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GLR Charts for Simultaneously Monitoring a Sustained Shift and a Linear Drift in the Process Mean

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Abstract

This paper considers the problem of monitoring the mean of a normally distributed process variable when the objective is to effectively detect both a sustained shift and a linear drift. The design and application of a generalized likelihood ratio (GLR) chart for simultaneously monitoring a sustained shift and a linear drift are evaluated. The GLR chart has the advantage that when we design this chart, we do not need to specify the size of the parameter change. The performance of the GLR chart is compared with that of other control charts, such as the standard cumulative sum (CUSUM) charts and the cumulative score (CUSCORE) charts. And we compare the proposed GLR chart with the GLR charts designed for monitoring only a sustained shift and for monitoring only a linear drift. Finally, we also compare the proposed GLR chart with the chart combinations. We show that the proposed GLR chart has better overall performance for a wide range of shift sizes and drift rates relative to other control charts, when a special cause produces a sustained shift and/or a linear drift in the process mean.

Keywords: CUSUM chart, generalized likelihood ratio, linear drift, statistical process control, sustained shift.

1. Introduction

Statistical process control (SPC) has been widely used to monitor and improve the quality of the output of a process. One of the most important problems in SPC is the detection of a change in the process mean, and a common situation is that the process observations are assumed to be independent normal random variables. Control charts, such as Shewhart, cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) charts, are used to monitor a process to detect special causes that produce changes in the process mean. Recently, generalized likelihood ratio (GLR) charts have been investigated by many researchers. GLR charts have some advantages that the size of the parameter change does not need to be specified and these charts have been shown to be very effective in a wide variety of settings in SPC applications. Recent investigations of this chart include Hawkins *et al.* (2003), Runger and Testik (2003), Capizzi and Masarotto (2008), Zou *et al.* (2009), Reynolds and Lou (2010), Huang *et al.* (2012, 2013), Xu *et al.* (2012, 2013), Wang and Reynolds (2013), and Reynolds *et al.* (2013).

In monitoring the process mean μ, there are several types of mean changes, for example, a sustained shift in μ, a transient shift in μ, a linear/nonlinear drift in μ, and so on. In this paper, we consider a sustained shift and a linear drift. In the sustained shift in μ, we assume that a special cause produces a step shift in μ that remains until detected by a control chart and action is taken to remove the special cause. The problem, to detect any special cause that produces a sustained shift in μ, has

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been commonly studied by many researchers. Reynolds and Lou (2010) developed a GLR chart for detecting a sustained shift in μ and evaluated the performance of this GLR chart.

In some situations, a special cause may produce a linear drift in μ, rather than a sudden shift. Linear drifts are usually due to causes such as gradual deterioration of equipment, catalyst aging, waste accumulation or human causes. Recent investigations of linear drifts include Reynolds and Stoumbos (2001), Runger and Testik (2003), and Fahmy and Elsayed (2006). More recently, Zou *et al.* (2009) and Xu *et al.* (2013) proposed GLR charts for detecting a linear drift in μ and evaluated the performance of these GLR charts.

Although a number of different control chart procedures for monitoring sustained shifts and linear drifts have been investigated in many papers, most papers consider the use of control charts designed to detect a particular type of the mean change, which is either a sustained shift or a linear drift. In some situations, however, a special cause may produce both a sustained shift and a linear drift. This means that a sustained shift occurs at some time and a linear drift starts at the same time. For example, if a sudden damage of equipment occurs and a gradual deterioration starts due to this damage, a sustained shift and a linear drift can occur simultaneously.

The objective of this paper is to design and evaluate a GLR chart for simultaneously monitoring a process which experiences a sustained shift and a linear drift. We compare the performance of the proposed GLR chart with the performance of other control charts, such as the GLR charts designed for monitoring only a sustained shift and for monitoring only a linear drift, the CUSUM charts, the CUSCORE charts, and chart combinations.

In the next two sections, we define the process model considered in this paper, and explain the GLR charts designed for monitoring a sustained shift and for monitoring a linear drift. Then we propose the GLR chart for simultaneously monitoring a sustained shift and a linear drift. The performance comparison with other control charts using simulation is done in Section 4, and our conclusions are given in Section 5.

2. Process Model

Let X be the quality characteristic of interest which is assumed to have a $N(\mu_0, \sigma_0^2)$ distribution when the process is the in-control state. We assume that the in-control values μ_0 and σ_0 are known or have been estimated accurately enough during a Phase I period that any error in estimation can be neglected. Suppose that samples of size n are taken from the process. In this paper, for simplicity we assume $n = 1$, and let X_k represent the sample obtained at sampling time k. When $n > 1$, we can handle by letting X_k correspond to the sample mean and changing the in-control variance from σ_0^2 to σ_0^2/n in the formulas in this paper. We also take the unit of time to be the same as the time interval between two consecutive samples so that samples are one time unit apart.

