A Leakage-Based Solution for Interference Alignment in MIMO Interference Channel Networks

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Abstract

Most recent research on iterative solutions for interference alignment (IA) presents solutions assuming channel reciprocity based on the suppression of interference from undesired sources by using an appropriate decoding matrix also known as a receiver combining matrix for multiple input multiple output (MIMO) interference channel networks and reciprocal networks. In this paper, we present an alternative solution for IA by designing precoding and decoding matrices based on the concept of signal leakage (the measure of signal power that leaks to unintended users) on each transmit side. We propose an iterative algorithm for an IA solution based on maximization of the signal-to-leakage-and-noise ratio (SLNR) of the transmitted signal from each transmitter. In order to make an algorithm removing the requirement of channel reciprocity, we deploy maximization of the signal-to-interference-and-noise ratio (SINR) in the design of the decoding matrices. We show through simulation that minimizing the leakage in each transmission can help achieve enhanced performance in terms of aggregate sum capacity in the system.

Keywords: interference alignment, iterative IA solution, max SLNR, channel reciprocity

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1. Introduction

Performance in wireless systems is often limited by interference between multiple links, where the frequency spectrum is widely occupied by interference. Interference management has been widely recognized as one of the key issues that need to be addressed for wireless networks. Interference alignment (IA) is a promising technology that has recently been developed to maximize the overlap between the spaces of all interference in each receiver. Its goal is to align all the interference from transmitters into one half of the signal space in each receiver, leaving the other half available to the desired signal. In other words, IA aligns all the interference in a receiver into a common sub-space of the total received signal space, and keeps the interference space linearly independent of the desired signal sub-space. The maximum achievable degree of freedom (DoF) per user for a two-user symmetric multiple input multiple output (MIMO) $(\mathrm{M} \times \mathrm{N})$ interference channel (MIMO-IC) with zero-forcing receivers for each user is $\min\left(\mathrm{M}, \mathrm{N}, \frac{\max(\mathrm{M},\mathrm{N})}{2}\right)$ [1]. Thus, the sum capacity per user for a two-user interference channel (IC) is theoretically equal to $\frac{1}{2}\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$ [2], where SNR is signal-to-noise ratio, provided there is perfect channel state information (CSI).

The earliest application of IA appeared in analysis of the X-channel [3], where it was not an IA scheme, but rather an iteratively achievable scheme for the X-channel with dirty paper coding (DPC) and successive decoding. The nomenclature of IA was actually done by Jafar and Shamai [4], where the authors crystallized the idea of IA for a two-user X-channel in its essential linear form without using DPC, successive decoding, or an iterative solution. IA was generalized for a *K*-user parallel interference channel by Cadambe and Jafar [5]. They showed that even for $K > 2$ interfering users, the sum capacity per user is $\frac{1}{2}\log(SNR) + o(\log(SNR))$.

As soon as an IA scheme was introduced analytically, a lot of research was carried out to further develop and enhance the IA algorithms. Sung et al. [6] proposed non-iterative linear precoder design methods for interference channel systems based on the conventional IA algorithm. They determined the orthonormal basis vectors of each user's precoding matrix to achieve the maximum DoF, the optimized precoding matrices in the IA method. A generalized precoder design was presented [7], where the precoder design for both symmetric systems (where each of the transmitting and receiving node is equipped with the same number of antennas) and asymmetric systems (where the antennas in the transmitting node exceeds those at the receive node).

The IA algorithms presented [6] [7] are non-iterative, also known as linear, solutions to IA for MIMO-IC systems. So far, non-iterative solutions for precoding and decoding (receive combining) matrices for interference channels based on IA, are known only for the three users, where each user achieves $M/2$ DoF $(N_{r_k} = N_{t_k} = M)$ for even M. An alternative method for IA is by employing iterative algorithms [8-11] to alternately optimize the precoding and decoding matrices in the transmitter and receiver, respectively. A so-called 'distributed' IA was introduced by Gomadam et al. [8], where IA is achieved with only local channel knowledge at each node but requires reciprocity in the wireless networks. Some work [8] has been extended [9] to include a distributed IA algorithm aiming to orthogonalize signal and interference sub-spaces based on power from leakage interference at each receiver, and to maximum signal-to-interference-and-noise ratio (SINR)-based IA that takes into account the power at the desired transmit-receive channel. An algorithm that alternates minimization over the precoding matrices in the transmitters and decoding matrices (or, equivalently, the interference sub-spaces) in the receivers was proposed [10], which claims to remove the requirement of reciprocity from the wireless channels. An iterative algorithm that aims at finding the IA solution that maximizes the average sum-rate was presented [11]. It utilizes the alternating minimizing algorithm where, at each step, either the precoders or the decoders are moved along the direction given by the gradient of the sum-rate. Three algorithms span the trade-off between performance and complexity for the static MIMO-IC [12]: 1) a minimum interference-plus-noise leakage algorithm to account for colored noise; 2) a minimum mean squared error (MMSE) algorithm that attempts to minimize the expected sum of the norms of error; and 3) a sum SINR maximization algorithm that attempts to maximize the sum signal power across the network divided by the sum interference power, incorporating the inter-stream interference for the users. A simplified version of the maximum sum SINR was presented [13] without incorporating inter-stream interference and colored noise.

The first-ever experimental study of IA over measured MIMO-orthogonal frequency division multiplexing channels was conducted for a variety of outdoor and indoor measurement scenarios by Ayach et al. [13]. The study experiment verified the claimed performance in terms of DoF. An assessment of practical issues including performance in realistic propagation environments, the role of CSI in the transmitter, and the practicality of IA in large networks are discussed by Ayach et al. [14]. The performance of IA over weak-to-moderate MIMO-IC has been investigated [15]. Rao et al. investigated the dynamics of the interference topology in a limited feedback system with heterogeneous path loss and spatial correlations, and proposed a dynamic scheme in terms of transmit SNR, feedback bits, and interference topology parameters [16].

Almost all of the iterative algorithms discussed above have an inherit requirement for channel reciprocity, except for the alternating minimization algorithm [10], but it has worse sum capacity performance, especially at a low SNR range (the results in Section 5 validate it). In this paper, we present an alternative approach for IA solution by designing the precoding and decoding matrices based on the concept of the signal leakage at each transmit terminal. In this paper, we present an alternative approach for an IA solution by designing the precoding and decoding matrices based on the concept of signal leakage in each transmit terminal. In particular, the precoding and decoding matrices are developed to obtain an IA solution based on maximization of the signal-to-leakage-and-noise ratio (SLNR). Furthermore, we also adopt maximization of SINR while designing the decoding matrices for each user, such that the requirement for channel reciprocity is no longer required. The approach with maximum SLNR in each transmitter minimizes the leakage power towards undesired receivers, with interference plus noise information being relayed by the respective receivers. Limiting the leakage signal in a transmitter, directed towards an undesired receiver terminal, ultimately aids minimization of the SINR in each receiver. The concept of signal leakage in terms of SLNR was used [17] to design transmit beam-forming for downlink multi-user MIMO systems. The concept of SLNR is incorporated in an MMSE approach [18] to develop a low complexity design of the linear transmit filters for MIMO-IC networks with nearly no loss in terms of bit error rate compared to the conventional MMSE approach [19]. The authors in [18] extended their design to a coordinated multi-cell system with multiple users per cell (i.e., a MU-MIMO-IC network) [20], where they proved it had no loss in terms of sum-rate performance.

