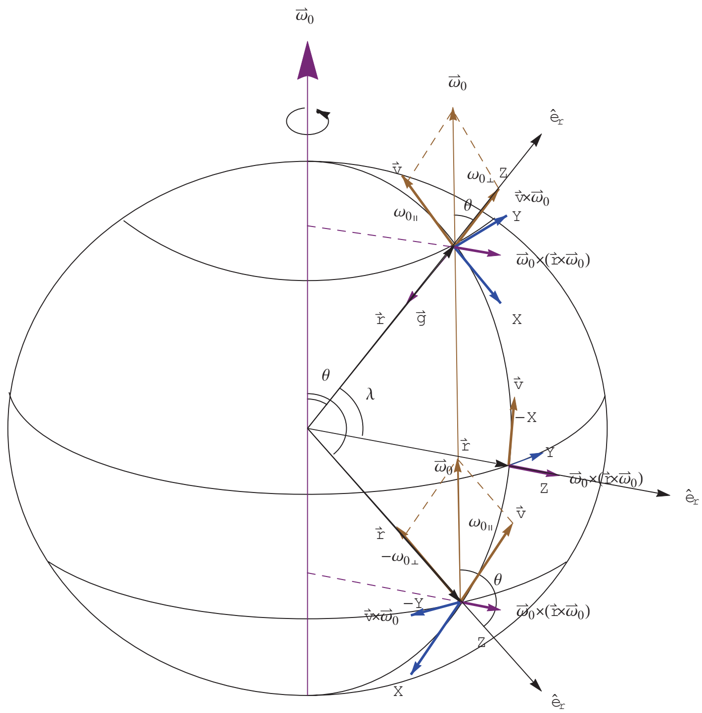


Fig. 1. Actual and fictitious forces on the rotating Earth's surface at the latitude of λ = π/2-θ in a topocentric frame of XYZ axis system. The angular velocity of the Earth is ω_0 and θ is the polar angle in the geocentric reference frame.

Recently, efficient visualization programs are utilized for the Coriolis force effects (Zimmerman & Olness 1995, Tam 1997, Yun 2005, Zeleny 2010). In particular, *Mathematica* is helpful for the convenient function of symbolic calculations and graphic manipulations. The *Mathematica* simulation of the atmospheric circulation matching the 3-Cell GCA model has been presented in our previous work (Yun 2006), however, the 3-Cell GCA model was shown with the illustration rather than with active platform. In this paper, we present anew *Mathematica* platform presenting the GCA simulation according to the 3-Cell GCA model in 2D or 3D graphics. In the platform, we can simulate the Coriolis effects at any point of the globe automatically and confirm the atmospheric circulation dynamics interactively.

2. LAGRANGE'S EQUATION IN A NON-INERTIAL FRAME OF REFERENCE

In the inertial frame, space should be homogeneous and time is isotropic to assert the invariance of the mechanical system. If we were to choose an arbitrary frame of reference, space would be inhomogeneous and anisotropic. Therefore, the equation of motion in the rotating Earth system should be described in the non-inertial frame of reference (Landau & Lifshitz 1976). For the validity of the principle of least action in the mechanical system independent of the frame of reference chosen, we must carry out the necessary transformation of the Lagrangian L_0 for the Lagrange's equation in the non-inertial frame of reference. This transformation is done in two steps. Firstly, we consider a frame of reference K' which moves with a translational velocity $\vec{V}(t)$ relative to the inertial frame K_0. Next, we bring a new frame of K which rotates relative to K' with angular velocity, $\vec{\omega}$. As a result, K executes both translational and rotational transformation to the inertial frame K_0 (fix star): $K_0 \xrightarrow{\vec{V}(t)} K' \rightarrow K' \overset{\omega}{\rightarrow} K$. The Lagrangian in K_0 frame is (Landau & Lifshitz 1976)

$$L_0 = \frac{1}{2} m v_0^2 - U \tag{1}$$

where, v_0 is the velocity of a particle in K_0 frame and U is a potential. The velocities $\vec{v}_0 = \vec{v}' + \vec{V}(t)$ come from a transformation $K_0 \xrightarrow{\vec{V}(t)} K'$ and the $\vec{v'} = \vec{v} + \vec{\omega}$ come from a transformation $K_0 \xrightarrow{\vec{V}(t)} K$. Finally, the Lagrangian in K frame is

$$L = \frac{1}{2} m v^2 + m \vec{v} \cdot \vec{\omega} \times \vec{r} + \frac{1}{2} m \left(\vec{\omega} \times \vec{r'}^2\right) - m \vec{W} \cdot \vec{r} - U \tag{2}$$

where, $\frac{d\vec{V}}{dt} = \vec{W}$. Then Lagrange's equation, which satisfy the principle of least action,