Consider the situation that after the change point τ^* between samples τ and $\tau + 1$, the distribution of X_k changes from the in-control $N(\mu_0, \sigma_0^2)$ distribution to an $N(\mu(k), \sigma_0^2)$ distribution. We assume that the out-of-control mean $\mu(k)$ for $k > \tau^*$ can be subjected to a sustained shift and/or a linear drift.

3. The GLR Charts

3.1. The GLR-S chart for monitoring a sustained shift

Reynolds and Lou (2010) evaluated a GLR chart designed to detect a sustained shift in μ (called the GLR-S chart) and showed that the overall performance of the GLR-S chart is at least as good as that of other control charts for detecting a wide range of sustained shifts.

When there has been a sustained shift to some unknown $\mu(k) = \mu_1 (= \mu_0 + \delta\sigma_0)$ for $k > \tau^*$, using the likelihood formula for detecting this shift, Reynolds and Lou (2010) used the following GLR-S statistic at sampling time k

$$R^S_{k,m_1,m_2} = \max_{\max(0,k-m_1)\le\tau\le k-m_2} \frac{k-\tau}{2\sigma_0^2} \left(\hat{\mu}_{1,\tau,k} - \mu_0\right)^2,$$

where $\hat{\mu}_{1,\tau,k}$ is the maximum likelihood estimator (MLE) of μ_1 obtained as

$$\hat{\mu}_{1,\tau,k} = \frac{1}{k-\tau} \sum_{t=\tau+1}^{k} X_t.$$

The parameter m_1 denotes the size of a moving window that may be useful to reduce the computational efforts, and the parameter m_2 is the minimum number of samples needed to estimate the parameters during monitoring by the GLR chart. Reynolds and Lou (2010) used $m_1 = 400$ and $m_2 = 1$ for evaluating the performance. The GLR-S chart will give a signal at sampling time k if $R^S_{k,m_1,m_2} > h_{GLR-S}$, where the control limit h_{GLR-S} is pre-specified based on the desired in-control performance.

3.2. The GLR-D chart for monitoring a linear drift

Xu *et al.* (2013) evaluated a GLR chart designed to detect a linear drift in μ (called the GLR-D chart). Zou *et al.* (2009) also proposed a GLR chart for monitoring a linear drift in μ. There are differences between these two charts as follows. First, Zou *et al.* (2009) assumed that the change point τ^* is always an integer value, whereas Xu *et al.* (2013) assumed that τ^* is a value satisfying $\tau \le \tau^* < \tau + 1$. Second, the GLR chart proposed by Zou *et al.* (2009) is a one-sided chart for detecting drifts in only one direction, whereas the GLR-D chart proposed by Xu *et al.* (2013) is a two-sided chart that can detect drifts in either direction.

Consider a situation that the process mean at sampling time $k > \tau^*$ becomes

$$\mu(k) = \mu_0 + \beta\,(k - \tau^*)\,\sigma_0,$$

where β is the standardized rate of drift. Xu *et al.* (2013) proposed the following GLR-D statistic at sampling time k

$$R^D_{k,m_1,m_2} = \max_{\max(0,k-m_1)\le\tau\le k-m_2} \frac{1}{2\sigma_0} \left\{ 2\hat{\beta}_{\tau,k} \sum_{t=\tau+1}^{k} (X_t - \mu_0)\, t - 2\hat{\beta}_{\tau,k}\hat{\tau}^*_{\tau,k} \sum_{t=\tau+1}^{k} (X_t - \mu_0) - \sigma_0 \hat{\beta}^2_{\tau,k} \sum_{t=\tau+1}^{k} (t - \hat{\tau}^*_{\tau,k})^2 \right\},$$

where $\hat{\beta}_{\tau,k}$ and $\hat{\tau}^*_{\tau,k}$ are the MLEs of β and τ^*, respectively, satisfying the equations

$$\hat{\beta}_{\tau,k} = \frac{\sum_{t=\tau+1}^{k} (t - \hat{\tau}^*_{\tau,k})(X_t - \mu_0)}{\sigma_0 \sum_{t=\tau+1}^{k} (t - \hat{\tau}^*_{\tau,k})^2}, \tag{3.1}$$

and

$$\hat{\tau}^*_{\tau,k} = \frac{\sigma_0 \hat{\beta}_{\tau,k} \sum_{t=\tau+1}^{k} t - \sum_{t=\tau+1}^{k} (X_t - \mu_0)}{\sigma_0 \hat{\beta}_{\tau,k}\,(k - \tau)}. \tag{3.2}$$

The parameter m_2 is usually chosen to be at least the number of the parameters monitored by the GLR chart (see Lai (1995)), therefore the minimum value of m_2 in the GLR-D chart is 2. Xu *et al.* (2013), however, used $m_1 = 400$ and $m_2 = 4$, because these values would give good performance for the GLR-D chart over a wide range of drift rates. The GLR-D chart will give a signal at sampling time k if $R^D_{k,m_1,m_2} > h_{GLR-D}$, where the control limit h_{GLR-D} is pre-specified based on the desired in-control performance.