This paper is organized as follows. Section 2 presents the system model considered in this paper. Section 3 briefly summarizes some of the potential iterative interference alignment algorithms for MIMO-IC networks presented so far. In Section 4, we present a detailed description of the iterative IA algorithm based on maximization of SLNR. Section 5 presents the simulation results and their analysis for the proposed algorithm, along with the IA algorithms summarized in Section 5 for MIMO-IC networks. Finally, concluding remarks are in Section 6.

2. System and Channel Model

Consider a K-user symmetric MIMO-IC network where the k^{th} ($\forall k \in [1,2,\ldots,K]$) transmit-receive pair is equipped with N_{t_k} transmit and N_{r_k} receive antennas, respectively, to transmit $d_k \leq \min(N_{t_k}, N_{r_k})$ spatial streams. In particular, there are K simultaneous links, with transmitter k linked with receiver k in a 1-1 mapping fashion. Each N_{t_k}-antenna transmitter tries to communicate to its corresponding N_{r_k}receiving antennas simultaneously, and generates co-channel interference in all undesired receivers as shown in **Fig. 1**. We denote a symmetric MIMO-IC network as $(N_r \times N_t, K)^d$ where all users have the same antenna configuration, i.e., $N_{t_k} = N_t, N_{r_k} = N_r$, and $d_k = d$ for $k \in [1,2,\ldots,K]$. We assume the narrow-band block-fading MIMO interference channel model, where all the channel links in the network remain constant during one symbol transmission, but change from one symbol transmission to another. The channel output at receiver node k is described as follows:

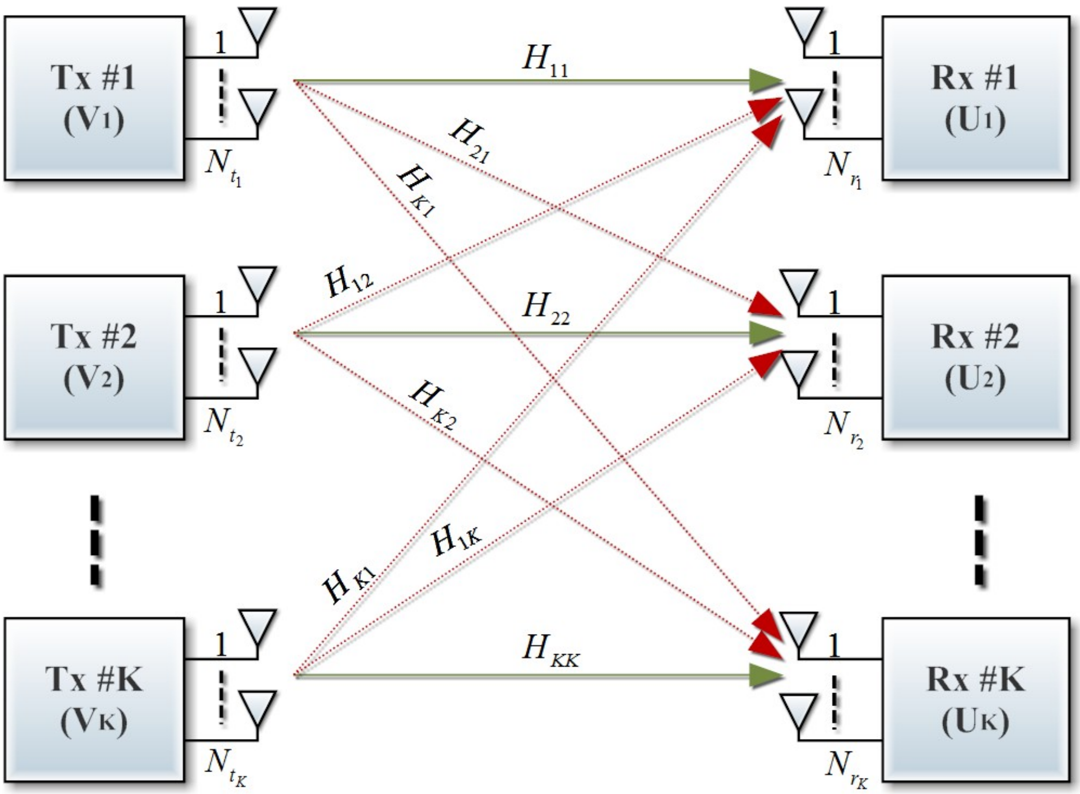

Fig. 1. **Structure of the K user MIMO interference channel.**

$$\boldsymbol{y}_k = \sum_{l=1}^{K} \mathbf{H}_{kl} \mathbf{V}_l \boldsymbol{x}_k + \boldsymbol{w}_k, \ \forall k \in [1,2,\ldots,K], \tag{1}$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_{r_k} \times N_{t_l}}$ is an independently and identically distributed (IID) MIMO complex channel-fading coefficient matrix between transmitter l and receiver k, $\mathbf{V}_l \in \mathbb{C}^{N_{t_l} \times d_l}$ is the precoding matrix for the l^{th} transmitter associated with transmitted symbols $\boldsymbol{x}_l \in \mathbb{C}^{d_l \times 1}$ with $d_l \le \min(N_{t_l}, N_{r_k})$; and d_l is the DoF associated with the l^{th} link. $\boldsymbol{w}_k \in \mathbb{C}^{N_{r_k} \times 1}$ denotes an IID complex Gaussian noise vector at receiver k with zero mean and noise variance σ_n^2, such that $\mathbb{E}[n_k n_k^*] = \sigma_n^2 \mathbf{I}_{N_{r_k}}$ $\forall k \in [1,2,\ldots,K]$. We further assume that the channels $\mathbf{H}_{kl}$ are each of full rank and mutually independent, and the transmitted symbols are IID. In this signal model, a perfect functioning of the carrier recovery symbol timing synchronization module is assumed. For simplicity, the transmit power at each link is assumed to be normalized to one.

Now, by denoting the decoding matrix of the k^{th} receiver as $\mathbf{U}_k \in \mathbb{C}^{N_{r_k} \times d_k}$, the receive filter output of receiver k can be written as

$$\widehat{\boldsymbol{x}}_k = \mathbf{U}_k^* \boldsymbol{y}_k = \mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k[n] \boldsymbol{x}_k + \mathbf{U}_k^* \sum_{l \neq k}^{K} \mathbf{H}_{kl} \mathbf{V}_l \boldsymbol{x}_l + \mathbf{U}_k^* \boldsymbol{w}_k. \tag{2}$$

The sum-rate performance of the k transmit-receive pair with IA in a MIMO-IC network is obtained from

$$R_{sum} = \sum_{k=1}^{K} \log_2 \left[\det\left(\mathbf{I}_{d_k} + \mathbf{S}_k \mathbf{Q}_k^{-1}\right)\right], \tag{3}$$

where $\mathbf{S}_k = \mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k^* \mathbf{H}_{kk}^* \mathbf{U}_k$ denotes the receive signal covariance matrix and $\mathbf{Q}_k = \sigma_k^2 \mathbf{U}_k^* \mathbf{U}_k + \sum_{l=1, l \neq k}^{K} \mathbf{U}_k^* \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^* \mathbf{U}_k$ denotes the interference-plus-noise covariance matrix in the k^{th} receiver. Some algorithms are designed to have orthogonal precoding and decoding matrices, where $\mathbf{U}_k^* \mathbf{U}_k = \mathbf{I}_{d_k}$.