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \vec{v}} = \frac{\partial L}{\partial r} :$$

$$m\frac{d\vec{v}}{dt} = -\vec{\nabla}U - m\vec{W} + m\vec{r} \times \vec{\dot{\omega}} + 2m\vec{v} \times \vec{\omega} + m\vec{\omega} \times (\vec{r} \times \vec{\omega}) \quad (3)$$

If we suppose that angular velocity is constant and neglect the translational velocity, we omit $m\vec{r} \times \vec{\dot{\omega}}$ and $m\vec{W}$. Then Eq. (3) is

$$m\frac{d\vec{v}}{dt} = -\vec{\nabla}U + 2m\,\vec{v} \times \vec{\omega} + m\vec{\omega} \times (\vec{r} \times \vec{\omega}) \quad (4)$$

We rewrite this again as

$$m\frac{d\vec{v}}{dt} = \vec{f}_{effective} \quad (5)$$

$$m\frac{d\vec{v}}{dt} = \vec{f}_{actual} + \vec{f}_{fictitious} \quad (6)$$

This is just a Newton's equation, which shows the motion of an object by the force $\vec{f}_{effective}$. Here $\vec{f}_{actual}$ includes the friction and the pressure gradient force, and the $\vec{f}_{fictitious}$ includes the Coriolis force $2m\vec{v} \times \vec{\omega}$ and the centrifugal force $m(\vec{\omega}) \times (\vec{r} \times \vec{\omega})$ which deflect the atmospheric flows on the rotating Earth. As shown in Fig.1, the Coriolis acceleration vector $\vec{v} \times \vec{\omega}$ direct to ±Y direction. Deriving process of the Lagrange's equation in the non-inertial frame is well described in other books (Symon 1971, Landau & Lifshitz 1976).

3. *MATHEMATICA* PROGRAMING FOR GERNERAL CIRCULATION OF THE ATMOSPHERE

3.1 Vector calculation of the effective force

To analyze the deflection effect of the atmospheric flow based on the 3-Cell GCA model at any point on the Earth's surface, we create a position function **point[t, θ]** in solving the Eq. (4) in *Mathematica* (Zimmerman & Olness 1995, Tam 1997). If a latitude ($\lambda = \pi/2 - \theta$) is given, the wind vector function $\vec{v}_0[\theta]$ is determined by the position function **point[t, θ]**. We assume that the initial wind blows along the meridian and its velocity is determined according to the 3-Cell GCA model. *Mathematica* coding for solving Eq. (4) is shown in In[1] - In[10] and its *Mathematica* solution is Out[11]: **point[t, θ] = {t v0x, -t^2ω0x Cos[θ], -1/2 gt^2}** while n Order = 0 for simplicity. Once **point[t, θ]** is given,

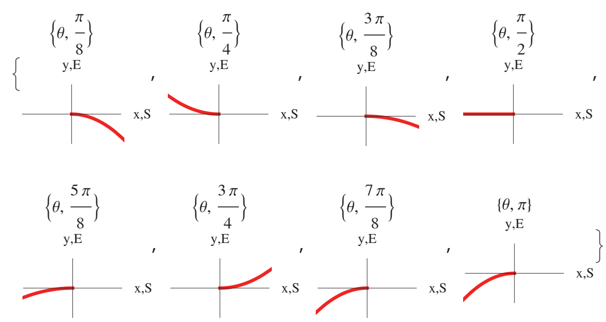


Fig. 2. The table of the deflections at eight points on the globe. The points are selected in the different atmospheric zone of the 3-Cell GCA model.

the **Parametric Plot** draw the path of the wind with a time domain vector array of the **point[t, θ]** in *Mathematica* such as that shown in Fig. 2. The Out[22] (Fig. 2) is a list of plot in deflection on the eight points on the globe selected respectively at the different atmospheric zone of the 3-Cell GCA model. As shown in Fig. 2, the winds deflect right in the Northern Hemisphere and deflect left in the Southern Hemisphere regardless of the wind direction. No deflection occurs at the equator ($\theta = \pi/2$) because the Coriolis force $m\,\vec{v} \times \vec{\omega} = 0$ at the equator such as that shown in Fig. 1. It was not until comprehension of the vectorial nature of the effective force in the non-inertial frame of reference that we could analyze the deflection effects of the atmospheric circulations effectively; we present some parts of the *Mathematica* coding to show the vector calculations and the results with their vector components.

