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Krylov-Schur 순환법을 이용한 다양한 2차원 구조의 도파관들에 관한 연구

(A Study on The eigen-properties on Varied Structural 2-Dim. Waveguides by Krylov-Schur Iteration Method)

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요 약

다양한 2차원 구조의 도파관들에 Krylov-Schur 순환법을 적용하였다. 이들의 고유특성들을 기술하는 방정식들은 삼각형 요소의 변-접선벡터에 기반을 둔 FEM으로 구성하였다. 고유-값들과 고유-모드들은 이들에 대한 Schur 행렬의 대각 성분들과 변환 행렬들로 부터 구하였다. 결과로써 이들 고유-값과 고유-모드 쌍들을 시각적으로 묘사하였다.

Abstract

Krylov-Schur iteration method has been applied to the 2-Dim. waveguides of the varied geometrical structure. The eigen-equations for them have been constructed from FEM based on the tangential edge vectors of triangular elements. The eigen-values and their modes have been determined from the diagonal components of the Schur matrices and its transforming matrices. The eigen-pairs as the results have been revealed visually in the schematic representations.

Keywords : eigen-pair, Krylov-Schur, FEM, Arnoldi decomposition, QR algorithm, unitary transform.

I. INTRODUCTION

Krylov-Schur iteration method has been one of the most important and actively developing algorithms for calculating the eigen-problems[1~2]. It has revealed the satisfying results for any physical experiments. It has been recognized that this algorithm would be indispensable tool to understand the physical properties of the propagating electromagnetic wave in any waveguide.

However, in accompanying with development of state of the art technology in the varied applying fields, there has been demanded persistingly reviewing and assessing the numerical algorithm to confirm its availability. Especially, such like the communication system, the structural complexity of the eigen-system would not be so simply constructed. The eigen-matric equation must be made more precisely and largely for describing their physical properties. But, as the numerical dimension is increased, it may not be neglected that the round off error would be amplified to corrupt the entire process as going with the calculation.

Previously, we have studied on the eigen-properties of a 2-Dim.(Dimensional) waveguides of the square form using Krylov-Schur iteration method[3]. The spectra of TM(Transverse Magnetic) and TE(Transverse Electric) eigen-modes and

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eigen-values have been revealed visually as the results. In the process of the calculation, it has been identified even more that this algorithm has been carried out robustly and drawn the eigen-pairs confidently comparing to any other iteration methods. From these reasons, it could be recognized ones again, the prominent ability of Krylov-Schur algorithm in calculating the large scale and non-symmetric eigen-problems.

To meet the demand of the periodic tendency, Krylov-Schur algorithm the same as previously studying, has been applied to three waveguides characterized with each different geometrical structure. Each structures have been the rectangular, circular and co-axial shapes. These would be more general structures accepted in the applying fields. The eigen-equations were constructed basing on FEM(Finite Element method). The mesh elements were simple triangles and the shape functions were constructed with constant tangential edge vectors. In this study, it has been aimed for certifying the availability of Krylov-Schur algorithm by revealing the eigen-properties of TM and TE modes for each waveguides of varied geometrical structures. As the results, the spectra for each eigen-pairs have been visualized with the schematic representations as like the previous study.

Ⅱ. FINITE ELEMENT FORMULATION

For the homogeneous waveguides, propagating TM and TE-modes would be described by the vector Helmholtz equation of dual form

$$\overrightarrow{\nabla_t}\times\left(\frac{1}{\nu}\overrightarrow{\nabla}_t\times\overrightarrow{F_t}\right)-k^2\zeta\overrightarrow{F_t}=0 \quad (1)$$