3. Interference Alignment Techniques

In this section, we present a brief summary of several iterative IA transmission strategies for the MIMO-IC network and encourage readers to refer to each respective reference for details. Non-iterative IA strategies have low complexity compared to iterative ones, and require global CSI, i.e., the CSI for every possible link, including interference links in an interference network, should be known to each and every transmitter. It is not generalized for a MIMO-IC network with more than three-users. On the other hand, iterative strategies have high complexity [6] (due to multiple iterations) and are mostly based on reciprocity of the channel. It requires only local CSI, i.e., each receiver only needs to know its desired channel's CSI plus the aggregate interference received. Thus, the solution for precoding and decoding matrices for $K > 3$ is achievable via iterative IA strategies.

According to Yetis et al. [21], the feasibility of a linear IA solution is achieved when there exists precoding matrices $\mathbf{V}_k: N_{t_k} \times d_k$ with $rank(\mathbf{V}_k) = d_k$ and decoding (receive combining) matrices $\mathbf{U}_k: N_{r_k} \times d_k$ with $rank(\mathbf{U}_k) = d_k$ that satisfy:

$$\mathbf{U}_k^* \mathbf{H}_{kl} \mathbf{V}_l = \mathbf{0}_{d_k \times d_l}, \ \forall l \neq k \tag{4}$$

$$\mathrm{rank}(\mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k) = d_k \ \forall k \in [1, \ldots, K]. \tag{5}$$

The condition in (4) is the condition for the existence of interference-free space of desired dimensions, and the condition in (5) ensures that the desired signal is visible and resolvable with the interference-free space. An immediate consequence of the formulation is the reciprocity of IA [9]. Based on these conditions, several iterative algorithms have been proposed so far to resolve the precoding matrices and decoding matrices. In this section, we briefly summarize some of the iterative algorithms, namely the distributed IA algorithm and maximum SINR [9], the alternating minimization algorithm [10], the maximum sum-rate [11], the MMSE algorithm [12], and the maximum sum SINR [13]. All these algorithms depend on channel reciprocity except for the alternating minimization algorithm, which the author claims removes the requirement for channel reciprocity [10].

3.1 Distributed IA Algorithm

The IA solution based on a distributed algorithm [9] is presented here. At each iteration, the algorithm reduces the interference leakage cost function at each receiver assuming channel reciprocity, provided $\mathbf{V}_k^* \mathbf{V}_k = \mathbf{U}_k^* \mathbf{U}_k = \mathbf{I}$ is defined as

$$I_L = \sum_{k=1}^{K} \mathrm{Tr}\left[\mathbf{U}_k^* \mathbf{Q}_k \mathbf{U}_k\right], \tag{6}$$

where

$$\mathbf{Q}_k = \sum_{l=1, l \neq k}^{K} \frac{P_k}{d_k} \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^*. \tag{7}$$

The distributed IA algorithm is as follows:

1. Start with arbitrary precoding matrices $\mathbf{V}_l \ \forall l$.
2. Compute the decoding matrices as $\mathbf{U}_k = \nu_d \left[\sum_{l=1, l \neq k}^{K} \frac{P_l}{d_l} \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^*\right] \ \forall k$
3. Reverse the direction and compute the precoding matrices as
   $\mathbf{V}_l = \nu_d \left[\sum_{k=1, k \neq l}^{K} \frac{P_k}{d_k} \mathbf{H}_{lk} \mathbf{U}_k^* \mathbf{U}_k \mathbf{H}_{lk}^*\right] \ \forall l.$
4. Repeat Step 2 and 3 until convergence.

3.2 Maximum SINR–based IA

The Max-SINR algorithm [9] targets maximizing the SINR of the k^{th} stream of receiver $k \ \forall k, l$, which is given by

$$\rho_{kl} = \frac{P_k}{d_k} \frac{\mathbf{U}^*_{k[:l]} \mathbf{H}_{kk} \mathbf{V}_{k[:l]} \mathbf{V}^*_{k[:l]} \mathbf{H}^*_{kk} \mathbf{U}_{k[:l]}}{\mathbf{U}^*_{k[:l]} \mathbf{Q}_k \mathbf{U}_{k[:l]}}, \tag{8}$$

where $\mathbf{U}_{k[:l]}$ and $\mathbf{V}_{k[:l]}$ denote the l^{th} column of $\mathbf{U}_k$ and $\mathbf{V}_k$, respectively. $\mathbf{Q}_k$ is the covariance matrix[1] of the noise, the inter-user, and the inter-stream interference that is given by

$$\mathbf{Q}_k = \sigma_k^2 \mathbf{I}_{N_{r_k}} + \sum_{l \neq k}^{K} \frac{P_t}{d_t} \sum_{d=1}^{d_k} \mathbf{H}_{kl} \mathbf{V}_{k[:d]} \mathbf{V}^*_{k[:d]} \mathbf{H}^*_{kl} - \frac{P_k}{d_k} \mathbf{H}_{kk} \mathbf{V}_{k[:l]} \mathbf{V}^*_{k[:l]} \mathbf{H}^*_{kk}. \tag{9}$$

3.3 Alternating Minimization IA

The alternating minimization algorithm [10] views the alignment problem as minimizing the leakage interference power at each receiver over the set of precoding and decoding (interference sub-spaces) matrices. The minimization problem is written as

$$\min_{\mathbf{V}_l \mathbf{V}_l^* = \mathbf{I}\ \forall l; \mathbf{U}_k \mathbf{U}_k^* = \mathbf{I}\ \forall k} \sum_{k=1}^{K} \sum_{l \neq k}^{K} \| \mathbf{H}_{kl} \mathbf{V}_l - \mathbf{U}_k \mathbf{U}_k^* \mathbf{H}_{kl} \mathbf{V}_l \|_F^2. \tag{10}$$

The precoders and decoders are iteratively refined. As a result, the pseudo-code for such a minimization is as follows:

1. Choose the set $\mathbf{V}_l$ $\forall l$ randomly.
2. Choose the columns of $\mathbf{U}_k$ to be the $N_{t_k} - d_s$ dominant eigenvectors of $\left(\sum_{l \neq k}^{K} \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^*\right) \forall k$.
3. Choose the columns of $\mathbf{V}_l$ to be the d_s least dominant eigenvectors of $\left(\sum_{k \neq l}^{K} \mathbf{H}_{kl}^* \left(\mathbf{I}_{N_{r_k}} - \mathbf{U}_k \mathbf{U}_k^*\right) \mathbf{H}_{kl} \forall l\right)$.
4. Repeat Step 2 and 3 until convergence.