Mathematica code #1

```
In[1] = Clear[Global`*"];
In[2] = IF[θ> 1/2π, ω[t_] := ω_S[t], ω[t_] :=ω_N[t]]; }
        {ω_N[t_] :={-ω0 Sin[θ], 0, ω0 Cos[θ]}; (N-Hemisphere)}
        {ω_S[t_] :={-ω0 Sin[θ], 0, -ω0 Cos[θ]}; (S-Hemisphere)
In[3] = fInertial = {0, 0, - mg} ;
In[4] = fCoriolis = 2 m Cross[r'[t], ω[t]];
In[5] = fCentrifugal =
        m Cross[ω[t], Cross[r[t], ω[t]]]
        //Simplify]
In[51] = v0[θ_]:= Which[θ==0, 10,
        1/8 π <= θ < π/6,  10,
        1/6 π <= θ < π/3,  -10,
        1/3 π <= θ < π/2,  10,
        1/2 π <= θ < π 2/3,  -10,
        2/3 π <= θ < π 5/6,  10,
        5/6 π <= θ < π 17/18,  -10,
        θ == π, -10 ];
In[6] = eq1 = -r''[t] - fInertial - fCoriolis - fCentrifugal
```

Table 1. The accelerations of the effective force calculated with those vector components at the eight cities. The calculation parameters: $\omega_0 = 7.292 \times 10^{-5}$ sec^{-1}, g=9.8 m/sec^2, and the unit of the acceleration is m/sec^2. Minus sign of the $\vec{v}_0x$ stands for the wind direction to the north unit of m/sec.

		Coriolis force ($\times 10^{-4}$)			Centrifugal Force ($\times 10^{-2}$)			Gravity			Effective force ($\times 10^{-3}$)		
City (latitude)	$\vec{v}_0x$	X	Y[a]	Z	X[b]	Y	Z	X	Y	Z	X	Y	Z[c]
Murmansk (68°58'N)	10	0	-13.612	0	1.1346	0	0.4370	0	0	-9.8	11.346	-1.3612	-9.7956
New York (40°42'N)	-10	0	9.5151	0	1.6744	0	1.9483	0	0	-9.8	16.774	0.9515	-9.7802
Honolulu (21°18'N)	10	0	-7.1424	0	1.4485	0	2.5788	0	0	-9.8	14.485	-0.7142	-9.7742
Equator (0°0'N)	±10	0	0	0	0	0	3.3924	0	0	-9.8	0	0	-9.7661
Lima (12°03'S)	-10	0	-3.0445	0	-0.6259	0	3.2446	0	0	-9.8	-6.9259	-0.3044	-9.7675
Santiago (33°27'S)	10	0	8.0314	0	-1.5594	0	2.3636	0	0	-9.8	-15.594	0.8031	-9.7764
Queen Mary (66°45'S)	-10	0	-13.399	0	-1.2304	0	0.5286	0	0	-9.8	-12.304	-1.3399	-9.7947
Daejeon (36°19'S)	-10	0	8.6379	0	1.6189	0	2.2023	0	0	-9.8	16.189	0.8637	-9.7779

[a]deflection to East (+) or West (+), [b]South (+) or North (-), [c]no factor

```
In[7] = initialRule = Thread/@ {r[0] -> 0,
        r'[0] -> {1, 0, 0}}//Flatten;
        nOrder = 0;
In[8] = eq2 = (Series[eq1/m, {t, 0, nOrder}] == 0)
        /.initialRule//Normal//Thread
        //Simplify
In[9] = vars = Table[D[{x[t],y[t],z[t],{t,i}]
        ,{i,2,nOrder+2}]/.t -> 0 //Flatten
In[10] = sol = Solve[eq2,vars]//First
In[11] = point[t_,θ_] = Series[r[t],{t,0,nOrder+2}]
         /.sol/.initialRule/.v0[θ] -> v0x//Normal]

Out[11] = {t v0x,-t^2 v0xω0 Cos[θ],-1/2gt^2}
```

