IEIE Transactions on Smart Processing and Computing

Parameter Estimation of Linear-FM with Modified sMLE for Radar Signal Active Cancelation Application

Seungkyu Choi and Chungyong Lee

School of Electrical and Electronic Engineering, Yonsei University / Seoul, South Korea {seekhide, cylee}@yonsei.ac.kr

***** Corresponding Author: Chungyong Lee

Received February 20, 2014; Revised April 20, 2014; Accepted August 28, 2014; Published December 31, 2014

** Regular Paper*

***Abstract*:** This study examined a radar signal active cancelation technique, which is a theoretical way of achieving stealth by employing a baseband process that involves sampling the incoming hostile radar signal, analyzing its characteristics, and generating countermeasure signals to cancel out the linear-FM signal of the hostile radar signal reflected from the airborne target. To successfully perform an active cancelation, the effects of errors in the countermeasure signal were first analyzed. To generate the countermeasure signal that requires very fast and accurate processing, the down-sampling technique with the suboptimal maximum likelihood estimation (sMLE) scheme was proposed to improve the speed of the estimation process while preserving the estimation accuracy. The simulation results showed that the proposed down-sampling technique using a 2048 FFT size yields substantial power reduction despite its small FFT size and exhibits similar performance to the sMLE scheme using the 32768 FFT size.

***Keywords*:** Parameter estimation, Active cancelation, Countermeasure signal, Linear-FM, Suboptimal MLE, Down-sampling

1. Introduction

Many studies have examined the passive cancelation technique of a reflected signal at an aircraft for very low observability, whereas there are few reports on the active cancelation technique. Theoretically, the active cancelation technique is considerably more flexible than the passive cancelation one because it only needs to duplicate the incoming hostile radar signal and reverse its phase by π to cancel out the reflected signal [1]. On the other hand, the active cancelation technique as a practical implementation of stealth must perform several processes simultaneously, which involve estimating the fundamental characteristics of the incoming hostile radar signal in real-time with precision and transmitting the countermeasure signal to the correct place with precisely the same power. If these are not performed well, the active cancelation technique can expose the aircraft to hostile radar.

As mentioned above, the first step of realizing the active cancelation technique is to analyze the incoming hostile radar signals because information on them can never be achieved in advance. For this purpose, a very fast and accurate estimation scheme is needed to extract the fundamental parameters of the incoming hostile radar signal, and then generate a countermeasure signal that corresponds to the reflected hostile radar signal by utilizing the estimated parameters, such as frequency, bandwidth, phase, and amplitude. In doing so, two essential premises need to be followed: real-time and accuracy. This means that the countermeasure technique should be able to respond and synthesize the related signals as quickly as possible with the signal characteristics as similar as possible to those of the original radar signal such that the detrimental side effects of the countermeasure signal is minimized. In addition, the effects of inevitable estimation errors and the limit of allowable errors in the countermeasure signal must also be considered.

In this study, the linear-FM signal, which is one of the essential signal waveform of a radar system, was adopted as a basic radar signal. Many studies have examined ways of estimating the parameters linear-FM signal [2-8] and polynomial phase signal [9-11], a superset of linear-FM signal. Among them, this study focused on the suboptimal MLE (sMLE) scheme [2] because it is based on the general FFT operation with almost the same mean-squareerror (MSE) performance as the Cramer-Rao bounds (CRB) at high SNR. For this reason, it matches the premises well and is applicable to the active cancelation technique as a parameter estimation scheme. On the other hand, it is important to consider that if the number of samples increases, the number of FFT sizes should also increase. Therefore, to alleviate the FFT size limitation, this paper proposes a down-sampling scheme that requires a small FFT size and preserves the estimation accuracy of the sMLE scheme. The simulation results are given to show the advantages of the proposed down-sampling scheme in terms of the power of the returned signal and the MSE performance of each parameter estimation.

2. Effects of Parameter Estimation Errors in a Countermeasure Signal

The pulsed linear-FM (LFM) signal adopted by hostile radars can be defined as

$$s(t) = Ae^{j\left(a_0 + a_1 t + a_2 t^2\right)}, 1 \le t \le T, \tag{1}$$

where A is the amplitude and variables $a_0 = \phi$, $a_1 = 2\pi f_0$, and $a_2 = 2\pi^2 B / T$ denote the simplified phase, frequency, and bandwidth parameters, respectively. Here, ϕ is the phase, f_0 denotes the center frequency, B depicts the sweep bandwidth, and T is the pulse width. The distortion of the hostile radar signal due to propagation was also assumed to be negligible and all the incoming signal to the surface of a target will be reflected perfectly. The reflected signal from the target can then be expressed as

$$x(t) = A_r e^{j\left(a_0 + a_1 t + a_2 t^2 + \pi\right)}, 1 \le t \le T, \tag{2}$$

where A_r is the amplitude of the reflected signal that can be derived by the one-way radar equation given in [12].

The ideal countermeasure signal for (2) can be given by

$$x_{cm}(t) = A_r e^{j\left(a_0 + a_1 t + a_2 t^2\right)}, 1 \le t \le T. \tag{3}$$

On the other hand, because the knowledge of the principal parameters of (2) to the target in advance is noncausal, the only viable action is that the target generates a countermeasure signal by extracting the unknown parameters, A_r, a_0, a_1, and a_2, out of the reflected signal. The sampled signal of (2) for estimating these parameters can be represented as

$$x[n] = A_r e^{j\left(a_0 + a_1 n t_s + a_2 n^2 t_s^2 + \pi\right)} + \varepsilon[n], 1 \le n \le N, \tag{4}$$

where $t_s = 1/f_s$ is a sampling interval and $N = f_s T$. Here, the noise $\varepsilon[n]$ is assumed to be an i.i.d complex Gaussian random variable with a zero mean and variance of σ^2, and the input signal-to-noise ratio (SNR) can be defined as

$$SNR = A_r^2 / \sigma^2. \tag{5}$$

As mentioned above, the objective is that the target generates the countermeasure signal such that it cancels out the reflected signal perfectly. Despite this, all parameter estimation schemes produce unavoidable estimation errors so that the countermeasure signal can never cancel the reflected signal out completely. By considering these parameter estimation errors, the returned signal to the hostile radar can be expressed as