3.4 Maximum Sum-rate IA

The maximum sum-rate algorithm [11] is a combined form of the alternating minimization algorithm and the gradient descent approach. At each step of the alternating minimization algorithm either the precoders or the decoders are moved along the direction given by the Grassmann manifold. The algorithm can be summarized as follows:

1. Choose the set $\mathbf{V}_l$ $\forall l$ randomly and proceed to find $\mathbf{U}_k \forall k$.
   a) Choose the columns of $\mathbf{U}_k$ to be the $N_{t_k} - d_k$ dominant eigenvectors of $\sum_{l \neq k}^{K} \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^* \forall k$.
   b) Compute the gradient of the sum-rate with respect to $\mathbf{U}_k$ and project the gradient in the Grassmann tangent space.

[1]It is to be noted that the noise (interference) covariance matrix $\mathbf{Q}_k$ might be different for dfferent algorithms.

   c) Compute the compact SVD of the Grassmann manifold.
   d) Obtain the new decoder by moving along the geodesic in the Grassmann manifold
2. Use $\mathbf{U}_k$ to obtain $\mathbf{V}_l\ \forall l$.
   a) Choose the columns of $\mathbf{V}_m$ to be the d_m least dominant eigenvectors of $\sum_{k\neq m}^{K}\mathbf{H}_{kl}^{*}\,\mathbf{U}_k\mathbf{U}_k^{*}\mathbf{H}_{kl}\ \forall l$.
   b) Perform Steps 1(b) to 1(d), but this time with respect to $\mathbf{V}_k$.
3. Repeat Step 1 and Step 2 until convergence.

3.5 MMSE-based IA

In the MMSE-based IA algorithm [12], the MMSE criterion minimizes the expected sum of the norms between each $\hat{x}_k$ (estimated data at the receiver) and x_k for all k, yielding the objective

$$\mathcal{J}_{MSE}=\sum_{k=1}^{K}\mathbb{E}\left\|\mathbf{U}_k^{*}\left(\mathbf{H}_{kk}\mathbf{V}_k\boldsymbol{x}_k+\sum_{l=1,l\neq k}\mathbf{H}_{kl}\mathbf{V}_l\boldsymbol{x}_l\right)-\boldsymbol{x}_k\right\|^{2}, \tag{11}$$

and an optimization problem of

$$\begin{array}{ll}\text{minimize} & \mathcal{J}_{MSE}\\ \text{subject to:} & \|\mathbf{V}_k\|_F^2\leq\rho_l,\ k\leq[1,\dots,K].\end{array} \tag{12}$$

Expanding the expectation and simplifying, the optimization is equivalent to

$$\begin{array}{ll}\text{minimize} & \displaystyle\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{U}_k^{*}\left(\tilde{\mathrm{R}}_k+\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k^{*}\mathbf{H}_{kk}^{*}\right)\mathbf{U}_k\right)\\ & -2\mathbb{R}\{\mathrm{tr}(\mathbf{U}_k^{*}\mathbf{H}_{kk}\mathbf{V}_k)\}\\ \text{subject to:} & \|\mathbf{V}_k\|_F^2\leq\rho_l,\ k\leq[1,\dots,K].\end{array} \tag{13}$$

The above problem is solved by relaxing the constraint to power inequality constraint $\|\mathbf{V}_k\|_{\mathrm{F}}^2\leq\rho_k,\ \forall k$ and restoring it to a solution satisfying the Karush-Kush-Tuker (KKT) condition. Thus at each step, the optimal precoders are

$$\mathbf{V}_l=\left(\mu_l\mathbf{I}_{N_r}+\sum_{k=1}^{K}\mathbf{H}_{kl}^{*}\,\mathbf{U}_k\mathbf{U}_k^{*}\mathbf{H}_{kl}\right)^{-1}\mathbf{H}_{ll}^{*}\mathbf{U}_l,\ \forall l, \tag{14}$$

where μ_l is the Lagrangian multiplier chosen to meet the power constraints. The optimal receive decoders are

$$\mathbf{U}_k = \left(\sum_{l=1}^{K} \mathbf{H}_{kl}\, \mathbf{V}_k \mathbf{V}_k^* \mathbf{H}_{kl}^*\right)^{-1} \mathbf{H}_{kk} \mathbf{V}_l,\ \forall k. \tag{15}$$

3.6 Maximum Sum SINR-based IA

This algorithm optimizes the sum signal power over the sum interference-plus-noise power originally proposed [12] for MIMO-IC network interoperating inter-stream interference. A simplified version of the algorithm was presented [13] without considering inter-stream interference. The max sum SINR algorithm can be summarized as follows [13]:

1. Choose the set $\mathbf{V}_l\ \forall l$ randomly.
2. Choose the columns of $\mathbf{U}_k\ \forall k$ as given by $\mathbf{U}_k = \nu_{\max}\left\{\left(\sum_{l\neq k}^{K} \mathbf{H}_{kl}\, \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^* + \sigma^2 \mathrm{I}_{\mathrm{N}_{\mathrm{r}_k}}\right)^{-1} \times \mathbf{H}_{kk} \mathbf{V}_k \mathrm{V}_k^* \mathbf{H}_{kk}^*\right\}\ \forall k.$
3. Choose the columns of $\mathbf{V}_l\ \forall l$ as given by $\mathbf{V}_l = \nu_{\max}\left\{\left(\sum_{k\neq l}^{K} \mathbf{H}_{kl}^*\, \mathbf{U}_k \mathbf{U}_k^* \mathbf{H}_{kl}\right)^{-1} \times \mathbf{H}_{kk}^* \mathbf{U}_k \mathbf{Y}_k^* \mathbf{H}_{kk}\right\}\ \forall l.$
4. Repeat Steps 2 and 3 until convergence.

4. Maximum SLNR-based IA

In this section, we present the solution for interference alignment based on the concept of minimization of signal power leakage towards unintended users and maximization of signal power within the desired signal sub-space in each transmitter for the K-user interference channel with multiple-antennas in each transmit and receive terminal. We start from the received signal (2) at k^{th} receiver. The SINR in the receiver is given by

$$\mathrm{SINR}_k = \frac{\mathrm{Tr}(\mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k^* \mathbf{H}_{kk}^* \mathbf{U}_k)}{\mathrm{Tr}(\sigma_k^2 \mathbf{U}_k^* \mathbf{U}_k + \sum_{l=1,l\neq k}^{K} \mathbf{U}_k^*\, \mathbf{H}_{kl} \mathbf{V}_l \mathbf{V}_l^* \mathbf{H}_{kl}^* \mathbf{U}_k)}. \tag{16}$$

The SINR expression in (16) for $k = 1, \ldots, K$ can be used as an optimization criterion for determining the precoding and decoding matrices such that the SINR is maximized for each receiver. An iterative IA solution for MIMO-IC network is given elsewhere [9], [12].