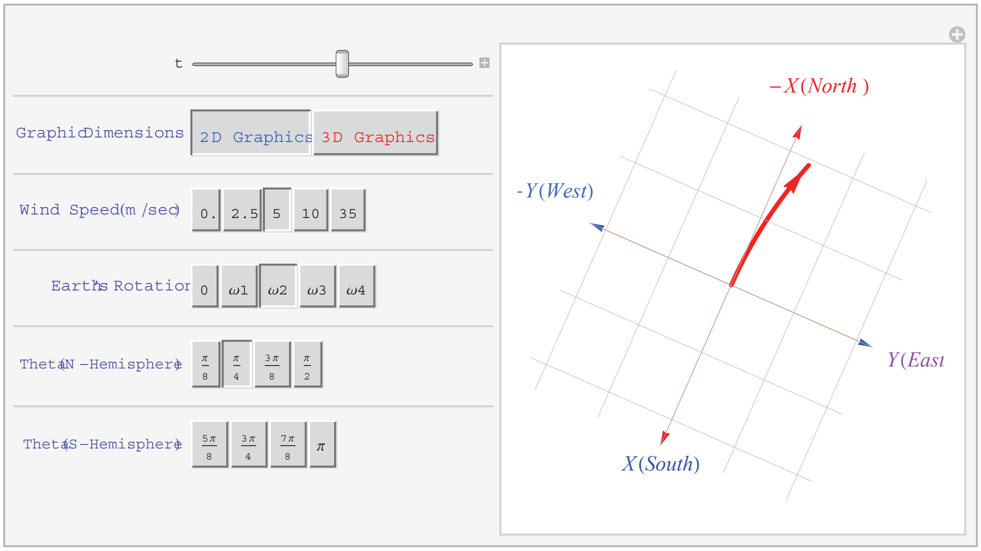
Mathematica code #2

```
In[22] = plot[θ_]:=
         ParametricPlot[point[t, θ][[{1,2}]]/.{ω0->1,
         g -> 9.8}//Evaluate, {t, 0, 510}
         ,PlotStyle -> {Thickness[0.0336], Hue[0.01]}
         ,AxesLabel -> {"x, S", "Y, E " }
         ,PlotRange -> {{-50, 50}, {30, -30}, {0, -50}}
         ,PlotLabel -> {"theta", θ}
         ,Ticks -> False, ImageSize->85,
         ,DisplayFunction -> Identity];
         Table[plot[θ], { θ, (1/8)π, (8/8)π, (1/8)π }]

Out[22] =
```

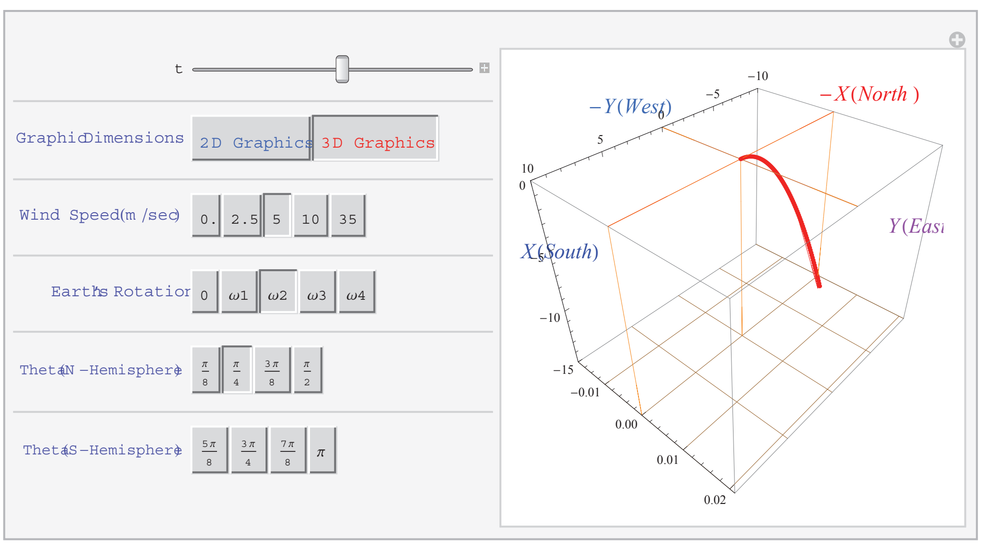
Mathematica code #3

```
In[31] = gomega = {-ω0Sin[θ], 0, ω0Cos[θ]};
         rveector = {0, 0, rho}; }
         velocity = {v0[θ], 0, 0}; }
         gravity = 0, 0, -9.8}; }
         corioacc [θ_]=
         2 Coross [velocity, gomega]//Thread}
         centriacc [θ_]=}
         Cross[gomega, Cross[rvector, gomega]]
         //Thread}
         effectacc[θ_]=
         {corioacc[θ] + centriacc[θ] + gravity}//Thread

Out[31] = {0, -2ω0 Cos[θ]v0[θ], 0}
Out[32] = {rhoω0^2Cos[θ]Sin[θ], 0, rhoω0^2Sin[θ]^2}
Out[33] = {rho ω0^2 Cos[θ]Sin[θ],
          -2ω0Cos[θ]v0[θ], -9.8+rhoω0^2Sin[θ]^2}
```