$$\begin{aligned} y[n] &= \left(A_r + A_r^e\right) e^{j\left(a_0 + a_0^e + \left(a_1 + a_1^e\right) n t_s + \left(a_2 + a_2^e\right) n^2 t_s^2\right)} \\ &\quad + A_r e^{j\left(a_0 + a_1 n t_s + a_2 n^2 t_s^2 + \pi\right)} \\ &= 2A_r \cos\left(\frac{a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2 - \pi}{2}\right) \\ &\quad \times e^{j\left(a_0 + a_1 n t_s + a_2 n^2 t_s^2 + \frac{a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2 + \pi}{2}\right)} \\ &\quad + A_r^e e^{j\left(a_0 + a_0^e + \left(a_1 + a_1^e\right) n t_s + \left(a_2 + a_2^e\right) n^2 t_s^2\right)}, \end{aligned} \tag{6}$$

where A_r^e, a_0^e, a_1^e, and a_2^e are the amplitude, phase, frequency, and bandwidth parameters errors, respectively. Generally, the second term can be ignored, because A_r^e is much smaller than A_r and the accuracy of A_r^e is very high, as proven by the simulation. Nevertheless, it is possible that $2A_r \cos\left(\frac{a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2 - \pi}{2}\right)$ in the first term is equal to $2A_r$, unless a_0^e, a_1^e and a_2^e are small enough. For this reason, the parameter estimation errors in the countermeasure signal can have an adverse effect: the amplitude of the returned signal can be larger than $2A_r$. If a_0^e, a_1^e, a_2^e are small enough, however, the error component $a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2$ will be infinitesimally small so that the amplitude of the returned signal can be smaller than the detection threshold of the hostile radar. As a result, the hostile radar will not be able to identify the target's existence. Therefore, it is necessary that the parameter estimation for the countermeasure signal be highly accurate.

Then, what level of estimation error is acceptable for each parameter? To achieve the beneficial effects with the countermeasure signal, the absolute value of $2A_r \cos\left(\frac{a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2 - \pi}{2}\right)$ must be smaller than A_r:

$$\left|2A_r \cos\left(\frac{a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2 - \pi}{2}\right)\right| \le A_r. \tag{7}$$

To satisfy this requirement, the absolute value of the error component E needs to be smaller than $\pi/3$, which is expressed as

$$|E| = \left|a_0^e + a_1^e n t_s + a_2^e n^2 t_s^2\right| \le \pi/3. \quad (8)$$

If it is assumed that the estimation errors of the remainder are zero, the upper bound of the maximum tolerable error of each parameter can be calculated as follows:

$$\begin{aligned} &\left|a_0^e\right| \le \frac{\pi}{3} \to \left|\phi^e\right| \le \frac{\pi}{3}, \\ &\left|a_1^e\right| \le \frac{\pi}{3nt_s} \le \frac{\pi}{3T} \to \left|f_0^e\right| \le \frac{1}{6T}, \\ &\left|a_2^e\right| \le \frac{\pi}{3n^2 t_s^2} = \frac{\pi}{3T^2} \to \left|B^e\right| \le \frac{1}{6\pi T}, \end{aligned} \quad (9)$$

where f_0^e, B^e and ϕ^e denote the center frequency, sweep bandwidth and phase error, respectively.

3. Sub-optimal Maximum Likelihood Estimation Scheme of Linear-FM Signal

The motivation behind the sMLE scheme [2] lies on the optimization problem of the MLE scheme that maximizes the function $L(a_1, a_2)$ as follows:

$$(\hat{a}_1, \hat{a}_2) = \arg\max_{a_1, a_2} L(a_1, a_2), \quad (10)$$

where $L(a_1, a_2) = \left|\sum_{n=1}^{N} x[n] e^{-j\left(a_1 n t_s + a_2 n^2 t_s^2\right)}\right|^2$. (11)

As the 2-dimensional search problem of (10) cannot be solved using a simple 1-dimensional FFT method, the sMLE scheme exploits the discrete ambiguity function (DAF) extracted from (11) [2]:

$$\mathrm{D}(\mathbf{x}, w, \tau) = \sum_{n=1}^{N-\tau} x[n+\tau] x^*[n] e^{-jwnt_s}, \quad (12)$$

where $(\cdot)^*$ denotes the complex conjugate operation and $\tau = N/2$. The main advantage of using the DAF, instead of $L(a_1, a_2)$, is that it enables the division of the 2-dimensional search problem into two sequential 1-dimensional search problems such that the FFT operation becomes applicable.

Next, the signal inside the DAF is defined as $z[n]$, which is represented as a sinusoidal signal whose frequency component is only composed of a_2 as follows:

$$\begin{aligned} z[n] &= x[n+\tau] x^*[n] \\ &= A_r^2 e^{j\left(2a_2 \tau t_s n t_s + a_1 \tau t_s + a_2 \tau^2 t_s^2\right)} + \varepsilon'[n] \\ &= A_r^2 e^{j\left(\bar{a}_2 n t_s + \bar{\phi}\right)} + \varepsilon'[n], \; 1 \le n \le N/2, \end{aligned} \quad (13)$$

where $\bar{a}_2 = 2a_2\tau t_s$, $\bar{\phi} = a_1 \tau t_s + a_2 \tau^2 t_s^2$, and $\varepsilon'[n] = x^*[n]\varepsilon[n+\tau] + x[n+\tau]\varepsilon^*[n] + \varepsilon^*[n]\varepsilon[n+\tau]$. The DAF can be considered such that $z[n]$ is passing through a DFT operator, which can be written as

$$\mathrm{D}(\mathbf{z}, w, \tau) = \sum_{n=1}^{N-\tau} z[n] e^{-jwnt_s}, \quad (14)$$

and the optimization problem of (10) can be simplified into the following problem

$$\hat{w} = \arg\max_{w} \left|\mathrm{D}(\mathbf{z}, w, \tau)\right|^2. \quad (15)$$

