J Electr Eng Technol Vol. 9, No. 2: 478-488, 2014
http://dx.doi.org/10.5370/JEET.2014.9.2.478

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

Minimization of Torque-Ripple in Switched Reluctance Motors Over Wide Speed Range

Milad Dowlatshahi†, Seyed Morteza Saghaiannejad*, Jin-Woo Ahn and Mehdi Moallem***

Abstract – Torque pulsation mechanism and highly nonlinear magnetic characterization of switched reluctance motors(SRM) lead to unfavorable torque ripple and limit the variety of applications in industry. In this paper, a modification method proposed for torque ripple minimization of SRM based on conventional torque sharing functions(TSF) to improve maximum speed of torque ripple-free operation considering converter limitations. Due to increasing phase inductance in outgoing phase during the commutation region, reference current tracking can be deteriorated especially when the speed increased. Moreover, phase torque production in incoming phase may not be reached to the reference value near the turn-on angle in which the incremental inductance would be dramatically decreased. Torque error for outgoing phase can cause increasing the resultant motor torque while it would be negative for incoming phase and yields reducing the motor torque. In this paper, a modification method is proposed in which phase torque tracking error for each phase under the commutation added to the other phase so that the resultant torque remained in constant level. This yields to extend constant torque region and reduce peak phase current when the speed increased. Simulation and experimental results for four phase 4 KW, 8/6 SRM validate the effectiveness of the proposed scheme.

Keywords: Torque ripple minimization, Switched reluctance motor, Torque sharing functions

1. Introduction

Switched reluctance motor (SRM) is a double salient motor with no winding or permanent magnet on rotor. It has rugged and fault tolerant structure with the most robust and reliable constructions [1]. Due to the simple structure, low manufacturing and repairing costs as well as ability to operate in wide range of temperature, SRM has been taking into accounts more recently. One of the most drawbacks of switched reluctance motor operation is high torque ripple compared than other ac type machines [2]. Torque pulsation mechanism and highly saturated nature of magnetic characterization can cause undesirable torque ripples and deteriorate drive performance especially in high precise industrial applications. In order to eliminate torque ripples many investigations have been undertaken recently and many torque control techniques introduced [3-13]. Even though design approaches were taking into consideration in [3, 4] to produce constant torque and minimizing torque ripples, control techniques have been more considered to control the motor torque in conventional switched reluctance motors. One of the most convenient methods is to coordinate incoming and outgoing phase torques so that the resultant torque remains constant during the commutation between phases. Torque Sharing Function(TSF) is a well-known torque control alternative in which reference torques for individual phases defined assuming the total torque stays at constant level. Direct instantaneous torque control (DITC) has also the simple torque control concept. However, implementation of switching rules is complicated. Some of the conventional TSFs such as linear, cosine, cubic and exponential types reported in literature [5]. One of the most drawbacks of TSFs method in industrial applications is the limitation of maximum speed with acceptable torque ripples. Except retaining the motor torque at constant level, effective and peak phase currents, power loss and voltage requirements during the commutation would also be some possible secondary objective functions to make an optimal TSF. [6] introduced θ_c^i , the rotor position where two adjacent phases produce the same torque at the same phase current and demonstrated that balanced commutation can guarantees the higher efficiency and reduced power loss during the commutation area. However, minimum required commutation interval depends on rotor speed. In other words, increasing the operating speed would cause to extending the minimum required commutation area. [5] defined a function so that the center of commutation varies from θ_c^i to θ_c^λ based on motor speed to extend the commutation area and satisfy the voltage limitations. [7, 8]

† Corresponding Author: Dept. of Electrical and Computer Engineering, Isfahan University of Technology, Iran. (Dolatshahi@ec.iut.ac.ir)
* Dept. of Electrical and Computer Engineering, Isfahan University of Technology, Iran. (saghaian@cc.iut.ac.ir, moallem@cc.iut.ac.ir)
** Dept. of Mechatronics Engineering, Kyungsung University, Korea. (jwahn@ks.ac.kr)
Received: June 3, 2013; Accepted: September 16, 2013

proposed some pre-calculated TSFs in which copper loss minimization were realized and converter voltage saturation were taken into account. However, these methods require complicated offline numerical manipulations. vujicic [9], proposed a family of TSFs to extend the torque ripple-free speed range. However, the method was based on motor parameters and efficiency improvement can't be guaranteed especially when the speed increased. Negative torque compensation using a modified TSF also presented by [10], in which reference phase torque in incoming phase regulated to compensate the negative torque rises from the tail of outgoing phase current. However, receding turn-off angle can eliminate negative torque especially in mid-speed range of operation. In this paper, a simple and convenient method is introduced to modify the conventional TSFs in order to minimize torque ripple, improve the efficiency and enhance torque-speed characteristics. Phase torque tracking error in incoming phase conduction, compensated by outgoing phase torque as well as excess of phase torque in outgoing phase caused by converter voltage saturation and limitation decreased the reference torque of incoming phase. Efficiency improvement, feasibility and effectiveness of the proposed scheme will be demonstrated by simulation and experiment results.