However, Fig. 2 does not show which fictitious forces cause the deflections unless we calculate those with their vector components respectively. Because the deflections of the winds over the globe come from the resultant effective force $\vec{f}_{effective}$ in Eq. (5), we must calculate accurately the ingredients of the effective force for the probable cause of deflections. For the examination of the deflection effects of the effective force, we calculate the constituent parts of the effective force accelerations with its vector components at the eight cities respectively, and summarize those in Table 1. The vector calculation of the fictitious forces with those components is easy in *Mathematica* as shown coding In[31]. The vector command of the *vector product* or *triple vector product* is coding as **cross[a, b]** or **cross[a, cross[b, c]]** in *Mathematica* as if we write down the formula in text. Resultant accelerations occur in X, Y, Z directions, however, only the Coriolis force acceleration occur in ±Y direction (In[31], Out[31]) which acts as a deflections force of the wind flowing ±X direction. The effective force accelerations in ±X, Z directions reduce or enhance of wind speed and gravity (In[31], Out[31-33]), those are not forces deflecting the wind direction for the wind speed in ±X, gravity in Z. These calculations confirm that the Coriolis force is the unique deflection force in the atmospheric circulation dynamics



(a) 2D Graphics snapshot



(b) 3D Graphics snapshot

Fig. 3. Snapshots of the *Mathematica* platform of the general atmospheric circulation to the 3-Cell model. (a) is the 2D Graphics presentation and (b) is that of 3D Graphics. Simulations begins on click ▶ in the popup when you click the ⊕ right of *t* panel. Anytime you may change the Graphics dimension and Graphics menu by selecting the panel menu. In addition, you may change the Graphics mode even if the simulation is in pause, then the platform shows the modified simulation.