An alternative approach to IA for MIMO-IC is to follow the minimization of the leakage power to unwanted user in each transmitter. From (2), the power of the desired signal for transmission k is given by $\|\mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k\|^2$ and the power of the interference that is caused by transmission k on the signal received by receiver l is given by $\|\mathbf{U}_l^* \mathbf{H}_{lk} \mathbf{V}_k\|^2$. Thus, the leakage for user k as total power leaked from this user to all other users is given by $\sum_{l=1,l\neq k}^{K} \|\mathbf{U}_l^* \mathbf{H}_{lk} \mathbf{V}_k\|^2$. Our intention here is to enhance the desired signal $\|\mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k\|^2$ to be larger compared to the power leakage to all other users, i.e., $\sum_{l=1,l\neq k}^{K} \|\mathbf{U}_l^* \mathbf{H}_{lk} \mathbf{V}_k\|^2$. These conditions can be presented in terms of a figure of merit called as SLNR defined as [22]

$$\mathrm{SLNR}_k = \frac{\|\mathbf{U}_k^*\mathbf{H}_{kk}\mathbf{V}_k\|^2}{\sum_{l=1,l\neq k}^{K}\|\mathbf{U}_l^*\mathbf{H}_{lk}\mathbf{V}_k + \mathbf{U}_l^* w_l \mathbf{I}_{N_r}\|^2}. \tag{17}$$

It is noted that unlike SINR_k, SLNR_k in (17) is a measure in the transmit terminal rather than in the receive terminal. The above expression in (17) can be simplified and rewritten as follows [17]

$$\mathrm{SLNR}_k = \frac{\mathrm{Tr}(\mathbf{V}_k^*\mathbf{H}_{kk}^*\mathbf{U}_k\mathbf{U}_k^*\mathbf{H}_{kk}\mathbf{V}_k)}{\mathrm{Tr}[\mathbf{V}_k^*\{\sum_{l=1,l\neq k}^{K}\mathbf{H}_{lk}^*\,\mathbf{U}_l\mathbf{U}_l^*\mathbf{H}_{lk} + \sigma_l^2\mathbf{U}_l\mathbf{U}_l^*\}\mathbf{V}_k]}, \tag{18}$$

where σ_l^2 is the noise variance at the l^{th} receive terminal.

Maximization of the SLNR for all K transmissions included in IA for MIMO-IC systems simultaneously improves the SINR for every user, since minimum leakage from each transmitter to the undesired receivers ensures that interference in each receiver is minimized. Using this concept of leakage, we devised the IA problem for MIMO-IC in terms of the optimization problem as follows:

$$\begin{aligned} &\underset{\mathbf{V}_k\in\mathbb{C}^{N_t\times d_k},\mathbf{U}_l\in\mathbb{C}^{N_r\times d_l}}{\arg\max} && \mathrm{SLNR}_k \\ &\text{subject to:} && \mathbf{V}_k^*\mathbf{V}_k = \frac{P_k}{d_k},\ \ k\in[1,\dots,K], \\ & && \mathbf{U}_l^*\mathbf{U}_l = \mathbf{I},\ \ l\in[1,\dots,K] \end{aligned} \tag{19}$$

where, P_k is the total power allocated to the k^{th} transmission.

Deriving a closed-form solution to the above optimization problem in (19) is difficult due to the interdependence of precoder, decoder, and receiver interference-free sub-space. A simple approach to solve (19) would be to iteratively find the solution using an alternating minimization over the precoding and decoding matrices. We start with arbitrary receive decoding matrices $\mathbf{U}_l\in\mathbb{C}^{N_r\times d_l}\forall l$, assuming readily available decoding matrices reduces the complexity to obtain the solution of the optimization problem in (19). According to the Rayleigh-Ritz quotient result [23], the optimal precoding matrices for the above problem in (19) are given by,

$$\begin{aligned} \mathbf{V}_k,\ \forall k = &\ \text{max generalized eigenvector } \times \\ &\left(\mathbf{H}_{kk}^*\mathbf{U}_k\mathbf{U}_k^*\mathbf{H}_{kk}, \sum_{l=1,l\neq k}^{K}(\mathbf{H}_{lk}^*\mathbf{U}_l\mathbf{U}_l^*\mathbf{H}_{lk} + \mathbf{U}_l\mathbf{U}_l^*)\right). \end{aligned} \tag{20}$$

Since, the right-hand side of the second term in (20) can be inverted, it can be written as

$$\mathbf{V}_k = \nu_{\max}^{d_k}\left\{\left(\sum_{l=1,l\neq k}^{K}(\mathbf{H}_{lk}^*\mathbf{U}_l\mathbf{U}_l^*\mathbf{H}_{lk} + \sigma_l^2\mathbf{U}_l\mathbf{U}_l^*)\right)^{-1}\mathbf{H}_{kk}^*\mathbf{U}_k\mathbf{U}_k^*\mathbf{H}_{kk}\right\}\forall k, \tag{21}$$

where $\nu_{\max}^{d_k}(A)$ gives the eigenvectors corresponding to the d_k largest eigenvalues of A, say $\lambda_{\max}(1: d_k)$. That is, the optimal precoding matrices are obtained as the d_k eigenvectors that correspond to the d_k largest eigenvalues of $\mathbf{H}_{kk}^*\mathbf{U}_k\mathbf{U}_k^*\mathbf{H}_{kk}$ and $\left(\sum_{l=1,l\neq k}^{K}\mathbf{H}_{lk}^*\,\mathbf{U}_l\mathbf{U}_l^*\mathbf{H}_{lk} + \sigma_l^2\mathbf{U}_l\mathbf{U}_l^*\right)^{-1}$, where the SLNR is maximized and given by d_k maximum eigenvalues, $\mathrm{SLNR}_k^{1:d_k} = \lambda_{\max}(1: d_k)$.

The goal of interference alignment is to choose precoding matrices $\mathbf{V}_k\forall k$ such that each receiver can decode its own signal by allowing all interfering users to share a portion of the sub-space in the user's receive space. Maximization of the SLNR in each transmit terminal minimizes the signal leakage to unwanted receivers participating in an IA along with that user. But this still does not align the interference from the remaining $(K-1)$ transmitters in the receiving terminal. For this, one can follow a similar approach to obtain the decoding matrices $\mathbf{U}_l\forall l$ for the reciprocal network, assuming channel reciprocity, or follow the maximization of the SINR, which does not require channel reciprocity to obtain the optimum decoding matrices. Since we are interested in removing the requirement for channel reciprocity, we follow the maximization of the SINR to obtain the decoding matrices $\mathbf{U}_l\forall l$ for each receiver. While doing so, both the precoding and decoding matrices are designed in a single forward link and, unlike existing designs, do not require a reverse link to obtain the decoding matrices, but do require CSI feedback on the transmit side. The decoding matrices are obtained from

$$\mathbf{U}_l = \nu_{\max}^{d_k}\left\{\left(\sum_{k=1,k\neq l}^{K}(\mathbf{H}_{kl}\mathbf{V}_k\mathbf{V}_k^*\mathbf{H}_{kl}^*) + \sigma_l^2\mathbf{I}_{N_r}\right)^{-1}\mathbf{H}_{kk}^*\mathbf{U}_k\mathbf{U}_k^*\mathbf{H}_{kk}\right\},\ \forall l. \qquad (22)$$

Algorithm 1 Max SLNR algorithm

1. Initialize: Start with an arbitrary decoding matrices $\mathbf{U}_l \in \mathbb{C}^{N_r\times d_l}\forall l$.
2. Begin iteration.
3. Compute $\mathbf{V}_k|\mathbf{U}_l\forall l \in [1,\dots,K]$ using (21).
4. Compute $\mathbf{U}_l|\mathbf{V}_k\forall k \in [1,\dots,K]$ using (22).
5. Repeat Step 3 and Step 4 until convergence.