where $k=\omega\sqrt{\epsilon_0\mu_0}$ is the propagation wave number and, for the TE mode $\overrightarrow{F_t}=\overrightarrow{E_t}$(transverse electric field strength), $\nu=\mu_r$(relative permeability μ/μ_0), $\zeta=\epsilon_r$ (relative permittivity ϵ/ϵ_0) and, for the TM mode $\overrightarrow{F}_t=\overrightarrow{H_t}$(transverse magnetic field strength), $\nu=\epsilon_r$, $\zeta=\mu_r$[4]. The eigen-equations have been obtained from FEM. It has been carried out using the Galerkin method of weighted residuals to construct the linear equation. The shape function has been constructed with the constant tangential edge vectors of the triangular element. Triangle barycentre coordinate are related to the vertex of triangle

$$\begin{pmatrix}\mathcal{L}_1\\ \mathcal{L}_2\\ \mathcal{L}_3\end{pmatrix}=\frac{1}{2A}\begin{pmatrix}a_1\,b_1\,c_1\\ a_2\,b_2\,c_2\\ a_3\,b_3\,c_3\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix} \quad (2)$$

where

$$a_i=x_jy_k-x_ky_j,\, b_i=y_j-y_k,\, c_j=x_k-x_j$$

(i,j,k are cyclic ordered vertex indexes) and A is an area of the triangular element. With these coordinates, the tangential edge vectors $\overrightarrow{W}_{ti}$ are made

$$\overrightarrow{W}_{ti}=l_i(\mathcal{L}_j\overrightarrow{\nabla}_t\mathcal{L}_k-\mathcal{L}_k\overrightarrow{\nabla}_t\mathcal{L}_j) \quad (3)$$

where l_i is a length of the edge opposite to the vertex i , and i, j, k are indexes of the barycentric coordinates in cyclic ordering[5]. The field strength in the single triangular element can be described with these tangential edge vectors,

$$\overrightarrow{F}_t=\sum_{i=1}^{i=3}e_{ti}\overrightarrow{W}_{ti} \quad (4)$$

where e_{ti} are coefficients to be found in the diagonalizing processes of the eigen-equation. Using this shape function the Helmholtz equation for a triangular element can be made as following

$$[S_{el}]\,\{e_t\}=k^2[T_{el}]\,\{e_t\} \quad (5)$$

where

$$[S_{el}]=\frac{1}{\nu}\iint_A(\overrightarrow{\nabla}_t\times\overrightarrow{W}_{tn})\bullet(\overrightarrow{\nabla_t}\times\overrightarrow{W}_{tn})ds$$

and

$$[T_{el}]=\zeta\iint_A(\overrightarrow{W}_{tm}\bullet\overrightarrow{W}_{tn})ds.$$

Subsequently, these element matrix equations can be assembled over all triangular meshes of the waveguides to obtain a global eigen-matrix equation.

$$[S]\{e_t\} = k^2[T]\{e_t\} \tag{6}$$

In this equation, $[S]$ and $[T]$ are a $n \times n$ square matrices and $\{e_t\}$ is a $n \times 1$ column matrix where n is a total number of the edges composing the mesh of the waveguides[6]. When obtaining the eigen-pairs of the TE mode, the tangential components of the electric fields must satisfy the Dirichlet boundary condition. This is accomplished in the course of implementing the programs by cancelling the edge components of the triangular elements which coincide with the wall of the waveguides.

Ⅲ. KRYLOV-SCHUR ITERATION METHOD

As mentioned in the previous study, it has been well known that the Krylov- Schur iteration method is the one of most reliable technique for finding the prominent eigen-pairs[7]. The method would be more efficiently implemented in finding specific eigen-pairs by performing the shift-invert strategy previously as following

$$\frac{1}{k_c^2 - \sigma}\{e_t\} = \frac{[T]}{[S] - \sigma[T]}\{e_t\} = [M]\{e_t\} \tag{7}$$

The sparsity and symmetry of the eigen-equation would be lost, but by this strategy the convergent rate is more promoted around at the specific value σ. Subsequently, Krylov-Schur iteration method is performed on this square matrix. It could be definitely summarized as in the Fig.1. Arnoldi decomposition compress the matrix $[M]$ into the Hessenberg matrix of dimension 20×20 by the orthogonal matrix $[Ar]$ of the dimension $n \times 20$ [8].

