

# Design and Implementation of Periodic Disturbance Compensators for Track Following Servo Systems

†

Jun Jeong

(Received December 17, 2013 ; Revised January 27, 2014 ; Accepted January 28, 2014)

**Key Words** : Periodic Disturbance( ), Compensator( ), Gain Determination( ), AFC( ), IMP( ), Track Following Servo( )

## ABSTRACT

Periodic disturbance compensators are widely used in track following servo systems. They are commonly designed and implemented by adaptive feedforward compensators or internal model based compensators. In track following servo systems, the gains of the compensators should be determined considering the change of the sensitivity transfer function and the implementation methods should be selected considering the system environment. This paper proposes a guide for determining gains of the periodic disturbance compensators. Various simulation and experimental results are presented to see the effect of gains. In addition, this paper introduces the various types of implementation methods and compares their merits and demerits.

## Nomenclature

	$x, r$ :
	$y$ :
$C_B$ :	1.
$C_P$ :	
$P$ :	
$S, S_B$ :	(AFC, adaptive feedforward cancellation)
$T$ :	(IMP, internal
$a, b$ :	model principle) 가
$g, \phi$ :	(1).
$u_p$ :	
$w$ :	(2) 가

† Corresponding Author ; Member, Department of Automation System, Dongyang Mirae University  
 E-mail : junjeong@dongyang.ac.kr  
 Tel : +82-2-2610-5242, Fax : +82-2-2610-1852

‡ Recommended by Editor Hyung-Jo Jung  
 © The Korean Society for Noise and Vibration Engineering

(3-5) AFC IMP가  
 ,  
 가 (3)

$$\begin{aligned}
 & A_u \sin(\phi) \\
 & (3) \quad (2) \quad \text{AFC} \\
 & \cos(\omega T k) \quad \sin(\omega T k) \\
 & A_u \cos(\phi) \quad A_u \sin(\phi) \\
 & \text{(adaptation law)} \\
 & (1) \\
 & u_{p,k} = A_u \cos(\omega T k + \phi) \\
 & = A_u \cos(\phi) \cos(\omega T k) - A_u \sin(\phi) \sin(\omega T k) \quad (3)
 \end{aligned}$$

2.2

IMP AFC  $g \phi$   
 가  
 IMP

Fig. 1

$$\begin{aligned}
 & (1) \\
 & g \quad \phi \quad \text{가} \quad g \\
 & \text{가} \quad \phi
 \end{aligned}$$

Fig. 1

$$\begin{aligned}
 & \text{IMP} \quad (1) \quad \text{AFC} \quad (2) \\
 & \phi \quad (4) \\
 & \phi = \angle \frac{P(z)}{1 + P(z)C_B(z)} \text{ at } \omega \quad (4)
 \end{aligned}$$

Fig. 1  $d_p$ 가  $y$

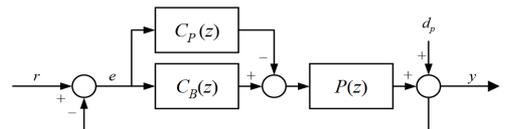


Fig. 1 Control block diagram containing a periodic disturbance compensator

2.

2.1

IMP AFC

(PDC, periodic disturbance compensator) (IMP) (AFC)

$$C_P(z) = \frac{U_P}{Y} = g \frac{\cos(\phi)z^2 - \cos(\omega T + \phi)z}{z^2 - 2\cos(\omega T)z + 1} \quad (1)$$

$$\begin{aligned}
 u_{p,k} &= \hat{a}_k \cos(\omega T k) + \hat{b}_k \sin(\omega T k) \\
 \hat{a}_k &= \hat{a}_{k-1} + g y_k \cos(\omega T k + \phi) \\
 \hat{b}_k &= \hat{b}_{k-1} + g y_k \sin(\omega T k + \phi)
 \end{aligned} \quad (2)$$

가  $2\pi/\omega$

$$\begin{aligned}
 & (3) \quad \text{가} \\
 & (1) \quad (11)
 \end{aligned}$$

IMP  
 $\cos(\omega T k) \quad \sin(\omega T k)$   
 (estimation)  $A_u \cos(\phi)$

(5)

$$\frac{S(z)}{S_B(z)} = \frac{1 + C_B(z)P(z)}{1 + (C_B(z) - C_P(z))P(z)} \quad (5)$$