The derivation for the optimal decoding matrices $\mathbf{U}_l\forall l$ for a given set of precoding matrices $\mathbf{V}_k\forall k$ can be obtained using a similar approach to maximization of SINR. Thus, the obtained decoding matrices $\mathbf{U}_l\forall l$ are passed on to update the precoding matrices. The iteration continues in this manner until the algorithm converges. Using SLNR maximization to design precoding matrices, and SINR maximization to design decoding matrices, removes the strict requirement for channel reciprocity. The iterative procedure described above is summarized in Algorithm 1

The goal here is to achieve interference alignment by progressively reducing the signal leakage power in each transmitter followed by the suppression of the leakage interference in each receiver and also to maximize the desired signal power within the desired signal sub-space in both cases. The precoding and decoding matrices are designed to maximize the SLNR and the SINR, respectively, and to reduce the transmit leakage power and receive leakage interference, respectively. At each iteration, the leakage is minimized, which implies that the algorithm must converge, but since there is an attempt to maximize the signal power within the desired signal sub-space, there might be delay in achieving convergence.

The max SLNR algorithm in Algorithm 1 must converge to certain precoding and decoding matrices, such that the SLNR in each transmitter reaches maximum, which implies that the desired signal power must be maximized and the leakage in each transmitter must be reduced. At each iteration, the signal power of the desired subspace is maximized and the leakage is minimized by optimally choosing the precoding and decoding matrices, as in (21) and (22) respectively. The leakage is bounded below by zero, and the noise in a receiver has a finite noise power. As a result, the algorithm must converge to a point where the leakage is zero and noise power (defined by the noise variance) is finite as the iteration increases. Thus, we can conclude that the algorithm converges to a set of precoding and decoding matrices such that the maximized SLNR in a transmitter becomes equivalent to the maximum SNR attained by enhancing the signal power in the desired sub-space.

5. Simulation and Analysis

In this section, we evaluate the performance of the proposed maximum SLNR interference alignment algorithm and compare it with some of the well-known algorithms on the topic, presented in Section 3, by using numerical simulation in MATLAB. For simplicity, we consider three-user interference alignment where all transmitters have N_t antennas, i.e., $N_{t_k} = N_t \forall k$, and all receivers have N_r antennas, i.e., $N_{r_k} = N_r \forall k$. The channel coefficients for all the transmit-receive links that include K desired links and $K(K-1)$ interfering links are assumed to be IID with zero mean, and a unit variance complex Gaussian random variable. Power constraint for each user is assumed to be identical, with $P_k = P = 1, \forall k$. The simulation parameters are summarized in **Table 1**.

Table 1. Parameters used in the simulations.

Parameter	**Value(s)**
# of users (K)	3
# of transmit and receive antennas ($N_r \times N_t$)	$(2 \times 2), (4 \times 4)$
Requested DoF/users ($d_k = d, \forall k$)	1, 2
Channel model	Rayleigh fading, fading coefficient ~ $\mathcal{CN}(0,1)$
Monte Carlo (MC) loops	500
Algorthms iterations (iter)	100, 400

The sum capacity performance of the proposed max SLNR algorithm along with the iterative algorithms mentioned in Section 3 for $(2 \times 2,1)^3$ and $(4 \times 4,2)^3$ MIMO-IC networks are presented in **Fig. 2** and **Fig. 3**, respectively.

For a $(2 \times 2,1)^3$ MIMO-IC network, we observe from **Fig. 2** that the performance of max SLNR is optimal and similar to that of max SINR, max sum SINR has slightly degraded performance for the SNR range of $0 - 20dB$, while the max sum-rate algorithm has more degraded performance at a low SNR, which improves as the SNR increases. The distributed IA algorithm has even more degradation for lower SNR values, which also improve with increasing SNR values. On the other hand, the MMSE algorithm has optimum performance at a low SNR, which first gradually and then rapidly degrades as SNR increases. Lastly, alternating minimization has the worst performance of all the others.

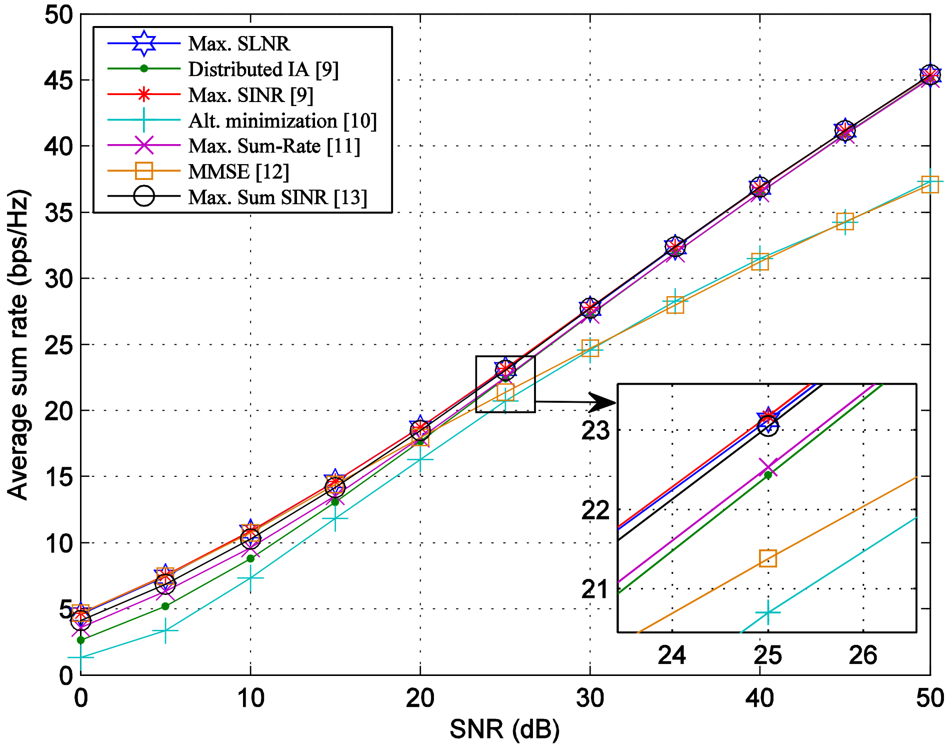

Fig. 2. **Sum capacity curves for various iterative IA algorithms for a $(2 \times 2, 1)^3$ MIMO-IC network (MC = 500, algorithm iterations = 100).**

For a $(4 \times 4{,}2)^3$ MIMO-IC network, **Fig. 3** shows that the performance of the proposed algorithm is the best for the SNR range of $0 - 25dB$, which is accompanied by the max SINR algorithm up to an SNR of $10dB$, after which the difference begins to grow large. Max sum SINR has consistently lower performance than the max SLNR algorithm. Distributed IA, max sum-rate, and alternating minimization algorithms have a similar trend, with the optimal performance for SNR above $25dB$, whereas the alternating minimization algorithm has the worst performance for a low SNR range. Meanwhile, the MMSE algorithm follows a trend similar to that of a $(2 \times 2{,}1)^3$ MIMO-IC network, except that here it does not have optimal performance at a low SNR range.