Because (15) is considered a traditional MLE scheme for a sinusoidal signal [13], we can simply find $\hat{w}$ by employing the FFT operation to $z[n]$. The resulting $\hat{a}_2$ is given by

$$\hat{a}_2 = \frac{\hat{w}}{2\tau t_s}. \quad (16)$$

Subsequently, $\hat{a}_2$ is substituted for a_2 in (10) and solve the 1-dimensional optimization problem

$$\hat{a}_1 = \arg\max_{w} L(w, \hat{a}_2), \quad (17)$$

where $L(w, \hat{a}_2) = \left|\sum_{n=1}^{N} \left(x[n] e^{-j\hat{a}_2 n^2 t_s^2}\right) e^{-jwnt_s}\right|^2$. As the term inside $|\cdot|^2$ is also the same as the DFT operation, we can solve (17) and obtain $\hat{a}_1$ by applying the FFT operation. To identify the remaining parameters, $\hat{A}_r$ and $\hat{a}_0$, we set $\hat{v}$ as

$$\hat{v} = \frac{1}{N} \sum_{n=1}^{N} x[n] e^{-j\left(\hat{a}_1 n t_s + \hat{a}_2 n^2 t_s^2\right)}, \quad (18)$$

such that both can be calculated simply as

$$\hat{a}_0 = \mathrm{Im}\{\log \hat{v}\} - \pi, \; \hat{A}_r = e^{\mathrm{Re}\{\log \hat{v}\}}. \quad (19)$$

According to the analysis presented in [2], the sMLE scheme has an advantage over the MLE scheme in terms of its low complexity at the expense of slight MSE performance degradation. To show this, the approximated MSE expressions of every parameter were introduced into the sMLE scheme, which is given by the following:

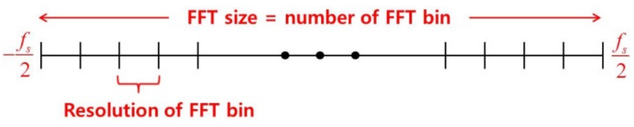

Fig. 1. Pictorial representation of the resolution of FFT bin.

$$\mathrm{var}(\hat{a}_0) \approx \left(\frac{14}{3} + \frac{4}{3\times\mathrm{SNR}}\right)\frac{1}{\mathrm{SNR}\times N},$$
$$\mathrm{var}(\hat{a}_1) \approx \left(\frac{17}{16} + \frac{1}{2\times\mathrm{SNR}}\right)\frac{96 f_s^2}{\mathrm{SNR}\times N^3},$$
$$\mathrm{var}(\hat{a}_2) \approx \left(1 + \frac{1}{2\times\mathrm{SNR}}\right)\frac{96 f_s^4}{\mathrm{SNR}\times N^5},\ \mathrm{var}(\hat{A}_r) \approx \frac{\sigma^2}{2N}. \quad (20)$$

4. Improved sMLE Scheme for an Active Cancelation Application

From the viewpoint of active cancelation applications, it is preferable to have the FFT size as small as possible to pursue the first premise in an active cancelation technique: real-time response. On the other hand, it is also more preferable that the FFT size be as large as possible, because the second premise is the pursuit of parameter estimation accuracy. Accordingly, there must be some trade-off between the real-time response and parameter estimation accuracy.

If the FFT size N_{FFT} is assumed to be bounded by $N_{FFT} \le N_{FFT}^{Th}$ for a quick response, the sMLE scheme itself cannot be applied in most cases for the following reasons. First and foremost, the sMLE scheme must employ N_{FFT} larger than the total number of samples $N/2$ to successfully extract $\hat{a}_2$ from $z[n]$. In other words, the sMLE scheme can be applied only if N_{FFT} satisfies the condition, $N/2 \le N_{FFT} \le N_{FFT}^{Th}$. The second reason is that although the above condition is satisfied, the resolution of the FFT bin might not be sufficiently high to satisfy the requirement of the estimation accuracy for $\hat{a}_1$ and $\hat{a}_2$.

As shown in Fig. 1, the resolution of the FFT bin is determined by two factors: the FFT size and the sampling frequency. Their inter-correlation is given as

$$FFT_{res} = \frac{f_s}{N_{FFT}}. \quad (21)$$

Considering the maximum tolerable error of $|a_1^e|$ and $|a_2^e|$ in (9), the FFT bin's resolution for successfully estimating both $\hat{a}_1$ and $\hat{a}_2$ is upper-bounded as follows:

$$FFT_{res} \le \frac{1}{6T}. \quad (22)$$

Here, the estimation errors are assumed to be generated only by the insufficient resolution of the FFT bin, not by the noise effects. Given a fixed f_s, the required number of N_{FFT} can be expressed as

$$6 f_s T \le N_{FFT} \le N_{FFT}^{Th}. \quad (23)$$

Note that if the FFT bin resolution compensation scheme, e.g., the quadratic interpolation scheme [14], is considered, the required number of N_{FFT} can be smaller than $6 f_s T$.

Overall, the problems mentioned above are exclusively affected by both the limited FFT size and the corresponding estimation accuracy degradation. Then, what can be done to overcome the aforementioned drawbacks of the sMLE scheme? To answer this, we will introduce a scheme that decreases the sampling frequency based on the given FFT size. This will enable preservation, even enhancement, of the resolution of the FFT bin.