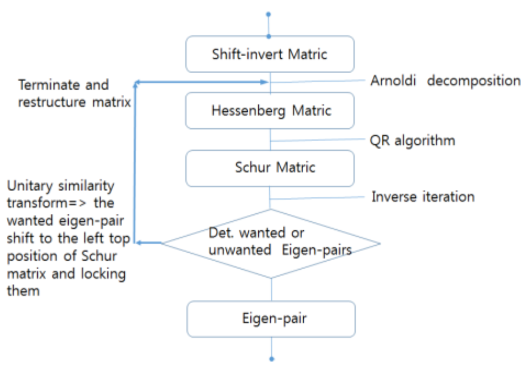

그림 1. Krylov-Schur 순환법의 계략도
Fig. 1. Block diagram for Krylov-Schur algorithm.

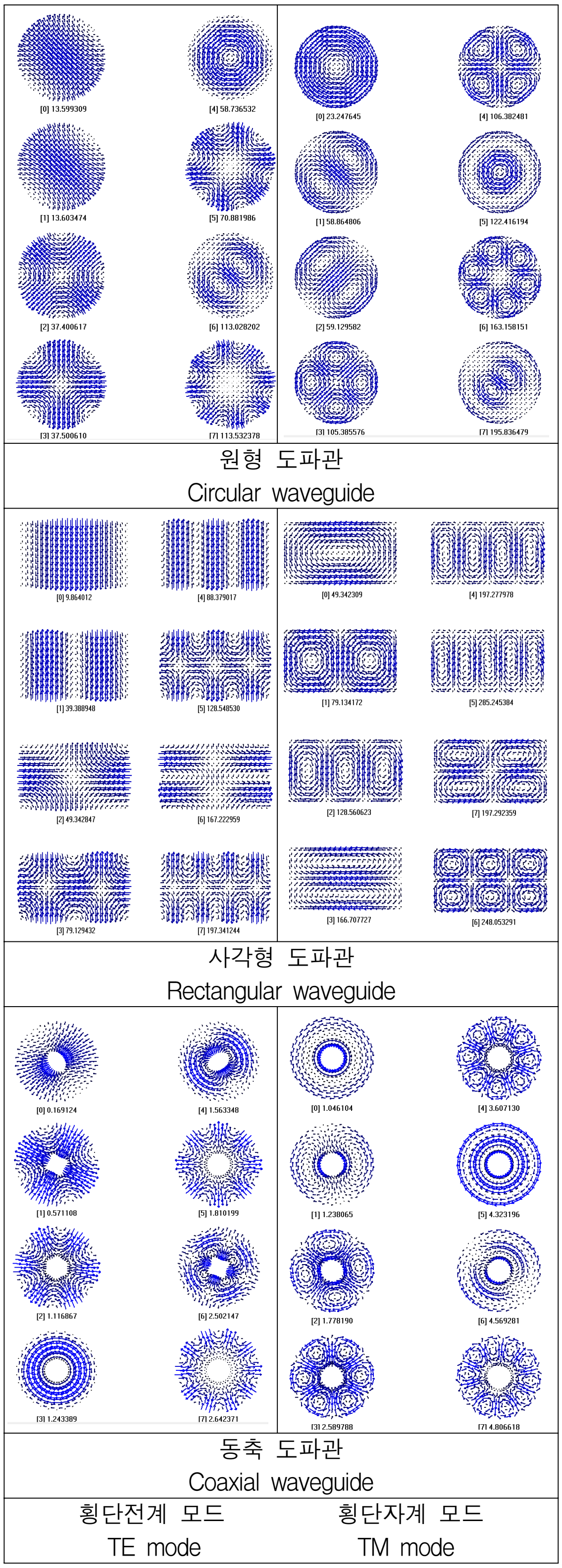

그림 2. 다양한 구조의 고유 쌍
Fig. 2. Eigen-pairs of varied waveguides.