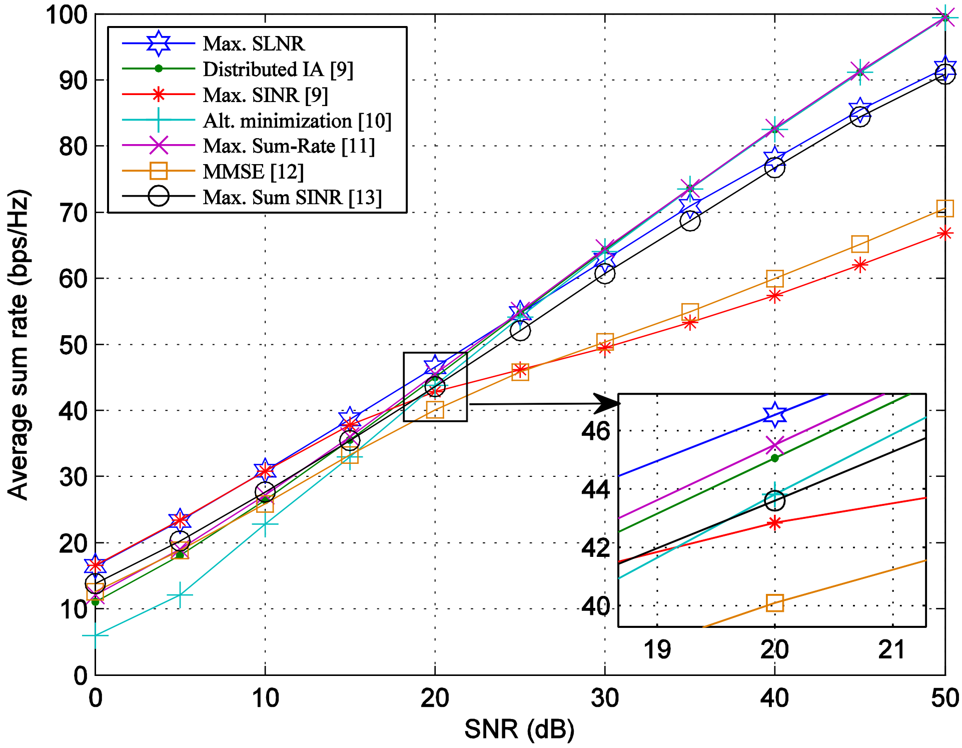

Fig. 3. Sum capacity curves for various iterative IA algorithms for a $(4 \times 4, 2)^3$ MIMO-IC network (MC = 500, algorithm iterations = 400).

Next, the convergence curves for max SLNR, distributed IA, max SINR, and max sum SINR are plotted for $(2 \times 2{,}1)^3$ and $(4 \times 4{,}2)^3$ MIMO-IC networks in **Fig. 4** and **Fig. 5** respectively. For a $(2 \times 2{,}1)^3$ MIMO-IC network, the convergence curves for all four algorithms are similar. The only difference is the sum rate values at which each algorithm converges, which are consistent with the results observed in **Fig. 2**. From **Fig. 5**, we observe that the max SINR achieves convergence most rapidly, but at a relatively low sum capacity, especially at high SNR values. The distributed IA algorithm achieves convergence fairly rapidly, with high sum capacity values in a high SNR range, while the other two algorithms achieve convergence at more or less the same instant for all SNR values. The convergence is delayed for the max SLNR and the max sum SINR due to an attempt to maximize the desired signal power within the desired signal sub-space.

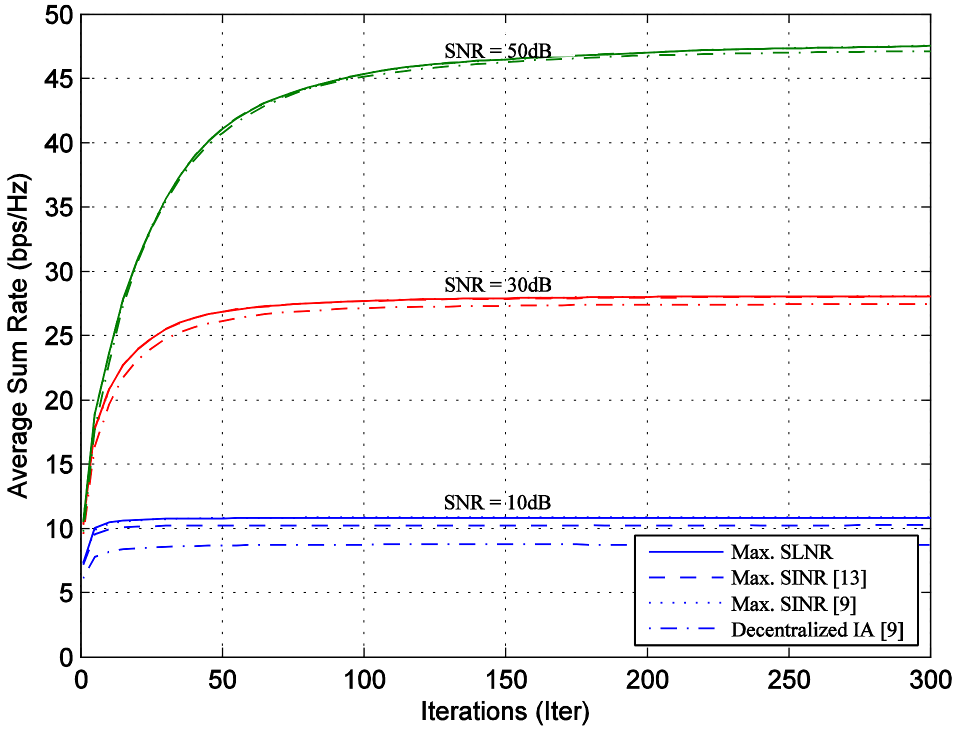

Fig. 4. Convergence curves for various iterative IA algorithms for a $(2 \times 2, 1)^3$ MIMO-IC channel (MC = 1000).