4.1 Down-sampling for bandwidth estimation

First of all, the simplest down-sampling scheme, which decreases only the sampling frequency, was applied. Down-sampling with a rate M_B was applied to (13) and the down-sampled signal can be expressed as

$$s[m] = A_r^2 e^{j(\bar{a}_2 m M_B t_s + \bar{\phi})} + \varepsilon'[mM_B],\ 1 \le m \le L, \quad (24)$$

where $L = N/2M_B$. Note that $s[m]$ can be considered a sinusoidal signal with length L and sampling frequency f_s/M_B. Here, $\varepsilon'[mM_B]$ can be modeled as a Gaussian distribution with a mean 0 and variance $(2A_r^2 + \sigma^2)\sigma^2$. Consequently, the resolution of the FFT bin, which was modified by down-sampling, is determined as

$$FFT_{res} = \frac{1}{M_B}\frac{f_s}{N_{FFT}}. \quad (25)$$

The scheme appears to offer some improvement on the resolution of the FFT bin and the corresponding FFT size as M_B becomes large. On the other hand, considering the MSE of $\hat{a}_2$, there is performance degradation proportional to the down-sampling rate, M_B, which is denoted as

$$\mathrm{var}(\hat{a}_2) \ge \left(1 + \frac{1}{2\times SNR}\right)\frac{96 f_s^4 M_B}{SNR\times N^3\left(N^2 - 4M_B^2\right)}. \quad (26)$$

Here, this study exploits the CRB expression of the sinusoidal signal [15], given as

$$\mathrm{var}(\bar{a}_2) \ge \frac{6 f_{s_c}^2}{N_c\left(N_c^2 - 1\right)SNR_c}, \quad (27)$$

where $N_c = \frac{N}{2M_B}$, $SNR_c = \frac{A_r^4}{\left(2A_r^2+\sigma^2\right)\sigma^2}$, $f_{s_c} = \frac{f_s}{M_B}$, and $\bar{a}_2 = a_2 N t_s$. Therefore, the simple scheme described above is unsuitable for reaching both of these active cancelation goals.

The above result provides an intuition that something needs to be done further to prevent sacrificing the estimation accuracy while decreasing the sampling frequency. To this end, the down-sampling scheme in [16] is exploited. This scheme forms the down-sampled sequence $s[m]$ by accumulating every M_B amount of samples of $z[n]$ as follows:

$$s[m] = A_r^2 \sum_{n=mM_B+1}^{(m+1)M_B} e^{j\left(\bar{a}_2 n t_s + \bar{\phi}\right)} + \sum_{n=mM_B+1}^{(m+1)M_B} \varepsilon'[n]$$
$$\simeq M_B A_r^2 e^{j\left(\bar{a}_2 m M_B t_s + \bar{\phi} + \frac{\bar{a}_2}{2}(M_B-1)t_s\right)} + \varepsilon''[m],\ 0 \le m \le L-1, \tag{28}$$

where $L = N/2M_B$ and $\varepsilon''[m] = \sum_{n=mM_B+1}^{(m+1)M_B} \varepsilon'[n]$. Here, $\varepsilon''[m]$ has a Gaussian distribution with a mean 0 and a variance $M_B\left(2A_r^2+\sigma^2\right)\sigma^2$. Following the same assumption of $s[m]$, as in (24), the resolution of the FFT bin modified by the accumulation is given by (25). Unlike the simple down-sampling scheme before, considering the MSE of $\hat{a}_2$, there was negligible performance degradation, because the proposed scheme has an SNR gain proportional to M_B, i.e., $SNR_c = \frac{M_B^2 A_r^4}{M_B\left(2A_r^2+\sigma^2\right)\sigma^2}$. The MSE of $\hat{a}_2$ of the proposed scheme can be expressed as

$$\operatorname{var}\left(\hat{a}_2\right) \ge \left(1+\frac{1}{2\times SNR}\right)\frac{96 f_s^4}{SNR\times N^3\left(N^2-4M_B^2\right)}. \tag{29}$$

The range of required M_B to reduce a certain amount of FFT size is then derived. Given the FFT size, we can calculate the range of M_B as follows:

$$\frac{6Tf_s}{N_{FFT}} \le M_B \le \frac{f_s}{2\pi B}, \tag{30}$$

where the left side comes from the maximum tolerable error bound, $\frac{f_s}{M_B N_{FFT}} \le \frac{1}{6T}$, and the right one is obtained from the Nyquist sampling theorem, $\frac{f_s}{M_B} \ge 2\pi B$, where $\pi B = \frac{\bar{a}_2}{2\pi}$. Therefore, the proposed down-sampling scheme can save the FFT size proportional to the down-sampling rate M_B that is bounded by (30). For example, if the number of samples is $N/2 = 5{,}000$, then the sMLE scheme should employ a FFT size larger than $N_{FFT} = 16{,}384$; however, the proposed down-sampling scheme can be applied by $N_{FFT} = 1{,}024$ with negligible performance degradation because the number of samples is reduced to $N/2M_B = 50$ by the down sampling rate, $M_B = 100$. Moreover, the computational complexity of the sMLE scheme can also be relieved using the proposed scheme, because as is already known, the computational complexity of the FFT is $N_i \log\left(N_i\right)$, where N_i is the input number of samples. In addition, the well-known error compensation scheme for the FFT-based scheme, the quadratic interpolation scheme [14], can also be applied to the proposed scheme to further enhance the estimation accuracy.

4.2 Down-sampling for frequency estimation

As in the sMLE scheme, the estimated $\hat{a}_2$ was first applied to (4) and its signal is denoted as

$$x^{(1)}[n] = x[n]e^{-j\hat{a}_2 n^2 t_s^2}$$
$$= A_r e^{j\left(a_1 n t_s + a_0 - a_2^e n^2 t_s^2 + \pi\right)} + \varepsilon[n]e^{-j\hat{a}_2 n^2 t_s^2}, \tag{31}$$

where $\hat{a}_2 = a_2 + a_2^e$ and $1 \le n \le N$. Because $x^{(1)}[n]$ can also be considered to be a sinusoidal signal with length N and sampling frequency f_s, FFT can be applied to (31) for finding $\hat{a}_1$. Originally, the required FFT size for estimating $\hat{a}_1$ should be larger than N. Nevertheless, a smaller FFT size can also be used at the expense of MSE performance degradation. In other words, although it is still possible to employ a small FFT size to estimate $\hat{a}_1$ with partial samples, the related MSE performance will be impaired due to the reduced number of samples and the degraded FFT bin resolution. For this reason, the estimation of $\hat{a}_1$ using the sMLE scheme itself has a limitation to employ the given FFT size.