2. Conventional Torque Sharing Functions

2.1 Torque production principles in SRM

Conventional switched reluctance motor has double salient structure including concentrated windings on stator poles. Torque production principle in this type of motor is based on tending to minimize magnetic reluctance for flux linkage paths which is some different from other types of ac motors. Energy conversion from magnetic to mechanical type has strong relationship with rotor position and phase current in SRM. The produced torque of each phase due to flowing of i_k at rotor position of θ can be calculated from Eq. (1):

$$T_{e,k} = \frac{\partial \int \lambda_k\left(i_k,\theta\right) di_k}{\partial \theta} \qquad k = 1,2,.....,m \tag{1}$$

where m is the number of phases. Due to highly nonlinear and saturated nature of magnetic characterization in SRM expressing a distinct relationship between phase flux, current and rotor position is substantial. The flux linkage of each phase can be represented by the following equation based on the corresponding phase current and rotor position:

$$\lambda_k\left(i_k,\theta\right) = \lambda_s\left(1 - e^{-i_k \cdot f_k(\theta)}\right) \tag{2}$$

Where λ_s is the saturated flux when rotor and stator poles are aligned properly and $f_k\left(\theta\right)$ can be determined by (3):

$$\begin{aligned} f_k\left(\theta\right) &= a_0 + a_1 \cos\left(N_r\left(\theta - 2\pi\left(k-1\right)/N_s\right)\right) + ... \\ &... + a_n \cos\left(n.N_r\left(\theta - 2\pi\left(k-1\right)/N_s\right)\right) \end{aligned} \tag{3}$$

In the above equation N_r and N_s are the number of rotor and stator poles, respectively. Considering (2), (3) and integrating (1) the phase torque can be obtained by the following equation:

$$T_{e,k}\left(i_k,\theta\right) = \frac{\lambda_s}{f_k^2\left(\theta\right)} \cdot \frac{\partial f_k\left(\theta\right)}{\partial \theta}\left[1 - \left(1 + i_k . f_k\left(\theta\right)\right) e^{-i_k \cdot f_k(\theta)}\right] \tag{4}$$

As it may seen from (4), considering nonlinear and saturation effects in magnetic characterization of an SRM can cause to complex calculation for phase torques. Hence, in many references, (5) is used to obtain phase torque approximately which comes from linear relationship between phase flux and current shown by (6).

$$T_{e,k}\left(i_k,\theta\right) = \frac{1}{2}\frac{\partial L_k\left(\theta\right)}{\partial \theta} . i_k^2 \tag{5}$$

$$\lambda_k\left(i_k,\theta\right) = L_k\left(\theta\right) . i_k \tag{6}$$

$$U_k = R.i_k + \frac{d\lambda_k\left(i_k,\theta\right)}{dt} \tag{7}$$

Considering (5), positive phase torque can be produced when the slope of inductance profile is positive. Moreover, current flow direction has no influence on the sign of the resultant torque. Fig. 1 illustrates the principle operation of switched reluctance machine in motoring and regenerating modes.

Phase current starts to be excited during the positive slope of inductance profile and it should be damped before

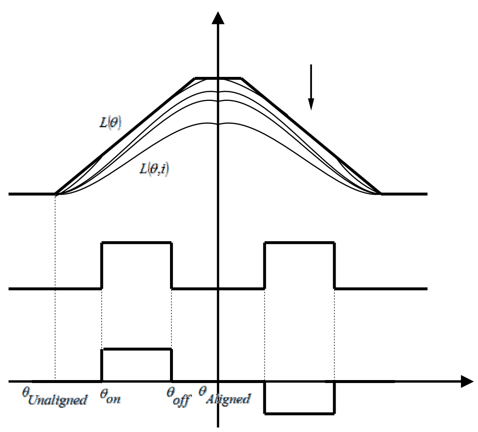


Fig. 1. Linearized and nonlinear inductance, phase current and produced torque profile for illustrating the motoring and generating operation modes of SR machine