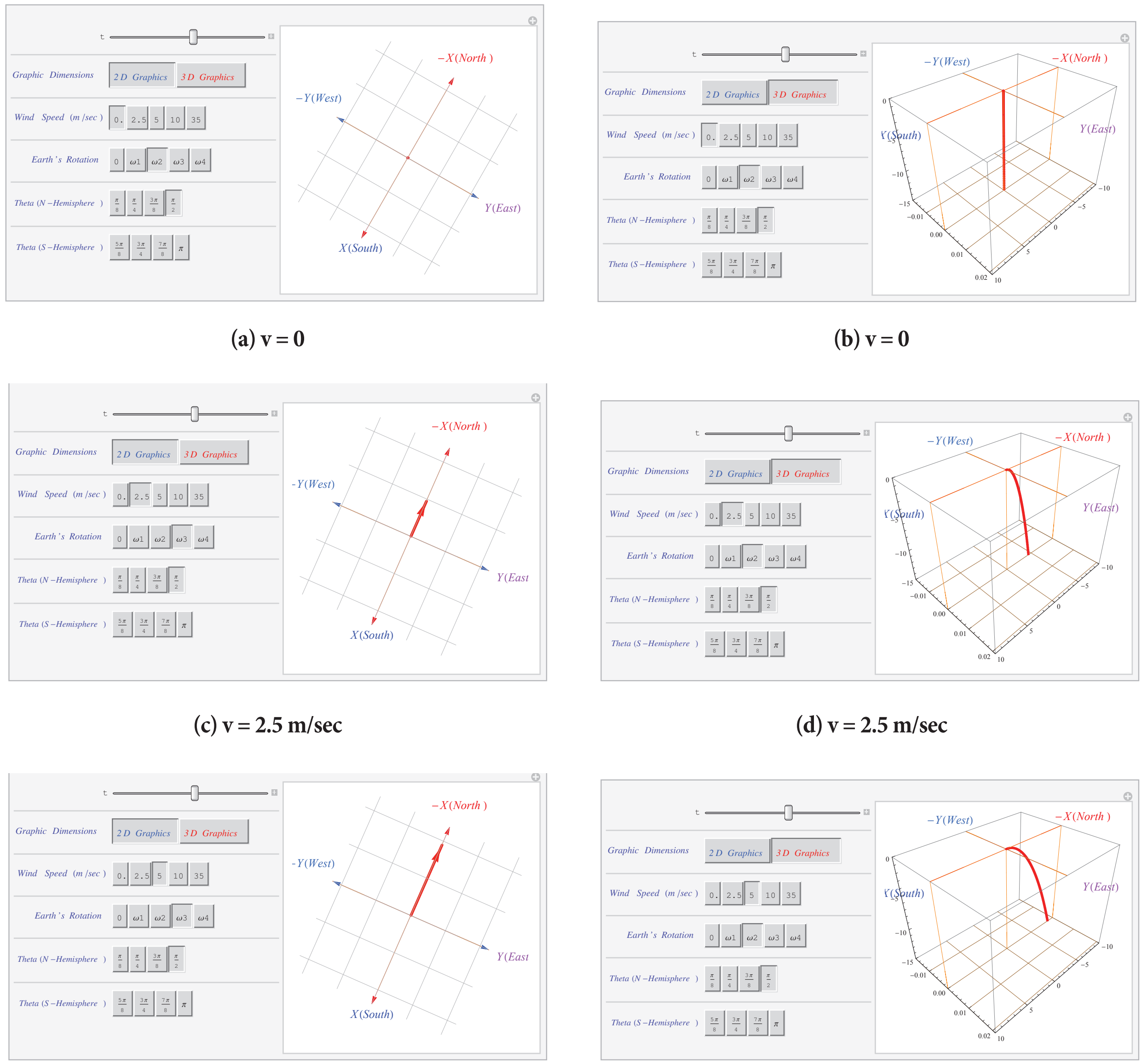
even though its relative intensity is so weak; *Coriolis force: centrifugal force: gravity* = 10^{-4} : 10^{-2} : 1 as shown in Table 1. Cyclone is an area of low pressure around which the winds blow parallel to the pressure gradient force balanced to the Coriolis force. A hurricane is a severe tropical cyclone which occurs over the northern Atlantic and eastern North Pacific oceans, and it is called the typhoon in the western North Pacific. This same type of cyclone has a different name in a different region of the world, however, all the climate events occur by the Coriolis force effect (Ahrens 2001, AMNH-Weather and Climate Events 2014).

3.2 *Mathematica* platform for the 3-cell atmospheric general circulation model

Understanding the vectorial nature of the effective forces on the Earth's surface is a keyword of the atmosphere circulation dynamics. However, it is not easy to evaluate the

(a) 2D Graphics (b) 3D Graphics

Fig. 4. The wind deflections at the six points of the globe in both 2D Graphics (a) and 3D Graphics (b). The six points are selected for the GCA 3-Cell model from the North pole: θ = 1/8 π: North-Polar Easterlies, 2/8 π: South-West Westerlies, 3/8 π: North-East Trade Winds, 5/8 π: South-East Trade Winds, 6/8 π: North-West Westerlies, 7/8 π: South-Polar Easterlies in both columns.