3.3. The GLR-SD chart for simultaneously monitoring a sustained shift and a linear drift

In this subsection, we propose a GLR-SD chart designed to detect a sustained shift and a linear drift simultaneously. In the situation that a special cause may produce a sustained shift, a linear drift, or both a sustained shift and a linear drift in the process mean, we can use a combination of two or more charts designed to detect each type of shift. A disadvantage of using multiple charts is that designing such a scheme requires that control-chart parameters be determined for multiple charts. The GLR-SD chart, however, has an advantage that by using a single chart we can detect a sustained shift and/or a linear drift in the process mean.

Consider a situation that a special cause may produce a sustained shift and/or a linear drift in the process mean. That is, the process mean at sampling time $k > \tau^*$ becomes

$$\mu(k) = \mu_0 + \delta\sigma_0 + \beta(k - \tau^*)\sigma_0, \tag{3.3}$$

where δ is the size of the sustained shift and β is the standardized rate of drift. In the case of linear drifts in the process mean, Zou *et al.* (2009) assumed that the change point τ^* is an integer value, that is $\tau^* = \tau + 1$, as mentioned previously, and Reynolds and Lou (2010) assumed that τ^* has a uniform distribution on $[\tau, \tau + 1]$ in evaluating the performance of the GLR-S chart for linear drifts. In the GLR-D chart, Xu *et al.* (2013) used the MLE of τ^* in the chart statistic based on the assumption that τ^* is a value satisfying $\tau \le \tau^* < \tau + 1$. Their assumption is more reasonable in practice, but this makes computing the chart statistic difficult because the MLEs of β and τ^* in (3.1) and (3.2) must be obtained by using iterative algorithms. We think that the effect of the assumption about τ^* would not be serious unless the sampling interval is very large, and the effect can be made up for by estimating the drift rate β. In this paper, for the simple calculation of the GLR-SD chart statistic we assume that the change point τ^* is the middle point of the interval $[\tau, \tau + 1]$, that is $\tau^* = \tau + 0.5$. Under this assumption, the process mean at sampling time $k > \tau^*$ in (3.3) can be represented as

$$\mu(k) = \mu_0 + \delta\sigma_0 + \beta(k - \tau - 0.5)\sigma_0. \tag{3.4}$$

Under the alternative hypothesis that a special cause produces a change such as (3.4) in the process mean, the likelihood function at sampling time k is

$$\begin{aligned} &L(\tau, \delta, \beta \mid X_1, X_2, \ldots, X_k) \\ &= \left(2\pi\sigma_0^2\right)^{-\frac{k}{2}} \exp\left[-\frac{1}{2\sigma_0^2}\left\{\sum_{t=1}^{\tau}(X_t - \mu_0)^2 + \sum_{t=\tau+1}^{k}(X_t - (\mu_0 + \delta\sigma_0 + \beta(t - \tau - 0.5)\sigma_0))^2\right\}\right]. \end{aligned}$$

After some calculations, the MLEs of δ and β can be obtained as

$$\hat{\delta}_{\tau,k} = \frac{\sum_{t=\tau+1}^{k}(t-\tau-0.5)\sum_{t=\tau+1}^{k}(t-\tau-0.5)(X_t-\mu_0) - \sum_{t=\tau+1}^{k}(t-\tau-0.5)^2\sum_{t=\tau+1}^{k}(X_t-\mu_0)}{\sigma_0\left[\left\{\sum_{t=\tau+1}^{k}(t-\tau-0.5)\right\}^2 - (k-\tau)\sum_{t=\tau+1}^{k}(t-\tau-0.5)^2\right]}$$

and

$$\hat{\beta}_{\tau,k} = \frac{\sum_{t=\tau+1}^{k}(t-\tau-0.5)(X_t-\mu_0)}{\sigma_0\sum_{t=\tau+1}^{k}(t-\tau-0.5)^2} - \hat{\delta}_{\tau,k}\frac{\sum_{t=\tau+1}^{k}(t-\tau-0.5)}{\sum_{t=\tau+1}^{k}(t-\tau-0.5)^2},$$

respectively.