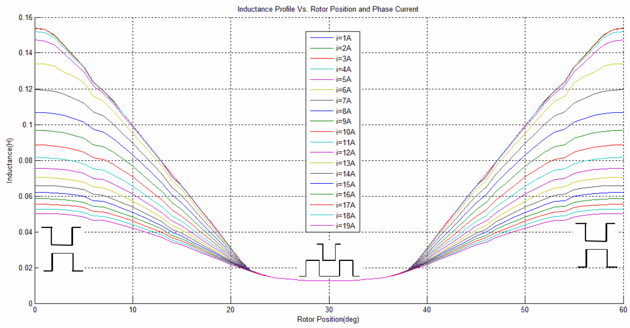


Fig. 2. Measured inductance profile for a 4kW, 8/6 SRM versus rotor position for different phase currents using voltage integration method

entering to the negative torque generating area for operating in motoring condition. Fig. 2 shows the measured inductance profile for a 4KW, 8/6 SRM which the detail nominal specifications listed in appendix. The position where the stator and rotor pole located so that the corresponding phase flux path has the maximum reluctance is named unaligned position. In this condition, not only phase inductance has the lowest value but also the slope of inductance in its neighboring would be zero. In other words, exciting the phase in which the rotor position is near unaligned position would cause to increasing phase current strongly, while phase torque production can be negligible. Similarly, the status where the stator and rotor poles are faced together is aligned position in which phase inductance has the maximum value and torque production capability is inconsiderable.

2.2 Torque sharing functions (TSF)

Simple concept as well as feasibility to implement in torque control and ripple minimization of almost every kinds of SRM caused TSF to be considered dramatically in both research and industrial applications. In this method torque interchanging between two adjacent phases is done in such a way that the resultant motor torque remains in constant reference value. The total produced motor torque is obtained from the summation of phase torques. Generally, the conduction period of each phase is divided into two intervals. One is single phase conduction and the other is two phase or commutation between adjacent phases. In TSF method, the reference torque of incoming phase considered as a specific function so that the resultant

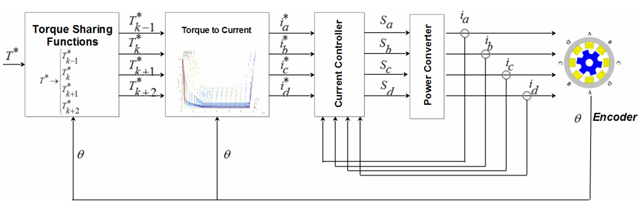


Fig. 3. Torque control block diagram used in TSF method for four-phase SRM

produced torque by outgoing and entering phase stay constant. Fig. 3 represents the control block diagram used in TSF method for four-phase SRM. The input reference torque divided into individual phase torques through the TSF functions. Phase reference currents obtained from "torque to current" look up tables at each rotor position and PWM or hysteresis control block applied to get the desired currents in conventional current control schemes.

The "torque to Flux" look up tables can also be used to control phase flux in inner control loop. Reference phase torque for each phase can be made up by shifting the predefined TSFs. Phase torque commands include zero torque region where the corresponding phase current should be zero and the nonzero area. This region can be divided into two major areas which are commutation mode and single conduction mode. In the commutation area two adjacent phases should produce positive torque based on the distinctive functions. Whereas, in the single conduction mode only one phase must provide the reference torque and the corresponding reference phase current obtained from "torque to current" look up table or utilizing the invertible SRM model [9] and find out the relevant current. Fig. 4 shows the conventional profiles of four typical TSFs which are linear, cosine, cubic and exponential forms.

Reference torques are defined for incoming and outgoing phases during the commutation interval as following equations:

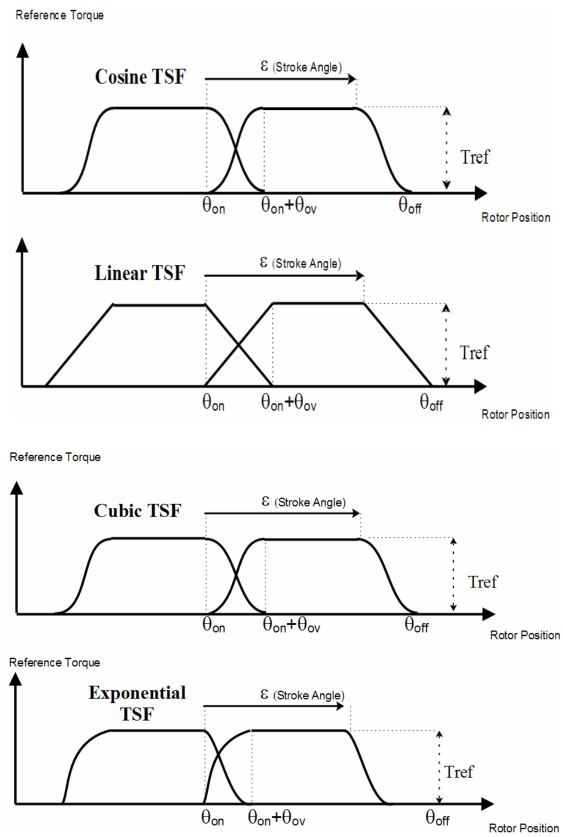


Fig. 4. Conventional TSF profiles: Cosine, linear, cubic and exponential

$$T_{Ph}^{*} = T_{ref} . f_{Linear}$$

$$f_{Linear} = \begin{cases} \frac{(\theta - \theta_{on})}{\theta_{ov}} & \theta_{on} \le \theta \le \theta_{on} + \theta_{ov} \\ 1 & \theta_{on} + \theta_{ov} \le \theta \le \theta_{off} - \theta_{ov} \\ \frac{(\theta_{off} - \theta)}{\theta_{ov}} & \theta_{off} - \theta_{ov} \le \theta \le \theta_{off} \end{cases} \quad (8)$$

Where θ_{on} , θ_{ov} and θ_{off} are the starting, overlap and ending of commutation angles, respectively. In this case, reference torque for incoming phase comes up to reach to the desired value by linear function over the overlap duration. At the same time, the reference torque in outgoing phase will reduce so that the total motor torque remains constant at desired value. Eqs (9-11) represents the function definitions considered for cosine, cubic and exponential TSFs.

$$T_{Ph}^{*} = T_{ref} . f_{Cosine}$$

$$f_{Cosine} = \begin{cases} Sin^2\left(\frac{\pi}{2}\left(\frac{\theta - \theta_{on}}{\theta_{ov}}\right)\right) & \theta_{on} \le \theta \le \theta_{on} + \theta_{ov} \\ 1 & \theta_{on} + \theta_{ov} \le \theta \le \theta_{off} - \theta_{ov} \\ Sin^2\left(\frac{(\theta_{off} - \theta)}{\theta_{ov}}\right) & \theta_{off} - \theta_{ov} \le \theta \le \theta_{off} \end{cases} \quad (9)$$

$$T_{Ph}^{*} = T_{ref} . f_{Cubic}$$

$$f_{Cubic} = \begin{cases} \frac{3}{\theta_{ov}^2}(\theta - \theta_{on})^2 - \frac{2}{\theta_{ov}^3}(\theta - \theta_{on})^3 & \theta_{on} \le \theta \le \theta_{on} + \theta_{ov} \\ 1 & \theta_{on} + \theta_{ov} \le \theta \le \theta_{off} - \theta_{ov} \\ 1 - \frac{3}{\theta_{ov}^2}(\theta - \theta_{off} + \theta_{ov})^2 - \frac{2}{\theta_{ov}^3}(\theta - \theta_{off} + \theta_{ov})^3 & \theta_{off} - \theta_{ov} \le \theta \le \theta_{off} \end{cases} \quad (10)$$

$$T_{Ph}^{*} = T_{ref} . f_{Expon}$$

$$f_{Expon} = \begin{cases} 1 - Exp\left(-\frac{(\theta - \theta_{on})^2}{\theta_{ov}}\right) & \theta_{on} \le \theta \le \theta_{on} + \theta_{ov} \\ 1 & \theta_{on} + \theta_{ov} \le \theta \le \theta_{off} - \theta_{ov} \\ Exp\left(-\frac{(\theta_{off} - \theta_{ov} - \theta)^2}{\theta_{ov}}\right) & \theta_{off} - \theta_{ov} \le \theta \le \theta_{off} \end{cases} \quad (11)$$

In order to increase the efficiency of TSF method, it would be desired to operate in maximum torque per Ampere (MTPA) condition. Many investigations have been undertaken to find the optimum overlap period and turn-on angle such that minimum copper loss obtained during the commutation period. [6] Introduced θ_c^i , the angle where two adjacent phases produce the same torque at same current. It is shown that if turn-on and turn-off angle are determined such that the intersection point of reference torques take place in θ_c^i the optimum performance and minimum torque ripple factor would be achieved. Torque ripple factor obtained from the following equation.