Based on the explanation above, this section introduces a frequency estimation scheme that can yield small MSE performance degradation, even if the available FFT size is limited. At first, N_r samples, which are smaller than the FFT size that is currently applied, are extracted and FFT operation can then be performed to find a coarse estimation of $\hat{a}_{1c} = a_1 + \hat{a}_{1c}^e$. Using $\hat{a}_{1c}$, a signal is generated, conjugated and then multiplied by $x^{(1)}[n]$, given as

$$y[n] = e^{j\hat{a}_{1c} n t_s},\ 1 \le n \le N,$$
$$y^{(1)}[n] = x^{(1)}[n]y^*[n] \tag{32}$$
$$= A_r e^{j\left(-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)n t_s + a_0 + \pi\right)} + \varepsilon'[n],$$

where $\varepsilon'[n] = \varepsilon[n]e^{-j\hat{a}_2 n^2 t_s^2} e^{-j\hat{a}_{1c} n t_s}$. Note that $y^{(1)}[n]$ can be treated as a sinusoidal signal with frequency $-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)$. Here, unlike the $-a_2^e n^2 t_s^2$ in (31), the component $-a_2^e n^2 t_s^2$ cannot be ignored because the difference between $\hat{a}_{1c}^e$ and $a_2^e n t_s$ is small. By taking into consideration the problem of obtaining $\overline{a}_2$ from the signal (13), the estimation of $-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)$ becomes the same problem of estimating $\overline{a}_2$. Therefore, the same down-sampling scheme can be employed to overcome the FFT size problem.

Similar to the proposed bandwidth estimation scheme, every M_f sample of $y^{(1)}[n]$ is accumulated and then operated by FFT to make a good estimation of $\hat{a}_{1f} \simeq -\left(\hat{a}_{1c}^e + a_2^e n t_s\right) + \hat{a}_{1f}^e$, given as

$$\begin{aligned} s^{(1)}[m] &= A_r \sum_{n=mM_f+1}^{(m+1)M_f} e^{j\left(-\left(\hat{a}_{1c}^e + a_2^e n t_s\right) n t_s + a_0 + \pi\right)} + \varepsilon''[n] \\ &\simeq M_f A_r e^{j\left(-\left(\hat{a}_{1c}^e + a_2^e n t_s\right) m M_f t_s + \overline{\phi}\right)} + \varepsilon''[m],\ 0 \le m \le L-1, \end{aligned} \quad (33)$$

where $L = N/M_f$, $\overline{\phi} = a_0 + \pi + \dfrac{-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)}{2}\left(M_f - 1\right)t_s$, and $\varepsilon''[m] = \sum_{n=mM_f+1}^{(m+1)M_f} \varepsilon'[n]$. Here, $\varepsilon''[m]$ has a Gaussian distribution with mean 0 and variance $M_f \sigma^2$. The estimated $\hat{a}_1$ can then be given as $\hat{a}_1 = \hat{a}_{1c} + \hat{a}_{1f} = a_1 - a_2^e n t_s + \hat{a}_{1f}^e$. Note that $\hat{a}_1$ includes the component, $a_2^e n t_s$, which is related to the bandwidth estimation error. Because $\hat{a}_{1f}^e$ and $-a_2^e n t_s$ are obtained through the FFT operation with a resolution of $\dfrac{1}{M_f}\dfrac{f_s}{N_{FFT}}$ and $\dfrac{1}{M_B}\dfrac{f_s}{N_{FFT}}$, respectively, the total FFT bin resolution can be modeled as

$$FFT_{res} = \max\left(\frac{1}{M_f}\frac{f_s}{N_{FFT}}, \frac{1}{M_B}\frac{f_s}{N_{FFT}}\right). \quad (34)$$

Subsequently, to derive the MSE of $\hat{a}_1$, the MSE of $\hat{a}_{1f}^e$ is first calculated by exploiting the CRB expression in (27) as

$$\operatorname{var}\left(\hat{a}_{1f}^e\right) \ge \frac{6 f_s^2}{SNR \times N\left(N^2 - M_f^2\right)}. \quad (35)$$

Assuming that $n = N$ produces the largest $-a_2^e n^2 t_s^2$, the MSE of $\hat{a}_1$ can then be expressed as

$$\begin{aligned} \operatorname{var}(\hat{a}_1) &\ge \operatorname{var}\left(\hat{a}_{1f}^e\right) + \operatorname{var}(\hat{a}_2)\left(N/f_s\right)^2 \\ &= \frac{96 f_s^2}{SNR \times N}\left(\frac{1}{16\left(N^2 - M_f^2\right)} + \left(1 + \frac{1}{2SNR}\right)\frac{1}{\left(N^2 - 4M_B^2\right)}\right). \end{aligned} \quad (36)$$

Compared to the MSE expression for the sMLE scheme in (20), the proposed down-sampling scheme exhibits negligible performance degradation.

As in the estimation of $\hat{a}_2$, the next step is to obtain the range of required M_f to reduce the FFT size by a certain amount. Given the FFT size, the suitable value of M_f can be chosen under the following constraint:

$$\frac{6Tf_s}{N_{FFT}} \le M_f \le \frac{\left|-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)\right|}{\pi}, \quad (37)$$

where the lower bound is the result of $\dfrac{f_s}{M_f N_{FFT}} \le \dfrac{1}{6T}$ and the upper bound is obtained from $\dfrac{f_s}{M_f} \ge 2\dfrac{\left|-\left(\hat{a}_{1c}^e + a_2^e n t_s\right)\right|}{2\pi}$. If it is assumed that an estimation error only occurs due to the low resolution of the FFT bin, the upper bound can then be represented by $\dfrac{N_{FFT}}{2}$.