Under the null hypothesis that there has been no change in the process, the likelihood function at sampling time k can be represented as

$$L(\infty,0,0\,|\,X_1,X_2,\ldots,X_k) = \left(2\pi\sigma_0^2\right)^{-\frac{k}{2}}\exp\left[-\frac{1}{2\sigma_0^2}\sum_{t=1}^{k}(X_t-\mu_0)^2\right].$$

A log likelihood ratio statistic for testing whether there has been a sustained shift and/or a linear drift in the process mean is

$$\begin{aligned}
R_{k,m_1,m_2}^{SD} &= \ln\frac{\max_{\max(0,k-m_1)\le\tau\le k-m_2} L(\tau,\delta,\beta\,|\,X_1,X_2,\ldots,X_k)}{L(\infty,0,0\,|\,X_1,X_2,\ldots,X_k)} \\
&= \max_{\max(0,k-m_1)\le\tau\le k-m_2}\frac{1}{2\sigma_0^2}\left\{2\sum_{t=\tau+1}^{k}(X_t-\mu_0)\left(\hat{\delta}_{\tau,k}\sigma_0+\hat{\beta}_{\tau,k}(t-\tau-0.5)\sigma_0\right)\right. \\
&\quad \left. -\sum_{t=\tau+1}^{k}\left(\hat{\delta}_{\tau,k}\sigma_0+\hat{\beta}_{\tau,k}(t-\tau-0.5)\sigma_0\right)^2\right\}. \qquad (3.5)
\end{aligned}$$

This GLR-SD chart will give a signal at sampling time k if $R_{k,m_1,m_2}^{SD} > h_{GLR-SD}$, where the control limit h_{GLR-SD} is pre-specified based on the desired in-control performance.

4. Evaluating the Performance of the GLR-SD Chart

4.1. Performance measures

A widely used way to measure control chart performance is to use the average time to signal (ATS). The in-control ATS is defined as the expected time from the beginning of process monitoring until a signal is given when the process is in control, and this metric is used as the measure of the rate of false alarms. When there is a sustained shift and/or a linear drift in μ, we use the steady-state ATS (SSATS) as the measure of the time required to detect the shift. The SSATS is based on the assumption that control-chart statistics have reached their steady-state distributions by the time the shift occurs.

In the evaluations and comparisons done here, we choose the control limits of the charts so that the in-control ATS is 1481.6 time units, and we obtain SSATS values using simulation with 100,000 runs. The actual in-control ATS values given in the Tables vary slightly from 1481.6 due to simulation error. Note that, with an in-control ATS of 1481.6, when the sample size $n = 1$ and the sampling interval $d = 1$ (time unit), the ARL (average run length) and ANOS (average number of observations to signal) will both be 1481.6, but when $n = 4$ and $d = 4$, the ANOS will be 1481.6, while the ARL will be 370.4. Here we assume that $n = 1$ and $d = 1$.

The simulated values of the SSATS given in this paper are based on the assumption that $\tau = 100$. If a false alarm occurred in a sequence of in-control observations, then this sequence of observations was discarded and a new sequence was generated for evaluating steady-state properties.

4.2. Choosing the chart parameters of the GLR-SD chart

There are two chart parameters, m_1 and m_2, which should be chosen to apply the GLR-SD chart. It was known that the GLR chart with a large value of m_1 is efficient for detecting small shifts whereas the GLR chart with a small value of m_1 is efficient for detecting large shifts. Reynolds and Lou (2010) and Xu *et al.* (2013) used $m_1 = 400$ in the GLR-S chart and in the GLR-D chart, respectively, because using this value gives performance that is essential the same as with no window which produces the best overall performance. For the numerical results in this paper, we also use $m_1 = 400$ in the GLR-SD chart statistic in (3.5).

For the value of m_2, Reynolds and Lou (2010) used $m_2 = 1$ in the GLR-S chart, whereas Xu *et al.* (2013) used $m_2 = 4$ in the GLR-D chart. Xu *et al.* (2013) stated that $m_2 = 4$ achieves the best overall performance for small and moderate drift rates and still works well for very large drift rates. It seems that more observations are needed to efficiently estimate the drift rate in the GLR-D chart when compared to estimating the amount of the shift in the GLR-S chart.

To evaluate the effect of the choice of m_2 in the GLR-SD chart, Table 1 gives SSATS values for sustained shifts and/or linear drifts in μ for m_2 values of 2, 3, and 4. The size of the sustained shift is expressed in terms of $\delta = (\mu - \mu_0)/\sigma_0$. Table 1 shows that the effect of the choice of m_2 seems not to be very significant as we expected, and the trend of SSATS values according to m_2 is mainly affected by the size of the sustained shift, δ. We see that using $m_2 = 2$ gives relatively a little bad performance for detecting small sustained shifts, but gives good performance for detecting large sustained shifts. Increasing the value of m_2 improves a little the ability of the GLR-SD chart to detect small sustained shifts. We use $m_2 = 2$ in the GLR-SD chart for the numerical results in this paper, and we expect that the GLR-SD chart with this value will have better overall performance.