(a) v = 0

(b) v = 0

(c) v = 2.5 m/sec

(d) v = 2.5 m/sec

(e) v = 5.0 m/sec

(f) v = 5.0 m/sec

Fig. 5. Wind deflections at the equator in 2D (left column) graphics and 3D graphics (right column) for the different wind velocities.

wind deflection effects at any point on the Earth's surface, because the deflection is varying on the resultant wind vector and associated effective force vectors at the point belong to the atmospheric pressure zone matching to 3-Cell GCA model. The fundamental concept of the inertial force effects in the non-inertial frame of reference is essential to analyze the GCA dynamics. Visual representation of such vectorial nature of the GCA will be in valuable to the researchers working in this field and help teach physics or meteorology students.

We provide Coriolis effects platform to simulate the wind progressing on the rotating Earth's surface matching the 3-Cell GCA model using the function **Manipulate** in Mathematica. Fig. 3a is a 2D Graphics snapshot of the platform and Fig. 3b is a 3D Graphics snapshot of the platform. The platform draws the path of the wind through the **point[t, θ]** with a time domain of vector array of the solution of Eq. (4) using the **Parametric Plot** of *Mathematica*. On starting the program, platform will show Fig. 3a. Simulation performs the built-in program when you click the ▶ appearing while you spread the ⊕ of *t* panel of the platform. While the program is executing, you can change the parameters of *Wind Speed, Earth's Rotation*, and *Theta*, then the change will be effective immediately by the platform. When you stop the simulation, then the platform present the trace so far, and you can change the parameters of the platform to see another simulation. We look at the different deflection in a simulation if we only click the different point in the status of pausing platform. Fig. 4 shows the deflections at six points both 2D and 3D Graphics at once by clicking the points on the platform panel. It shows promptly winds of the GCA 3-Cell model on the one platform: two polar easterlies, two westerlies, and two trade winds. The performance mode of the platform will change to the 2D Graphics or 3D Graphics anytime you want. Manipulation of both modes enables you to analyze not only 2-dimensional deflections but also 3-dimensional deflections. For example, at the equator we can analyze the X, Z deflection as the wind velocity is varying along the ±X direction and confirm the deflection effects such that as shown in Fig. 5. Because the program simulate the position function **point[t, θ]** of solution of the vector differential equation with effective force, the simulation performs the physics behavior of three vectors – wind velocity ($\vec{v}$), angular velocity of the Earth ($\vec{\omega}$), and position vector ($\vec{r}$) – and their associated effects accurately. Hence, the program differs from the animation that animate the assign functions with the **Animate** function from the convenient graphic tool (Zeleny 2010). In *Mathematica*, we can save the snapshots of the simulations and print it. This program also executes well on later version of 8.0 *Mathematica*.

4. SUMMURY

The Earth is a planet of a sphere which revolves the fix star while rotating itself with a constant angular velocity. The frame of reference on the rotating Earth is a non-inertial frame of reference since the frame is in acceleration continuously. Therefore, the equation of motion should be modified through the coordinate's transformations by the least action principle in the mechanical system of Eq. (4): $m\frac{d\vec{v}}{dt}/dt=-\vec{\nabla}U+2\,m\ \vec{v} \times \vec{\omega} + m\vec{\omega} \times (\vec{r} \times \vec{\omega})$. Here the Coriolis force $2\,m\,\vec{v} \times \vec{\omega}$ and centrifugal force $m\vec{\omega} \times (\vec{r} \times \vec{\omega})$ are the most important fictitious forces which play a significant role in a variety of natural processes, most prominently the atmospheric circulation dynamics. In particular, the Coriolis force is also responsible for the circular motion of the tropical cyclone, the tropical typhoon and the hurricane. Physical comprehension and manifesting ability for the non-inertial frame of reference on the rotating Earth becomes a merit of asset to physicist and natural scientist. The conception of the inertial forces on the Earth has been recognized recently because the Coriolis force is not only the most important effective force in the GCA dynamics but also an indispensable task for the scientist in long-range missile launching, satellite operation and GPS position sensors in modern technology. We demonstrate the Coriolis effects platform to simulate the wind progressing on the rotating Earth's surface matching the 3-Cell GCA model in *Mathematica*. The platform draws the realistic path of the wind, through **point[t, θ]** with the time domain of the vector array of the solution of Eq. (4) using the **Parametric Plot** in *Mathematica*. We expect this platform will be a helpful tool for the physicist and the scientist to analyze the atmospheric general circulation dynamics.

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