$$T.R\% = \frac{T_{Max} - T_{Min}}{T_{ave}} * 100 \quad (12)$$

Neglecting the stator resistance with some simple manipulations, it can be shown that the minimum overlap period required for commutation between two adjacent phases for reference torque T at the speed of ω obtained from (13):

$$\theta_{ov,\min} = \max\left\{\sqrt{\frac{T}{a_k(\theta)}} . \frac{L_k(\theta)\omega}{V_{dc}}\right\} \quad (13)$$

In the above equation:

$$a_k(\theta) = \frac{1}{2}\frac{\partial L(\theta, i)}{\partial \theta} \quad (14)$$

The greater motor speed, the bigger overlap commutation interval should be. For an instance, the maximum overlap interval in the four-phase SRM is 15° and as it can be seen from (13) converter voltage should be increased when the speed ascended. Finding the optimum turn-on and turn-off angles for each conventional TSFs has been investigated in [12, 13] using an offline procedure. However, the results are depending on motor technical specifications and would be changed in other SRM. Moreover, the pre-calculated operations have to be undertaken to attain the optimum performance. In the next section, an efficient modification method will proposed in order to have acceptable torque ripple and extend torque-speed characterization so that it can be applied on each former TSFs using some online adjustment rules.

3. Modified Torque Sharing Functions

One of the most drawbacks of TSF methods in torque ripple control problem as well as maximizing torque per ampere ratio is that the maximum speed, in which the admissible torque ripple performance can be obtained, is limited. This value will be much lower than the base speed especially at nominal torque value. It should be noted that some constraints on voltage converter will affect on rising and falling time and may cause deteriorations in torque control performance. In other words, if it is desired to achieve the rated torque at the rated speed, the turn-on and turn-off angle must be changed in advance and it would cause to decrease torque per ampere ratio dramatically.

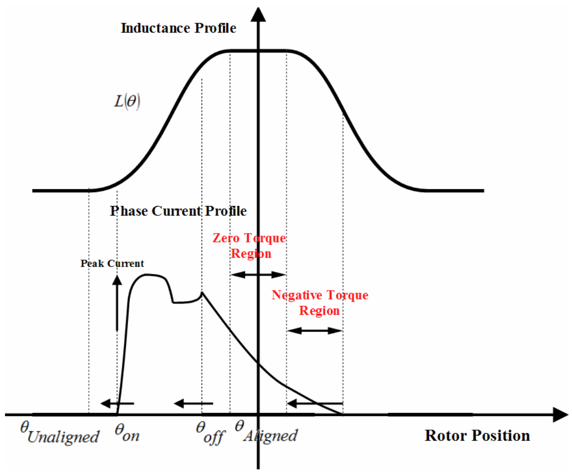


Fig. 5. Effects of receding turn-on and turn-off angles on peak phase current and negative torque region

Moreover, receding turn-on angle may cause to increase the peak current at the starting conduction of excited phase in order to produce the desired torque. This may pull down the efficiency and imposes higher copper loss in commutation intervals. Setting back the turn-off angle can prevents tail current to enter to the negative torque region as it is illustrated by Fig. 5.

Approaching θ_{on} to $\theta_{Unaligned}$ will provide the wider commutation area when the speed increased. However, this will cause to decrease the phase inductance at the starting of phase conduction and increase peak current as a consequence. In addition, phase torque production capability will comes down dramatically. This can ne lead to unsuccessful torque tracking in almost all types of conventional TSFs. Similarly, reducing turn-off angle would cause to preventing negative torque generation. Neglecting stator resistance, phase voltages are calculated from the following equation:

$$v_k \approx \frac{\partial \lambda_k}{\partial \theta}.\omega \tag{15}$$

According to Eq. (15) maximum speed with torque ripple-free operation can be determined from (16).

$$\omega_{\max} = \min\left(\frac{+V_{DC}}{Max\left(\frac{\partial \lambda}{\partial \theta}\right)}, \frac{-V_{DC}}{Min\left(\frac{\partial \lambda}{\partial \theta}\right)} \right) \tag{16}$$

In the modified proposed method, deficiency of produced phase torque in incoming phase will compensated by excess of phase torque generated by the error of outgoing phase current. Fig. 6 shows the concept of modification procedure for a simple linear TSF. In Fig. 6, "incoming effect" refers to the lack of torque production in incoming phase due to the low value of incremental inductance and generated torque near the unaligned rotor position.