4.3 Influence of the parameter estimation errors on the power of the returned signal

The error component to be considered is a_0^e. First of all, the estimated $\hat{a}_1$ and $\hat{a}_2$ can be restated as

$$\begin{aligned} \hat{a}_2 &= a_2 + a_2^e \\ \hat{a}_1 &\simeq a_1 + \left(-a_2^e n t_s + \hat{a}_{1f}^e\right) = a_1 + a_1^e. \end{aligned} \quad (38)$$

$\hat{v}$ can then be rewritten as

$$\hat{v} = \frac{1}{N}\sum_{n=1}^{N} A_r e^{j\left(a_0 + \pi - a_1^e n t_s - a_2^e n^2 t_s^2\right)}, \quad (39)$$

where the noise term in (18) is neglected for notational simplicity. Because $\hat{v}$ includes an accumulation process similar to that of the process in (28), the equation can be written as

$$\hat{v} \simeq A_r e^{j\left(a_0 + \pi + \frac{-\left(a_1^e + a_2^e n t_s\right)}{2}(N-1)t_s\right)}. \quad (40)$$

In this case, the estimated $\hat{a}_0$ is given by $\hat{a}_0 = a_0 + \dfrac{-\left(a_1^e + a_2^e n t_s\right)}{2}\left(N-1\right)t_s$ because $\hat{a}_0 = \operatorname{Im}\{\log \hat{v}\}$

$-\pi$. Consequently, considering the noise effect, the estimated $\hat{a}_0$ can be expressed as

$$\hat{a}_0 \simeq a_0 + \left(\frac{-\left(a_1^e + a_2^e n t_s\right)}{2}\left(N-1\right)t_s + \hat{a}_0^e \right) = a_0 + a_0^e, \quad (41)$$

where $\hat{a}_0^e$ can be viewed as the error caused by the neglected noise term.

Consider E in total. By putting the above results in E, its results can be calculated as

$$\begin{aligned} E &\simeq \frac{-\left(\left(-a_2^e n t_s + \hat{a}_{1f}^e\right) + a_2^e n t_s\right)}{2}\left(N-1\right)t_s + \hat{a}_0^e \\ &+\left(-a_2^e n t_s + \hat{a}_{1f}^e\right) n t_s + a_2^e n^2 t_s^2 \qquad (42) \\ &= \frac{\left(-\hat{a}_{1f}^e\right)}{2}\left(N-1\right)t_s + \hat{a}_0^e + \hat{a}_{1f}^e n t_s . \end{aligned}$$

Assuming that E has the maximum value at $n = N$, (42) can be simplified to $E \simeq \frac{\hat{a}_{1f}^e}{2} N t_s + \hat{a}_0^e$. This shows that every parameter estimation error is mutually destructive, not mutually constructive. In addition, the influence of $\frac{\hat{a}_{1f}^e}{2} N t_s$ on E will be very small, because the accuracy of $\hat{a}_{1f}^e$ depends on the resolution of the FFT bin (34). Therefore, E will be affected mainly by $\hat{a}_0^e$, whose prime error sources are $\hat{a}_{1f}^e$ and a_2^e.

5. Simulation Results

The parameter values given in Table 1 were applied and a computer simulation was performed with 10^4 trials. In the conventional sMLE scheme, the minimum required number of FFT sizes was $N_{FFT} = 16,384$, because the total number of samples was $N = 10^4$. For this minimum FFT size case, however, the FFT bin resolution of both the bandwidth and frequency estimation can be determined to be $\frac{f_s}{N_{FFT}} = 1.2 \times 10^6$, which cannot satisfy the minimum FFT bin resolution bound of $\frac{1}{6T} = 3.3 \times 10^5$. Although it is important to choose a FFT size larger than $N_{FFT} = 65,536$ to satisfy the bound, assuming that the quadratic interpolation error compensation scheme is applied to both the bandwidth and frequency estimation process, a FFT size smaller than $N_{FFT} = 65,536$ can be employed. Accordingly, the FFT size was selected to be $N_{FFT} = 32,768$ for the sMLE scheme in the simulation. In the proposed scheme, if the FFT size is fixed to $N_{FFT} = 2,048$, the value of M_B can be calculated from $\frac{6Tf_s}{N_{FFT}} \le M_B \le \frac{f_s}{2\pi B}$. The value of $M_B = 50$ and $M_f = 50$ were chosen such that the bound $\frac{6Tf_s}{N_{FFT}} \le M_f \le \frac{N_{FFT}}{2}$ can be satisfied. With this choice of M_B and M_f , and even with the quadratic interpolation scheme, similar performance to the conventional sMLE scheme using $N_{FFT} = 32,768$ is expected.

Table 1. Simulation Parameters for the Active Cancelation Technique.

Parameters	Parameter Value
Radar Peak Power (Kw)	$P_t = 5$
Radar Antenna Gain (dB)	$G_t = 30$
Center Frequency (GHz)	$f_0 = 2$
Sweep Bandwidth (MHz)	$B = 20$
Phase	$\phi = \pi/6$
Pulse Width (ns)	$T = 500$
Range (Km)	$R = [6:3:24]$
Sampling Frequency (GHz)	$f_s = 10 f_0$
Aircraft Antenna Gain (dB)	$G_r = 30$
Noise Power at Aircraft (dBw)	$N_0 = -50$
RCS (m^2)	1
Total Number of Sample	$N = f_s T$
FFT Size	N_{FFT} = variable
Number of Estimation Pulses	$N_e = 1$
Number of Return Pulses	$N_p = 20$

In this simulation, it was assumed that the parameters of the reflected signal can be estimated within the number of estimation pulses, N_e , and immediately generate a countermeasure signal to counteract the following consecutive reflected signal N_p . To accomplish this, we also assumed that the pulse repetition interval of the hostile radar signal is known in advance. The power of the returned signal was calculated based on the assumption that N_p pulses are integrated coherently at the hostile radar.