4.3. Other control charts used for comparisons

One of the traditional approaches for monitoring a sustained shift in the process mean is to use a CUSUM chart for μ (called the CUSUM-S chart). The CUSUM-S chart is designed to detect a sustained shift from μ_0 to a specified μ_1, assuming that σ^2 remains constant at σ_0^2. See, for example, Page (1954), Hawkins and Olwell (1998), and Montgomery (2012).

The CUSUM statistic at sampling time k is given by

$$C_k = \max(0, C_{k-1} + M_k), \tag{4.1}$$

where $C_0 = 0$ and

$$M_k = \frac{\mu_1 - \mu_0}{\sigma_0^2}\left(X_k - \frac{\mu_0 + \mu_1}{2}\right).$$

The CUSUM-S chart will give a signal at sampling time k if $C_k > h_C$. Note that the CUSUM statistic can be expressed in a different form by dividing all terms in (4.1) by the constant $(\mu_1 - \mu_0)/\sigma_0^2$.

The CUSUM chart described above is a one-sided chart for detecting a sustained shift from $\mu = \mu_0$ to a specified μ_1. The two-sided CUSUM chart, that is to use two one-sided charts in combination with the same control limit h_C for both charts, can be used for detecting both increases and decreases in μ. The CUSUM charts used in the comparisons in this paper are all two-sided CUSUM charts.

A CUSCORE chart proposed by Box and Ramirez (1992) may be useful for detecting a linear drift. The CUSCORE statistic with handicap r_k (called the CUSCORE-D chart) at sampling time k is given by

$$S_k = \max(0, S_{k-1} + N_k),$$

Table 1: The effect of the choice of m_2 on the SSATS of the GLR-SD chart for sustained shifts and/or linear drifts in the process mean.

δ	β	GLR-SD		
		$m_2 = 2$	$m_2 = 3$	$m_2 = 4$
0.00	0.000	1481.29	1481.06	1483.21
0.00	0.005	104.82	104.76	104.34
0.00	0.010	67.33	67.19	66.96
0.00	0.020	42.84	42.85	42.69
0.00	0.050	23.56	23.54	23.44
0.00	0.100	14.89	14.89	14.83
0.00	0.300	7.10	7.09	7.07
0.50	0.000	50.25	50.33	49.73
0.50	0.005	34.50	34.41	34.10
0.50	0.010	28.20	28.07	27.89
0.50	0.020	21.91	21.84	21.70
0.50	0.050	14.67	14.63	14.53
0.50	0.100	10.31	10.31	10.25
0.50	0.300	5.54	5.54	5.52
1.00	0.000	13.80	13.77	13.65
1.00	0.005	12.74	12.71	12.59
1.00	0.010	11.97	11.94	11.82
1.00	0.020	10.78	10.74	10.66
1.00	0.050	8.61	8.63	8.57
1.00	0.100	6.84	6.84	6.81
1.00	0.300	4.22	4.22	4.22
1.50	0.000	6.55	6.56	6.53
1.50	0.005	6.40	6.38	6.35
1.50	0.010	6.23	6.21	6.19
1.50	0.020	5.93	5.93	5.89
1.50	0.050	5.28	5.27	5.27
1.50	0.100	4.56	4.56	4.55
1.50	0.300	3.18	3.20	3.23
2.00	0.000	3.88	3.89	3.91
2.00	0.005	3.82	3.85	3.86
2.00	0.010	3.78	3.80	3.81
2.00	0.020	3.68	3.70	3.73
2.00	0.050	3.44	3.47	3.50
2.00	0.100	3.14	3.16	3.19
2.00	0.300	2.41	2.45	2.50
3.00	0.000	1.83	1.93	2.00
3.00	0.005	1.81	1.91	2.00
3.00	0.010	1.81	1.91	1.99
3.00	0.020	1.79	1.89	1.97
3.00	0.050	1.74	1.84	1.92
3.00	0.100	1.66	1.77	1.84
3.00	0.300	1.43	1.54	1.61
5.00	0.000	0.67	0.80	0.92
5.00	0.005	0.67	0.79	0.91
5.00	0.010	0.67	0.79	0.92
5.00	0.020	0.67	0.79	0.91
5.00	0.050	0.66	0.78	0.90
5.00	0.100	0.65	0.76	0.89
5.00	0.300	0.61	0.71	0.83
	h	8.9135	8.8817	8.7989

where $S_0 = 0$, $N_k = (X_k - \mu_0 - r_k) f(k,\beta,\tau_1)$, and

$$f(k,\beta,\tau_1) = \begin{cases} \beta(k-\tau_1), & \text{for } k > \tau_1, \\ 0, & \text{otherwise.} \end{cases}$$

The handicap r_k is usually chosen to be $r_k = 0.5 f(k,\beta,\tau_1)$, and the CUSCORE statistic applies for a linear drift starting at a known time τ_1 that is initially set to $\tau_1 = 0$. The value of τ_1 should be reinitialized to be $\tau_1 = k$ every time $S_k = 0$. The CUSCORE-D chart will give a signal at sampling time k if $S_k > h_S$. For more details, see Runger and Testik (2003).