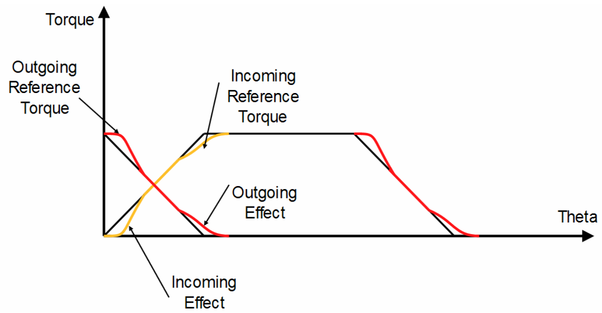


Fig. 6. Concept of proposed modification method for linear TSF

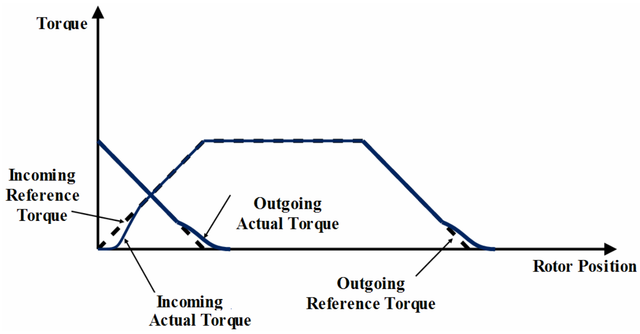


Fig. 7. Typical incoming and outgoing actual (solid line) and reference torques (dashed lines)

Similarly, "outgoing effect" refers to the excess of produced torque by outgoing phase due to high inductance value near the aligned rotor position which caused to negative phase current error in inner control loop represented by Fig. 3. Considering (15, 16) maximum allowable flux changing rate will be decreased when the motor speed ascended and as a consequence the maximum speed with torque ripple-free operation will reduced. Fig. 7 displays the typical actual and reference torques for linear TSF.

Fig. 8 shows the concept of incoming and outgoing effects from the rate of flux changing point of view. In this figure, P represents the peak needed flux changing rate for constructing the major part of reference torque, while D depends on minimum required value for phase voltage to stifle the outgoing phase current. If P and D values placed in inner area limited by red lines such as displayed in Fig. 8, the torque tracking control for two adjacent phases under the commutation in TSF method will be properly done. In other words, converter voltage limitation for torque tracking control will be satisfied if P and D absolute values be smaller than V_{DC}/ω. When the motor speed increased, the magnitude of border red lines decreased toward zero and voltage limitation may be violated by P or D respect to their greatest value. If D breaches the voltage limitation red lines, torque tracking control in outgoing phase would be deteriorated.

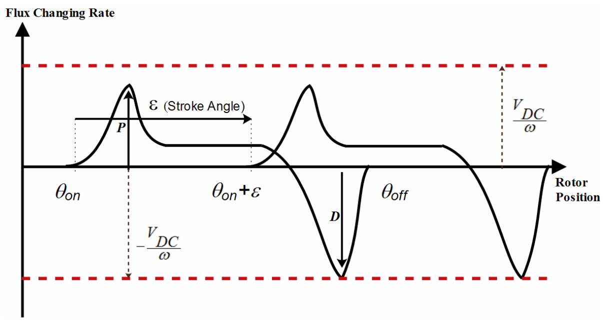


Fig. 8. Typical flux linkage changing rate respect to rotor position considering conventional TSFs

This may cause to negative torque error and affects as an overshoot on the resultant motor torque. Due to low capability of torque production near the aligned position and decreasing form of incremental inductance in this region the generated torque ripple would not be high. Contrariwise, if P infracts the converter voltage limitation, the produced torque in incoming phase can't reach to the reference value and it reflects as undershoot in motor torque. Since the major part of reference torque should be produced by incoming phase during this area, torque tracking error may cause to increase torque ripple factor (TRF) in resultant motor torque which is obtained by the following equation:

$$TRF\% = \frac{T_{Max} - T_{Min}}{T_{ave}} * 100 \tag{17}$$

The incoming and outgoing phase torques can be presented by (18) and (19) equations:

$$\Delta T_k = T_{k,TSF} - T_k \tag{18}$$

$$\Delta T_{k+1} = T_{k+1,TSF} - T_{k+1} \tag{19}$$

Where $T_{k+1,TSF}$, $T_{k,TSF}$ are the reference torques which calculated from (8)-(11), respectively. The proposed modification scheme can be presented by the following logical rules as shown in Table 1. in this method, reference phase torques are determined based on the condition weather actual motor torque is less than the desired value or greater. If the resultant torque is lower than required value, it had been caused by less produced torque in incoming phase due to lower value of incremental inductance at the start of commutation, the remained torque will be compensated by outgoing phase. Similarly, the excess of produced torque generated by outgoing phase will be compensated by incoming phase. This can reduce stress on entering phase to reach to the desired torque level. In Table 1 T^*_{k+1}, T^*_k are the modified reference torques for entering and outgoing phases and as it is illustrated from Fig. 6. The required reference torque in incoming phase will be reduced using the proposed TSFs. This can rebate the required flux as well as P in incoming phase.