In Fig. 2, the vertical axis represents the amount of power reduction with the original returned signal set as a reference. As expected, Fig. 2 shows that the performance of the proposed scheme with $N_{FFT} = 2,048$ exhibits similar performance as the sMLE scheme with $N_{FFT} = 32,768$. When $N_{FFT} = 8,192$ is applied to the bandwidth and frequency estimation process in the sMLE scheme, the resulting power reduction is smaller than the proposed scheme, as expected. Next, the average of absolute error of both the proposed and the conventional sMLE schemes were examined. As observed in Fig. 3, the total error of the proposed scheme using $N_{FFT} = 2,048$ is

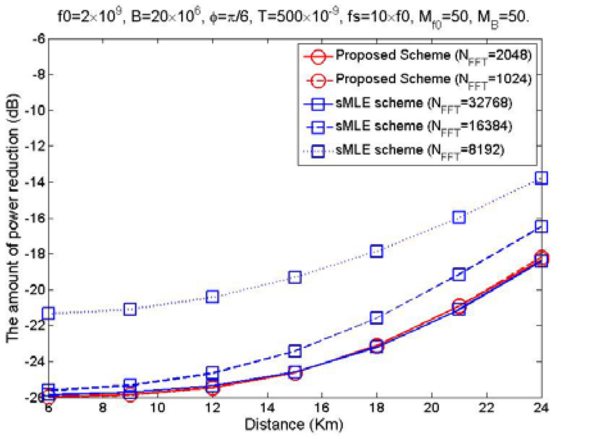

Fig. 2. Amount of power reduction by the countermeasure signals with reference to the original returned signal.

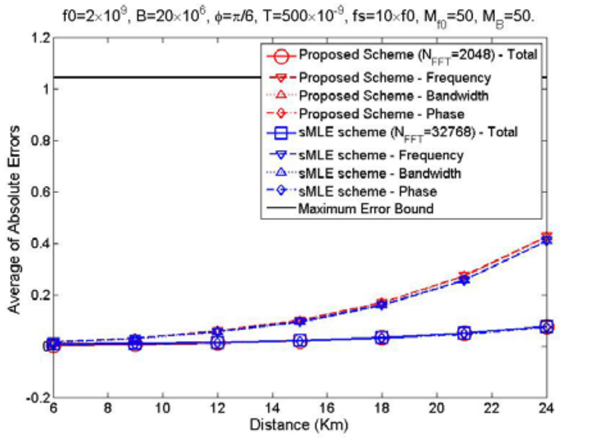

Fig. 3. Absolute value of total error and each parameter errors of the proposed scheme and the conventional sMLE scheme.

similar to that of the sMLE scheme using $N_{FFT} = 32,768$, corresponding precisely to the result in Fig. 2. In addition, E is similar to a_0^e in both cases, which supports the mathematical analysis.

Finally, the MSE performance of the proposed and the conventional sMLE schemes were examined. Figs. 4 and 5 show that the MSE performance of the bandwidth and the frequency estimation using the proposed scheme with the $N_{FFT} = 2,048$ approach to those of the MSE bounds, which are similar to those of the sMLE scheme with $N_{FFT} = 32,768$. In addition, even the MSE performance of the proposed scheme with only $N_{FFT} = 1,024$ was similar to that of the sMLE scheme with $N_{FFT} = 32,768$. In addition, when the number of the FFT sizes is below the minimum required number of FFT sizes, $N_{FFT} = 8,192$, the MSE performance of the sMLE scheme is deteriorated severely. Fig. 6 shows that the MSE performance of the estimated phase using the proposed schemes also shows the same trend as that of the proposed bandwidth and the frequency estimation schemes. Finally, Fig. 7 shows that

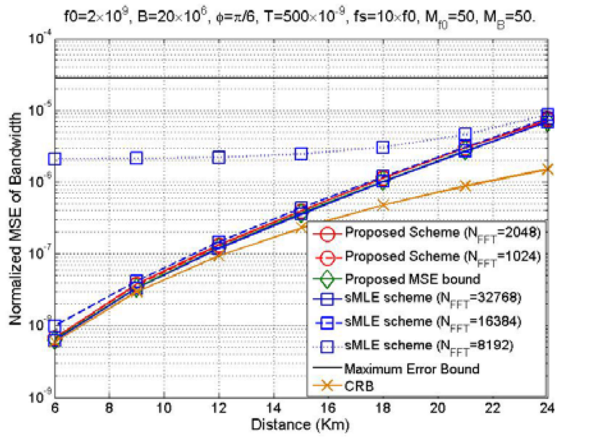

Fig. 4. Comparison of the bandwidth MSE performance between the proposed scheme and the conventional sMLE scheme.

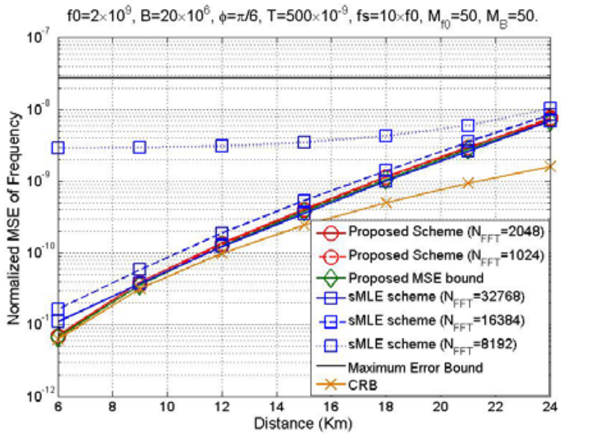

Fig. 5. Comparison of the frequency MSE performance between the proposed scheme and the conventional sMLE scheme.

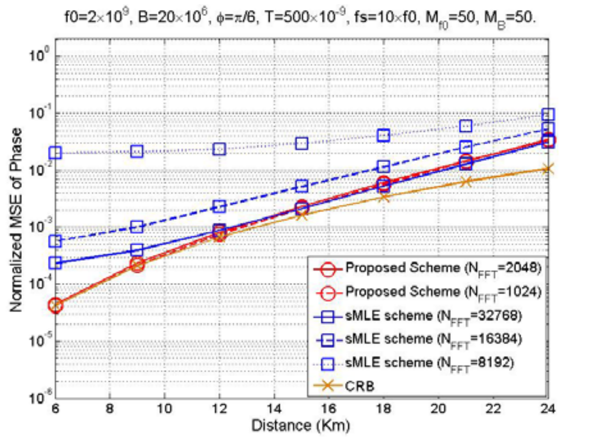

Fig. 6. Comparison of the phase MSE performance between the proposed scheme and the conventional sMLE scheme.

the proposed schemes and the conventional MLE scheme exhibit the same performance such that the effect of amplitude estimation error is negligible.