A two-sided CUSCORE chart, like a two-sided CUSUM chart, can be used for detecting both the upward and downward linear drifts, and this two-sided CUSCORE chart with the same control limit for both charts is used in the comparisons in this paper.

4.4. Simulation study

We now consider the performance of the GLR-SD chart relative to the GLR-S chart, the GLR-D chart, the CUSUM-S chart, the CUSCORE-D chart, and some chart combinations. As mentioned previously, we use $m_1 = 400$ and $m_2 = 2$ in the GLR-SD chart, $m_1 = 400$ and $m_2 = 4$ in the GLR-D chart, and $m_1 = 400$ and $m_2 = 1$ in the GLR-S chart. For applying the CUSUM-S chart and the CUSCORE-D chart, the chart parameters need to be pre-specified. Two sustained shifts are pre-specified for the CUSUM-S chart: $\delta_1 = 0.5$ and $\delta_1 = 2.0$. Two linear drifts are also pre-specified for the CUSCORE-D chart: $\beta_1 = 0.01$ and $\beta_1 = 0.2$. These pre-specified values are the same as those in Xu *et al.* (2013).

Table 2 gives SSATS values of some single charts. The results summarized in Table 2 are as follows. The GLR-SD chart has uniformly worse performance than that of the GLR-S chart. Reynolds and Lou (2010) stated that the GLR-S chart performs well in detecting linear drifts, although it is designed to detect sustained shifts. The GLR-SD chart is better than the GLR-D chart when there are both large sustained shifts ($\delta \geq 2.0$) and linear drifts. The GLR-SD chart is better than the CUSUM-S chart with $\delta_1 = 0.5$ except when there is no sustained shifts and there is both small sustained shifts and small linear drifts ($\delta = 0.5$ and $\beta \leq 0.02$). Similarly the GLR-SD chart is better than and the CUSCORE-D chart with $\beta_1 = 0.01$ except when there is no sustained shifts but small linear drifts ($\delta = 0.0$ and $\beta \leq 0.01$) and there is both small sustained shifts and small linear drifts ($\delta = 0.5$ and $\beta \leq 0.005$). The GLR-SD chart is uniformly better than the CUSUM-S chart with $\delta_1 = 2.0$ and the CUSCORE-D chart with $\beta_1 = 0.2$. It is noted that the GLR-SD chart is much better than the CUSUM-S chart and the CUSCORE-D chart when the size of shift and/or drift is far from the pre-specified values. We see that the control limit of the GLR-SD chart is bigger than those of the GLR-S chart and the GLR-D chart. It may make performance of the GLR-SD chart bad, but it does not give a significant meaning because chart statistics of three GLR charts are different from each other.

Table 3 gives SSATS values of some chart combinations, such as the GLR-S and GLR-D chart combination, and some CUSUM-S and CUSCORE-D chart combinations. The values used for the chart parameters in the CUSUM-S and CUSCORE-D chart combinations are $\delta_1 = 0.5, 2.0$ and $\beta_1 = 0.01, 0.2$. When using two charts together in combination, two control limits were adjusted so that these two charts have approximately the same in-control ATS values and the in-control ATS value of the combination is 1481.6. From Table 3, we see that the GLR-SD chart is worse than the GLR-S and GLR-D chart combination, however, it is better than the CUSUM-S and CUSCORE-D chart combination except for small sustained shifts ($\delta \leq 0.5$).

The conclusion that we obtain from Tables 2 and 3 is that the overall performance of the GLR-SD chart is comparable with other single charts and chart combinations in some cases and is significantly

Table 2: SSATS values for the GLR-SD chart, the GLR-S chart, the GLR-D chart, the CUSUM-S chart, and the CUSCORE-D chart for sustained shifts and/or linear drifts in the process mean.