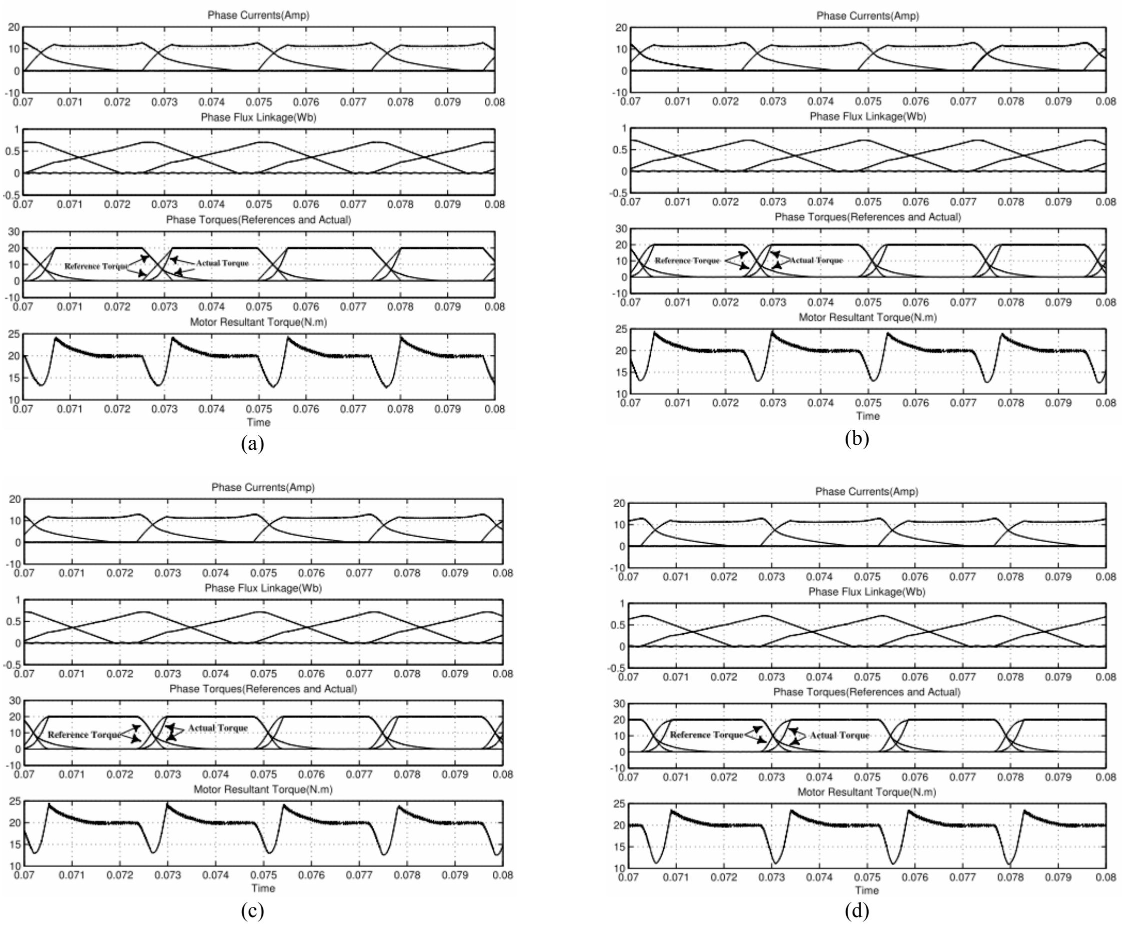
Table 1. Proposed modification rules during commutation

Condition	Step	Reference phase torques calculation
$T < T^*$	*First*	$T^*_{k+1} = T_{K+1,TSF}$
	Second	$T^*_k = T_{K,TSF} + \Delta T_{k+1}$
$T > T^*$	*First*	$T^*_k = T_{K,TSF}$
	Second	$T^*_{k+1} = T_{K+1,TSF} + \Delta T_k$

In other words, the excess torque from outgoing phase torque controller can be used so that the remaining torque generated by incoming phase becomes reduced. Moreover, the lack of torque generation in incoming phase which can be compensated by outgoing phase would cause to enhance the efficiency due to the higher incremental inductance and torque production capability in outgoing phase.