Fig. 2 Fig. 3

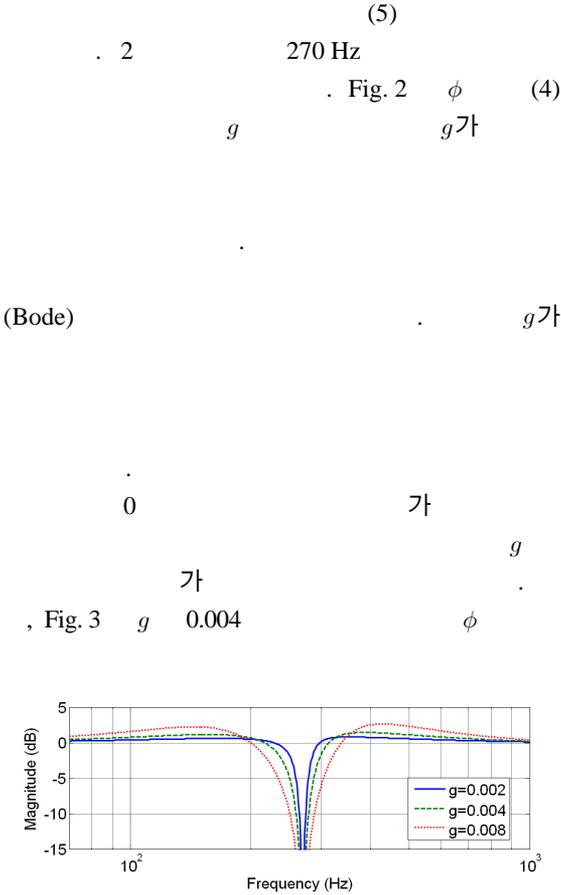


Fig. 2 The effect of gain  $g$  on the sensitivity transfer function

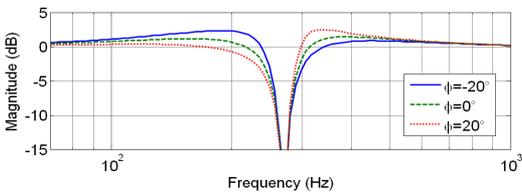


Fig. 3 The effect of gain  $\phi$  on the sensitivity transfer function

2.3

HDD 195,000 TPI(track per inch)  
 130  $\mu$ m  
 5,400 RPM(round per minute)  
 90Hz 160  
 14.4kHz  
 69.4 $\mu$

Fig. 4

HDD  
 30  
 HDD  
 HDD  
 Fig. 4 (a)  
 Fig. 5 90 Hz  
 Fig. 4 (b) (f)  
 90 Hz(1x), 180 Hz(2x), 270 Hz(3x), 360 Hz(4x),  
 450 Hz(5x)  
 가

가  
Fig. 6

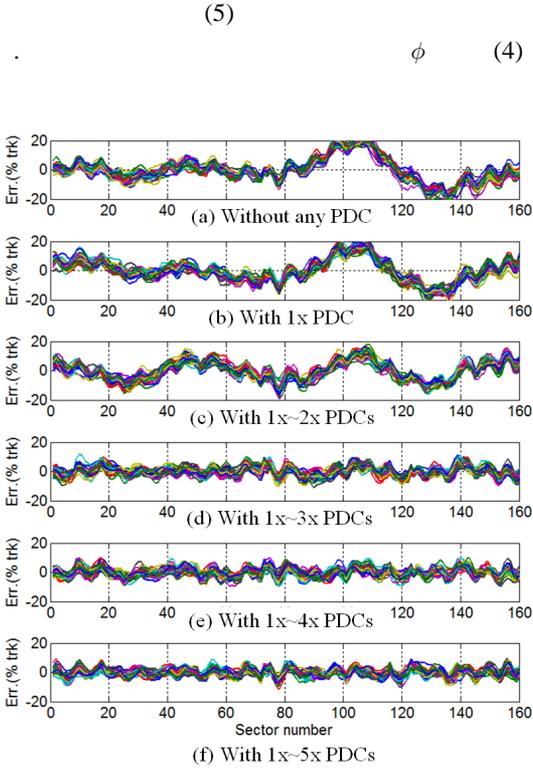


Fig. 4 Time domain track following servo results

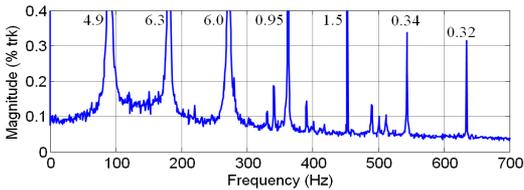


Fig. 5 FFT of track following errors without periodic disturbance compensator

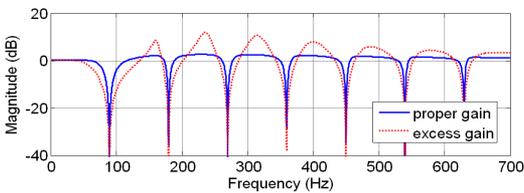


Fig. 6 Variation of the sensitivity transfer function due to the seven periodic disturbance compensators

가  
g  
g 90 Hz 2.91e-3, 0.73e-3,  
0.75e-3, 0.79e-3, 0.78e-3, 0.75e-3, 0.70e-3  
φ 45, 7.4, -21, -44, -61, -75, -88  
g