The proposed algorithm is no better than max SINR for a $(2 \times 2{,}1)^3$ MIMO-IC network, and the actual performance enhancement can be seen for a MIMO-IC network with higher order antennas. On the other hand, the max SLNR algorithm has consistent achievement compared to max sum SINR, which reflects the similarity between the two in designing the decoding matrices for each receiver. The extra noise term included in the max SLNR algorithm while designing the precoding matrices for each transmitter contributes to the performance enhancement being achieved. We also observe that more iterations are required for convergence of the max SLNR as the number of antennas and operating SNR increase. This is because the gap between the rate with the random initial matrices and local optimal point also grows with antenna order and operating SNR. This growth further increases as there is an attempt to maximize the desired power in the desired sub-space along with IA. Nevertheless, upon convergence, the sum capacity achieved by max SLNR approaches that of distributed IA, which is optimum in a high SNR range for a MIMO-IC network with higher order antennas. As for the performance evaluation of max SINR, we suspect that there might be some design flaw that fails to consider performance, especially for high operating SNR for a MIMO-IC network with higher order antennas.

The main features of all the IA algorithms that are compared via simulations in this section are summarized in **Table 2**. The complexity of the algorithms in **Table 2** are determined using their simulation times for $(2 \times 2, 1)^3$ and $(4 \times 4, 2)^3$ MIMO-IC channels, respectively using MATLAB.

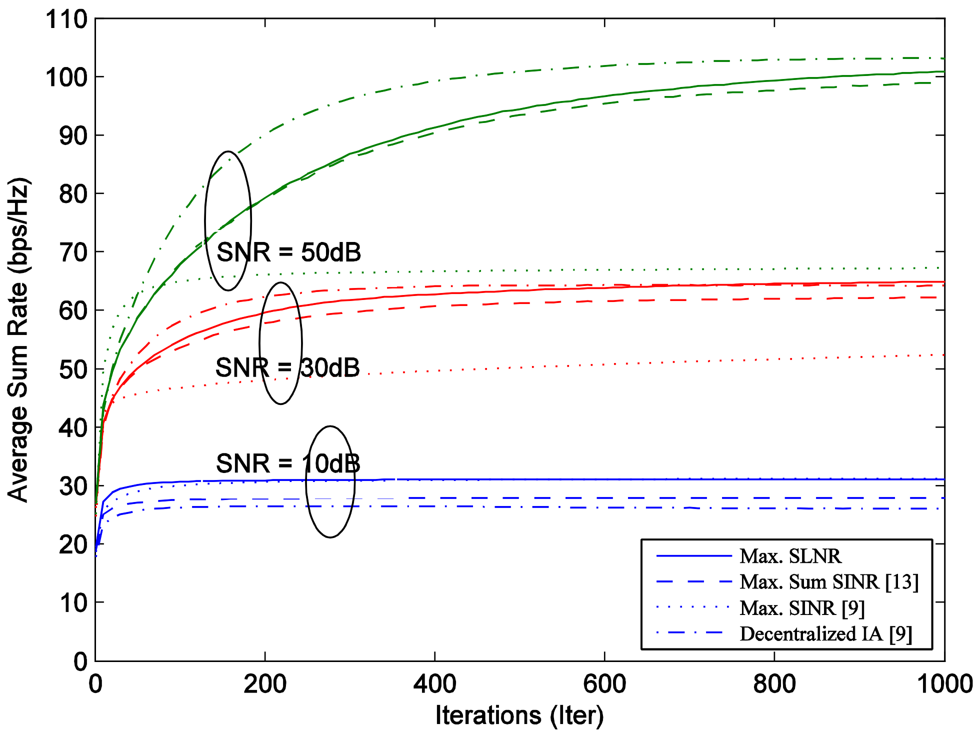

Fig. 5. Convergence curves for various iterative IA algorithms for a $(4 \times 4, 2)^3$ MIMO-IC channel (MC = 1000).

Table 2. Comparison of various IA algorithms

<table>
<tr><th rowspan="2">Algorithm</th><th rowspan="2">Pros</th><th rowspan="2">Cons</th><th colspan="2">Complexity (run time)</th></tr>
<tr><th>$(2 \times 2, 1)^3$</th><th>$(4 \times 4, 2)^3$</th></tr>
<tr><td rowspan="2">Distributed IA [9]</td><td rowspan="2">Simple to implement, and converges rapidly, even at high SNR.</td><td rowspan="2">Sum capacity moderate for low SNR range. Channel reciprocity required.</td><td colspan="2">Low</td></tr>
<tr><td>0.454 s</td><td>0.498 s</td></tr>
<tr><td rowspan="2">Max SINR [9]</td><td rowspan="2">Sum capacity good for low antenna order. Performs well at low SNR.</td><td rowspan="2">Sum capacity degrades moderately for high antenna order, especially for a high SNR region. Channel reciprocity required.</td><td colspan="2">Medium</td></tr>
<tr><td>0.492 s</td><td>1.552 s</td></tr>
<tr><td rowspan="2">Alt. minimization [10]</td><td rowspan="2">Optimal sum capacity in high SNR region for high antenna order. Does not require channel reciprocity.</td><td rowspan="2">Poor sum capacity except in high SNR region for high antenna order.</td><td colspan="2">Low</td></tr>
<tr><td>0.343 s</td><td>0.425 s</td></tr>
<tr><td rowspan="2">Max sum-rate [11]</td><td rowspan="2">Optimal sum capacity in high SNR region.</td><td rowspan="2">Moderate to poor sum capacity in low SNR region. Added complexity due to channel reciprocity required.</td><td colspan="2">High:</td></tr>
<tr><td>2.114 s</td><td>2.458 s</td></tr>
</table>

MMSE [12]	Enhanced performance in terms of transmission error minimization. Moderate to good sum capacity in low SNR range, especially for low antenna order.	Poor sum capacity in high SNR range. Channel reciprocity required.	High	
			4.0153 s	5.282 s
Max sum SINR [13]	Consistent sum capacity throughout the SNR ranges. Good sum capacity for high antenna order.	Slow convergence especially in a high SNR range for high antenna order. Channel reciprocity required.	Lower medium	
			0.518 s	0.584 s
Max SLNR	Best sum capacity in low and moderate SNR ranges. Does not require channel reciprocity. Constant performance for varying antenna order.	Slow convergence especially in a high SNR range for high antenna order.	Lower medium	
			0.567 s	0.675 s

6. Conclusion

In this paper, we present a new iterative interference alignment algorithm that does not require channel reciprocity, based on the maximization of SLNR and SINR in each transmit and receive terminal, respectively. We focus on reducing signal power leakage in each transmitter and suppress interference leakage in each receiver. The simulation results suggest that the proposed algorithm achieves the best performance from among the other iterative algorithms, especially for the lower and mid-range SNR values, irrespective of antenna order. The convergence of the proposed algorithm is delayed due to an attempt to maximize the desired signal power within the desired signal sub-space. Nevertheless, upon convergence, the proposed algorithm's sum capacity performance approaches that of the distributed algorithm at a high SNR for a $(4 \times 4,2)^3$ MIMO-IC network. Unlike max sum SINR, the proposed algorithm takes into account the extra noise term that the interfered users receive from a respective transmitter for precoding matrices construction in each transmitter. The extra noise term that is included in the max SLNR IA algorithm contributes to the performance achievement.

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