δ	β	GLR-SD	GLR-S	GLR-D	CUSUM-S		CUSCORE-D	
					$\delta_1 = 0.5$	$\delta_1 = 2.0$	$\beta_1 = 0.01$	$\beta_1 = 0.2$
0.00	0.000	1481.29	1482.30	1483.66	1483.73	1482.66	1482.07	1482.62
0.00	0.005	104.82	99.05	95.87	91.44	135.87	93.47	123.10
0.00	0.010	67.33	63.83	61.94	59.92	83.00	62.73	76.10
0.00	0.020	42.84	41.14	39.75	39.82	50.37	43.64	46.80
0.00	0.050	23.56	22.74	22.11	23.88	26.30	28.44	24.92
0.00	0.100	14.89	14.42	14.07	16.49	16.36	21.14	15.77
0.00	0.300	7.10	6.90	6.77	9.52	8.23	13.69	8.42
0.50	0.000	50.25	43.99	45.80	36.37	145.97	41.37	96.38
0.50	0.005	34.50	31.27	31.78	28.35	54.37	33.32	44.75
0.50	0.010	28.20	26.02	26.14	24.64	39.19	29.89	33.70
0.50	0.020	21.91	20.51	20.53	20.65	27.53	25.94	24.52
0.50	0.050	14.67	13.89	13.82	15.51	16.78	20.77	15.63
0.50	0.100	10.31	9.86	9.79	12.12	11.56	17.15	11.04
0.50	0.300	5.54	5.35	5.30	7.98	6.61	12.28	6.89
1.00	0.000	13.80	12.48	13.40	13.82	19.39	21.50	16.08
1.00	0.005	12.74	11.62	12.37	13.20	16.27	20.64	14.13
1.00	0.010	11.97	10.95	11.57	12.72	14.51	19.94	13.03
1.00	0.020	10.78	9.97	10.40	11.97	12.51	18.80	11.47
1.00	0.050	8.61	8.10	8.32	10.46	9.66	16.62	9.21
1.00	0.100	6.84	6.27	6.59	9.05	7.68	14.60	7.65
1.00	0.300	4.22	4.04	4.09	6.72	5.21	11.23	5.73
1.50	0.000	6.55	5.99	6.53	8.67	7.14	16.47	7.31
1.50	0.005	6.40	5.86	6.35	8.54	6.93	16.13	7.15
1.50	0.010	6.23	5.71	6.17	8.43	6.75	15.84	7.01
1.50	0.020	5.93	5.44	5.87	8.20	6.45	15.31	6.78
1.50	0.050	5.28	4.91	5.22	7.67	5.85	14.14	6.32
1.50	0.100	4.56	4.27	4.50	7.05	5.21	12.88	5.84
1.50	0.300	3.18	3.02	3.16	5.73	4.11	10.43	4.94
2.00	0.000	3.88	3.56	3.97	6.46	4.33	13.83	5.36
2.00	0.005	3.82	3.50	3.92	6.41	4.30	13.64	5.33
2.00	0.010	3.78	3.46	3.86	6.36	4.25	13.47	5.30
2.00	0.020	3.68	3.39	3.77	6.29	4.19	13.22	5.35
2.00	0.050	3.44	3.18	3.52	6.05	4.05	12.50	5.09
2.00	0.100	3.13	2.90	3.21	5.74	3.83	11.64	4.90
2.00	0.300	2.41	2.26	2.53	4.97	3.35	9.76	4.39
3.00	0.000	1.83	1.67	2.18	4.44	2.66	10.94	4.06
3.00	0.005	1.81	1.66	2.17	4.43	2.65	10.88	4.04
3.00	0.010	1.81	1.64	2.16	4.42	2.65	10.81	4.04
3.00	0.020	1.79	1.64	2.14	4.39	2.64	10.68	4.03
3.00	0.050	1.74	1.59	2.09	4.33	2.61	10.34	3.98
3.00	0.100	1.66	1.53	2.02	4.23	2.58	9.89	3.92
3.00	0.300	1.43	1.32	1.82	3.93	2.46	8.72	3.73
5.00	0.000	0.67	0.62	1.49	2.99	1.68	8.24	3.03
5.00	0.005	0.67	0.61	1.49	2.98	1.69	8.20	3.03
5.00	0.010	0.67	0.61	1.49	2.98	1.68	8.19	3.02
5.00	0.020	0.67	0.61	1.49	2.98	1.68	8.12	3.01
5.00	0.050	0.66	0.61	1.49	2.96	1.68	7.99	3.01
5.00	0.100	0.65	0.60	1.49	2.94	1.67	7.83	2.99
5.00	0.300	0.61	0.57	1.48	2.85	1.65	7.25	2.93
	h	8.9135	7.3288	6.8473	5.3514	6.4109	2.7220	4.8660

Table 3: SSATS values for the GLR-SD chart, the GLR-S and GLR-D chart combination, and the CUSUM-S and CUSCORE-D chart combination for sustained shifts and/or linear drifts in the process mean.