가 3 dB  
10 dB  
Fig. 7  
가  
(c) Fig. 6

Fig. 8 Fig. 7 (b) (c)  
160 Hz  
90 Hz φ  
90 Hz  
가  
0~700 Hz  
2.6 %

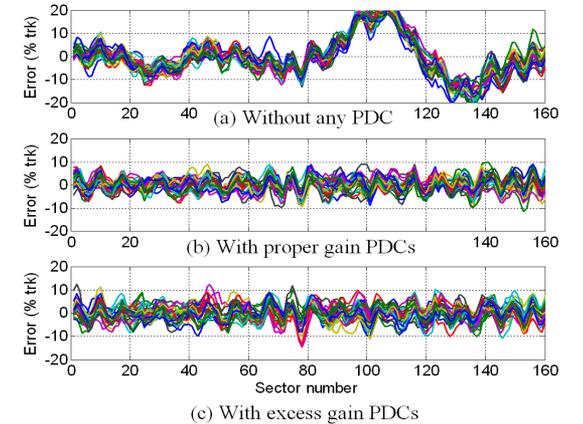


Fig. 7 Time domain track following servo results

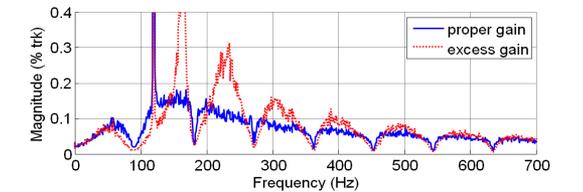


Fig. 8 FFT of track following errors with seven periodic disturbance compensators

3.2 % 가

$$x_k = A \cos(wTk + \phi) \tag{8}$$

(9)

(10)

3.

$$\begin{bmatrix} x_k \\ r_k \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{11}$$

(11)

3.1

(firmware)

. AFC (2)

IMP (1)

(6)

$$\begin{aligned} r_1(k) &\equiv A \sin(wT) \cos(wTk + \phi) \\ r_2(k) &\equiv A \sin(wT) \sin(wTk + \phi) \end{aligned} \tag{9}$$

$$\begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} = \frac{1}{\sin(wT)} \begin{bmatrix} 1 & 0 \\ \cos(wT) & -\sin(wT) \end{bmatrix} \begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} \tag{10}$$

$$u_{p,k} = g \cos(\phi) y_k - g \cos(wT + \phi) y_{k-1} + 2 \cos(wT) u_{p,k-1} - u_{p,k-2} \tag{6}$$

$$\begin{bmatrix} r_{1,k+1} \\ r_{2,k+1} \end{bmatrix} = \begin{bmatrix} \cos(wT) & -\sin(wT) \\ \sin(wT) & \cos(wT) \end{bmatrix} \begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y_k \tag{11}$$

x y (1) 가

(7)

$$u_{p,k} = g [\sin(\phi) - \cos(\phi)] \begin{bmatrix} r_{1,k+1} \\ r_{2,k+1} \end{bmatrix} \tag{12}$$

가 (12)

$$\begin{bmatrix} x_{k+1} \\ x_{k+2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \cos(wT) \end{bmatrix} \begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_k \tag{7}$$

(13)

(3)

$$u_{p,k} = g [-\cos(wT + \phi) \cos(\phi)] \begin{bmatrix} x_{k+1} \\ x_{k+2} \end{bmatrix} \tag{7}$$

$$\begin{aligned} r_1(k) &\equiv g A \sin(wT) \cos(wTk) \\ r_2(k) &\equiv g A \sin(wT) \sin(wTk) \end{aligned} \tag{12}$$

(7)

$x_k$

가

**Table 1** Comparisons of implementation methods

Eqn.	Implementation Form	Merits and Demerits	Floating point op. time (MC)		Fixed point op. time (MC)	Min. calc. ISR to $u_k$
			w/ FPU	w/o FPU		
(2)	$a_k = a_{k-1} + g y_k \text{costbl}[n+c]$ $b_k = b_{k-1} + g y_k \text{sintbl}[n+c]$ $u_k = a_k \text{costbl}[n] + b_k \text{sintbl}[n]$	M: Feedback off and on characteristics D1: FPU floating point operation time D2: Necessity of sine, cosine table	728	3945	1349	Multi.: 4 Add.: 3
(6)	$u_k = b_1 y_k + b_2 y_{k-1} - a_1 u_{k-1} - a_2 u_{k-2}$	M: None D: Truncation error propagation during feedback off	503	3788	1369	Multi.: 1 Add.: 1
(7)	$x_k = a_1 x_{k-1} - x_{k-2} + y_k$ $u_k = b_1 x_k + b_2 x_{k-1}$	M: Short operation time D: Overflow of state variables and truncation error propagation during feedback off	368	3285	1041	Multi.: 1 Add.: 2
(11)	$r_{1,k} = a_1 r_{1,k-1} - a_2 r_{2,k-1} - y_k$ $r_{2,k} = a_2 r_{1,k-1} + a_1 r_{2,k-1} - y_k$ $u_k = b_1 r_{1,k} + b_2 r_{2,k}$	M: Sine, cosine estimation of disturbance D: Overflow of state variables	470	5031	1385	Multi.: 1 Add.: 2
(14)	$r_{1,k} = a_1 r_{1,k-1} - a_2 r_{2,k-1} + b_1 y_k$ $r_{2,k} = a_2 r_{1,k-1} + a_1 r_{2,k-1} + b_2 y_k$ $u_k = r_{1,k}$	M: Sine, cosine estimation of control input D: Operation time	470	5271	1456	Multi.: 1 Add.: 1