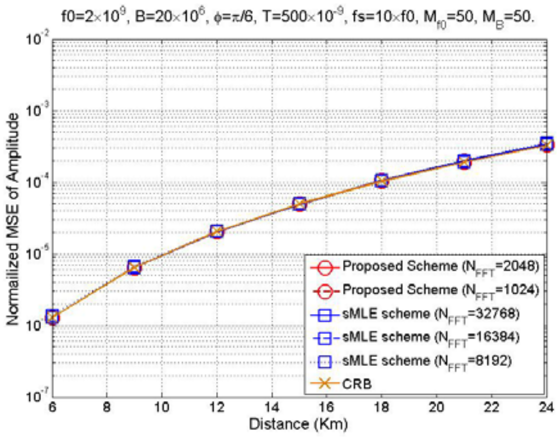

Fig. 7. Comparison of the amplitude MSE performance between the proposed scheme and the conventional sMLE scheme.

6. Conclusion

To realize an active cancelation technique, a highly-accurate and high-speed parameter estimation scheme for a linear-FM signal based on the sMLE scheme was considered. The proposed scheme, which employs a down-sampling technique to overcome the shortcomings of the sMLE scheme, enables a small FFT size to be applied while preserving the estimation accuracy. In the active cancelation technique, there are other parameters, such as the pulse width, pulse repetition frequency and the incident angle, which must be estimated simultaneously. Therefore, in-depth studies about the estimation of these parameters would be an interesting work in the future.

Acknowledgement

This work has been supported by the Low Observable Technology Research Center program of Defense Acquisition Program Administration and Agency for Defense Development

References

[1] Dimitris V. Dranidis, Airborne Stealth in a Nutshell-Part I (the Magazine of the Computer Harpoon Community), Accessed April 2014. Article (CrossRef Link)

[2] S. Peleg and B. Porat, "Linear FM signal parameter estimation from discrete-time observations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 27, pp. 607-616, Jul. 1991. Article (CrossRef Link)

[3] P. O'Shea, "Fast parameter estimation algorithms for linear FM signals," in *IEEE Int. Conf. Acoust., Speech and Signal Process.*, 1994. ICASSP-94, 19-22, April 1994. Article (CrossRef Link)

[4] G. Cornelia, M. Lucian, and R. Romulus, "Detection and estimation of linear FM signals," in *Int. Symp. Signals, Circuits, and Systems*, 2005, ISSCS 2005, 14-15, July 2005. Article (CrossRef Link)

[5] WANG Ling-huan, Ma Hong-guan, Li Zhao, AI Ming-shun, and ZHANG Xin-yu, "Parameter estimation of linear FM signal based on matching fourier transform," in *IEEE Int. Conf. Ind. Tech.*, 2008, ICIT 2008, 21-24, April 2008. Article (CrossRef Link)

[6] Zhe Yan, Jiao Zhang, Fei Fan, Shutao Che, and Hong Zhao, "Parameters extraction of linear FM signal using fractional autocorrelation," in *Int. Conf. Intelligent Control and Inform. Process.*, 2010, 13-15, Aug. 2010. Article (CrossRef Link)

[7] C. De Luigi and E. Moreau, "An iterative algorithm for estimation of linear frequency modulated signal parameters," *IEEE Signal Process. Lett.*, vol. 9, pp. 127-129, April. 2002. Article (CrossRef Link)

[8] P. M. Djuric and S. M. Kay, "Parameter estimation of chirp signals," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 38, pp. 2118-2126, Dec. 1990. Article (CrossRef Link)

[9] S. Peleg and B. Porat, "Estimation and classification of polynomialphase signals," *IEEE Trans. Inf. Theory*, vol. 37, pp. 422-430, Mar. 1991. Article (CrossRef Link)

[10] P. OShea, "A fast algorithm for estimating the parameters of a quadratic FM signal," *IEEE Trans. Signal Process.*, vol. 52, no. 22, pp. 385-392, Feb. 2004. Article (CrossRef Link)

[11] P.Wang, I. Djurovic, and J. Yang, "Generalized high-order phase function for parameter estimation of polynomial phase signal," *IEEE Trans. Signal Process.*, vol. 56, pp. 3023-3028, Jul. 2008. Article (CrossRef Link)

[12] Bassem R. Mahafza and Atef Z. Elsherbeni, *MATLAB Simulations for Radar Systems Design*. Chapman and Hall/CRC 2004.

[13] Louis L. Scharf, Statistical Signal Processing: *Detection, Estimation, and Time Series Analysis*. Addison-Wesley Publishing Company, Inc. 1991.

[14] E. Jacobsen and P. Kootsookos, "Fast, accurate frequency estimators," *IEEE Signal Processing Mag.*, vol. 24, pp. 123-125, May 2007. Article (CrossRef Link)

[15] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. 20, pp. 591-598, Sep. 1974. Article (CrossRef Link)

[16] Zhang Gang-bing, Liu Yu, Xu Jia-Jia, and Hu Guo-bing, "Frequency estimation based on discrete Fourier transform and least squares," in *Int. Conf. Wireless Commun. & Signal Process.*, 2009, WCSP 2009, 13-15, Nov. 2009. Article (CrossRef Link)

Seungkyu Choi received the B.S. and M.S. degrees in electrical and electronic engineering from Yonsei University, Seoul, Korea, in 2002 and 2009, respectively. He is currently working toward a Ph.D. degree in the Communication Signal Processing Laboratory, Yonsei University. His current research interests include statistical signal processing, MU-MIMO precoder design and energy-efficient resource allocation technique for massive MIMO system.

Chungyong Lee received his B.S. and M.S. degrees in electronic engineering from Yonsei University, Seoul, Korea, in 1987 and 1989, respectively, and a Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, U.S.A., in 1995. He was a Senior Engineer of Samsung Electronics Co., Ltd., Kiheung, Korea, from 1996 to 1997. In 1997, he joined the faculty of the School of Electrical and Electronic Engineering, Yonsei University, where he is currently a professor. His research interests include array signal processing and communication signal processing