δ	β	GLR-SD	GLR-S & GLR-D	CUSUM-S & CUSCORE-D			
				$\delta_1 = 0.5$ $\beta_1 = 0.01$	$\delta_1 = 2.0$ $\beta_1 = 0.01$	$\delta_1 = 0.5$ $\beta_1 = 0.2$	$\delta_1 = 2.0$ $\beta_1 = 0.2$
0.00	0.000	1481.29	1480.24	1481.37	1480.03	1482.69	1483.72
0.00	0.005	104.82	96.60	90.91	96.53	94.53	126.48
0.00	0.010	67.33	62.30	60.10	64.32	61.80	77.87
0.00	0.020	42.84	39.96	40.50	43.82	40.91	47.62
0.00	0.050	23.56	22.15	24.50	26.01	23.78	25.08
0.00	0.100	14.89	14.05	16.96	16.70	15.79	15.76
0.00	0.300	7.10	6.74	9.79	8.46	8.60	8.16
0.50	0.000	50.25	42.16	37.02	43.64	39.21	107.83
0.50	0.005	34.50	30.30	29.10	33.93	29.78	47.04
0.50	0.010	28.20	25.25	25.55	29.77	25.51	34.99
0.50	0.020	21.91	19.87	21.45	24.71	20.92	25.16
0.50	0.050	14.67	13.52	16.09	17.06	15.07	15.73
0.50	0.100	10.31	9.61	12.57	11.94	11.21	11.01
0.50	0.300	5.54	5.21	8.25	6.85	7.08	6.60
1.00	0.000	13.80	12.16	14.63	16.75	13.29	16.54
1.00	0.005	12.74	11.38	14.01	15.54	12.60	14.33
1.00	0.010	11.97	10.67	13.44	14.50	12.02	13.05
1.00	0.020	10.78	9.69	12.59	12.93	11.12	11.48
1.00	0.050	8.61	7.90	10.98	10.12	9.37	9.08
1.00	0.100	6.84	6.31	9.45	8.06	7.92	7.43
1.00	0.300	4.22	3.95	6.98	5.43	5.92	5.26
1.50	0.000	6.55	5.89	9.21	7.64	7.52	6.83
1.50	0.005	6.40	5.75	9.07	7.40	7.41	6.67
1.50	0.010	6.23	5.62	8.92	7.21	7.26	6.54
1.50	0.020	5.93	5.34	8.68	6.87	7.07	6.29
1.50	0.050	5.28	4.80	8.09	6.21	6.58	5.78
1.50	0.100	4.56	4.18	7.40	5.52	6.09	5.26
1.50	0.300	3.18	2.97	5.98	4.32	5.11	4.22
2.00	0.000	3.88	3.51	6.84	4.66	5.61	4.43
2.00	0.005	3.82	3.47	6.79	4.59	5.57	4.40
2.00	0.010	3.78	3.42	6.73	4.57	5.54	4.37
2.00	0.020	3.68	3.35	6.64	4.48	5.47	4.31
2.00	0.050	3.44	3.14	6.38	4.31	5.31	4.16
2.00	0.100	3.13	2.88	6.03	4.07	5.09	3.95
2.00	0.300	2.41	2.25	5.20	3.52	4.54	3.46
3.00	0.000	1.83	1.69	4.69	2.82	4.17	2.77
3.00	0.005	1.81	1.68	4.68	2.81	4.15	2.76
3.00	0.010	1.81	1.68	4.66	2.81	4.15	2.76
3.00	0.020	1.79	1.66	4.63	2.80	4.13	2.75
3.00	0.050	1.74	1.62	4.56	2.77	4.08	2.72
3.00	0.100	1.66	1.55	4.45	2.73	4.00	2.68
3.00	0.300	1.43	1.34	4.12	2.59	3.76	2.55
5.00	0.000	0.67	0.63	3.14	1.79	2.98	1.76
5.00	0.005	0.67	0.63	3.14	1.78	2.98	1.76
5.00	0.010	0.67	0.63	3.13	1.78	2.97	1.76
5.00	0.020	0.67	0.63	3.13	1.78	2.97	1.76
5.00	0.050	0.66	0.62	3.11	1.78	2.96	1.75
5.00	0.100	0.65	0.61	3.09	1.77	2.94	1.74
5.00	0.300	0.61	0.58	3.01	1.74	2.87	1.71
	h	8.9135	7.6124	5.7300	7.0855	5.9195	6.9480
			7.1690	3.1050	3.3123	5.4088	5.3715

better in other cases. We note that in using the GLR-SD chart we do not need to specify which type of parameter change (a sustained shift and/or a linear drift) may occur, because the GLR-SD chart can detect either one or both.

5. Conclusions

We considered the situation in which a special cause may produce both a sustained shift and a linear drift in the normal process mean. The GLR-SD chart for simultaneously monitoring a sustained shift and a linear drift was designed and evaluated in this paper.

We compared the performance of the proposed GLR-SD chart with that of other charts using the SSATS values when there were a sustained shift and/or a linear drift in the process mean. Simulation results showed that the GLR-SD chart is relatively effective for detecting sustained shifts and/or linear drifts, and is significantly better in some cases. The GLR-SD chart gives better overall performance than some CUSUM-S and CUSCORE-D chart combinations, as well as some single charts. It has the important advantage that we can efficiently detect a wide range of sustained shifts and/or linear drifts using only one chart, and it does not require the user to specify the size of the parameter change. As expected, the CUSUM chart and the CUSCORE chart perform poorly when the actual size of shift or drift is significantly different from the pre-specified values.

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