$$\begin{bmatrix} x_k \\ x_{k+1} \end{bmatrix} = \frac{1}{g \sin(\omega T)} \begin{bmatrix} \sin(\phi) & \cos(\phi) \\ \cos(\omega T + \phi) & -\sin(\omega T + \phi) \end{bmatrix} \begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} r_{1,k+1} \\ r_{2,k+1} \end{bmatrix} = \begin{bmatrix} \cos(\omega T) & -\sin(\omega T) \\ \sin(\omega T) & \cos(\omega T) \end{bmatrix} \begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} + \begin{bmatrix} g \cos(\phi) \\ -g \sin(\phi) \end{bmatrix} y_k \quad (14)$$

$$u_{p,k} = [1 \ 0] \begin{bmatrix} r_{1,k+1} \\ r_{2,k+1} \end{bmatrix} = r_{1,k+1}$$

(14) FPU  
가  
ISR(in-  
terrupt service routine)

Table 1

가  
, (1) (15)

3.2

$$\frac{U_P}{Yz} = g \frac{\cos(\phi)z - \cos(\omega T + \phi)}{z^2 - 2\cos(\omega T)z + 1} \quad (15)$$

가 HDD

가

가

가

가

(2), (6), (7), (11), (14)  
가

가

가

Table 1 10

가

가 (2)

C MC(ma- (3)

chine cycle)

32 y 0

(sine),

(cosine)

가

FPU(floating point unit)

가 STMicroelectronics ARM Cortex

y 0

STM32F407, IAR Systems 가

가

Embedded Workbench (7) FPU

(11) (14)

n

가 (2)

FPU 가 (16)

$$\begin{bmatrix} r_{1,k+1} \\ r_{2,k+1} \end{bmatrix} = \begin{bmatrix} \cos(nwT) & -\sin(nwT) \\ \sin(nwT) & \cos(nwT) \end{bmatrix} \begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} \quad (16)$$

(7) (11)

$g$

가

$g$

,  $g$

$g$

(overflow)가

, (11)

(14)

4.

가

가

$g$

$\phi$

가

가

, AFC

2012

References

(1) Bodson, M., Sacks, A. and Khosla, P., 1994, Harmonic Generation in Adaptive Feedforward Cancellation Schemes, IEEE Transactions on Automatic Control, Vol. 39, No. 9, pp. 1939~1944.

(2) Shin, H. C., Park, S. W., Park, T. W, Yang, H. S. and Park, Y. P., 2004, Compensation of the Repeatable Run Out Using Repetitive Controller in HDD, Transactions of the Korean Society for Noise and Vibration Engineering, Vol. 14, No. 2, pp. 136~143.

(3) Boo, C. H., Kang, C. I and Kim, H. C., 2004, Fast Compensator of Periodic Disturbance in Disk Drives, Journal of Control, Automation, and Systems Engineering, Vol. 10, No. 2, pp.153~163.

(4) Jeong, J., Kim, J. G., Park, N. C., Yang, H. S. and Park, Y. P., 2007, Disk Vibration and Eccentricity Compensation of Near Field Recoding Systems Base on the Internal Model Principle, Transactions of the Korean Society for Noise and Vibration Engineering, Vol. 17, No. 6, pp. 539~546.

(5) Park, C. H., Son, Y. S., Ham, S. Y., Kim, B. I. and Yun, D. W., Precise Speed Control of Direct Drive PMSM for the Cogging Torque Measurement System, Journal of the Korean Society for Precision Engineering, Vol. 26, No. 5, pp.79~86



**Jun Jeong** received the B.S., M.S. and Ph.D. degrees in Dept. of Mechanical Engineering from Yonsei university in 1994, 1996 and 2001 respectively. He is currently working in Dongyang Mirae University as a professor.

His research interests are HDD servo and active vibration isolation control.