QR algorithm with the 20×20 matrix $[Qr]$ is applied to the compressed square matrix resulting into the upper triangular Schur matrix[9]. To obtain the wanted eigen-value, inverse iteration method with $[Ir]$ is operated to this matrix[10]. The wanted eigen-values λ_w or not wanted is determined by the tolerant value Tol and the relation

$$\begin{aligned} &\| [M]([V_m][\phi_w]) - \lambda([V_m][\phi_w]) \|^2 \\ &\quad = \left|\left\{b_{m+1}^T\right\}\{\phi_w\}\right| \\ &\quad\quad \le \max\left\{u \| T_m \|_F, Tol\times\lambda_w\right\} \end{aligned} \tag{8}$$

Where u and $\| * \|_F$ are the unit round off and the Frobenius norm respectively. The eigen-values would be located randomly in the diagonal position of the Schur matrix. The unitary similarity transform is carried out to shift these components to left upper position by the matrix $[Tr]$[11]. The eigen-modes corresponding to these eigen-values are resulted from the above transforming matrices by multiplying them sequently

$$[e_t] = [Ar][Qr][Is][Tr] \tag{9}$$

After obtaining these pairs, they are locked and made no longer participate in the subsequent calculation for other remaindering eigen-pairs. The iterating calculations are continued subsequently restructuring the Schur matrix which is smaller than the original one by the locked components. The initial Arnoldi vector is assumed to be the last column vector of the last decomposition matrix.

표 1. 고유 - 모드에 따른 전파상수
Table 1. Propagation constants of the eigen-modes.

	Circular(kr)		Rect.(kr_w)		Coaxi.(Kr_o)	
Spect. Numb.	TE	TM	TE	TM	TE	TM
0	3.686	4.821	3.140	7.024	0.411	1.023
1	3.688	7.672	6.276	8.900	0.756	1.113
2	4.171	7.690	7.024	11.338	1.057	1.333
3	6.124	10.314	8.895	12.911	1.060	1.609
4	7.664	10.314	9.401	10.357	1.250	1.899
5	8.419	11.064	11.338	16.890	1.345	2.079
6	10.631	12,775	12.738	14.046	1.582	2.138
7	10.660	13.994	14.048	15.750	1,625	1.291

Ⅳ. RESULTS AND DISCUSSION

Krylov-Schur iteration method have been carried out as the previous study. The elementary meshes have been the same triangular forms without differentiating for each geometrical characteristics of waveguides. The vector shape function have been constructed basing on the constant tangential edges as previous study. The edge numbers have been slightly differed for each other depending on their geometrical structures. The numbers have been 1008, 759 and 570 for circular, rectangular and coaxial waveguides respectively. These numbers have not caused any troubles in implementing the inverse matrix by LU decomposition even using personal computer. The tolerant values have not been the same for each waveguides but characterized by the minimum value for them about $Tol \sim 10^{-4}$. The iterative loops have been the same for all waveguides and made sure to execute robustly for them. It has been assumed that the lateral boundaries were coated with perfect conductor. The spaces occupied by the waveguides were assumed to be linear and homogeneous. So, the calculations have not been worrying about any leakage and anisotropic field variation. The resulting eigen-pairs for each waveguides are revealed schematically in the Fig.2. The propagation constants $\sqrt{(k_i^2)}$ have been calculated from converting each eigen-values into $k_i^2 = \frac{1}{\lambda_i} + \sigma$ These values have been plotted into the table 1 where r, r_w, r_o are circular waveguide's radius, rectangular waveguides width and coaxial waveguide's outer radius respectively all having values '1'. Comparing to the results by C. J. Reddy et al.,[4] it could be confirmed that these eigen-pairs represent the eigen-properties of the varied waveguides very well.

Ⅴ. conclusion

The Krylov-Schur iteration method has been applied to the 2-Dim. waveguides of varied geometrical structures. The eigen-pairs satisfying the convergent condition have been obtained. It would be certified that these spectra reveal the characteristics of the eigen-properties of the waveguides. So, it was successfully confirmed that Krylov-Schur algorithm could be applied to varied physical structures.

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