4. Simulations and Experimental Results

In order to verify the proposed scheme, a four phase 8/6, 4KW SRM is considered to simulate and implement the introduced modification method. Each phase has 0.75 ohm resistance and the nominal rated voltage is 380 V. This motor is an industrial type of SRM with 26 N.m rated torque and has impressive nonlinearity in magnetic characteristic. The motor is considered to operate at some desired load torque and simulations were undertaken using MATLAB software based on the concept represented by Fig. 6. Turn-on and off angles are selected so that the desired motor speed without negative phase torque generation can be obtained. For an instance, phase conduction is considered to be started at 35 degree and ended at 54 degree for the speed of 1000 rpm. Flux linkage data calculated from the voltage integration method. In this case, a rectangle voltage applied to the phase at a specific rotor position and voltage, current and rising time recorded by the oscilloscope. These data used to calculate torque, flux and inductance profile [15]. This procedure were repeated from the unaligned toward aligned position by the step of 2 degree in rotor position. One phase excitation used to determine the aligned situation corresponding to the excited phase, while, C phase excitation used to obtain the unaligned position of the A phase and absolute encoder used to determine the exact location of rotor position within this interval. Fig. 8 shows the simulation results for the four conventional TSFs at the speed of 1000 rpm and 20 N.m load torque. Simulation results display phase currents, flux linkages, reference and actual phase and resultant torques from top to bottom respectively. In this figure, phase torque was calculated using the bicubic spline interpolation of the static torque measurement data. Torque ripple factor were obtained considering (17) as 60%, 56%, 57% and 62% for linear, cosine, cubic and exponential TSFs, respectively. As it can be seen from Fig. 9 starting effect in incoming phase has the major influence on



(a) (b) (c) (d)

Fig. 9. Simulation results at 1000 rpm and load torque of 20N.m using a) linear b) cosine c) cubic and d) exponential TSFs

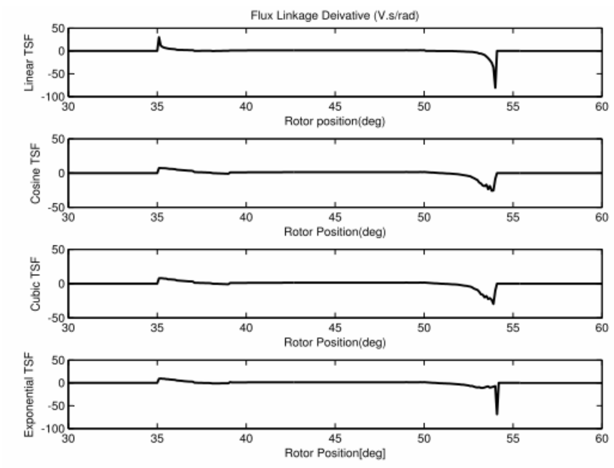


Fig. 10. Derivative flux linkage profile for load torque of 20N.m using linear, cosine, cubic and exponential TSFs, respectively

resultant torque compared than ending effect. Fig. 10 represents the flux linkage derivative profile for the conventional TSFs. Table 2 illustrates the peak and dip values of flux linkage derivative as well as TRF and average torque corresponding to each TSFs.

Table 2. Comparison of conventional TSFs without modifications

Type	Min. Flux Derivative	Max. Flux Derivative	*TRF%*	T_{ave}
Linear	-68.9	30.3	60	19.6
Cosine	-29.7	7.13	56	19.83
Cubic	-25.4	7.94	57	19.76
Exponential	-80.8	9.67	62	19.37

Considering the results represented by Table 1, cosine and cubic TSF have the minimum TRF and greatest average torque. Exponential TSF has the sharpest flux linkage derivative profile which can make the larger torque error due to the high torque error in incoming phase torque control. However, in cosine and cubic TSF, the wider tip and dip shape in flux linkage derivative profile makes the positive torque error in incoming phase and negative torque error in outgoing phase. This can somewhat reduce the dip shape in motor torque caused by the incoming phase torque error. Fig. 11 shows the simulation results considering the proposed modification scheme in 1000 rpm and load torque of 20Nm. Torque ripple factor reduced to less than 7.5% considering the introduced method represented by (18). Fig. 12 depicts the comparison of torque-speed characteristics between four-conventional

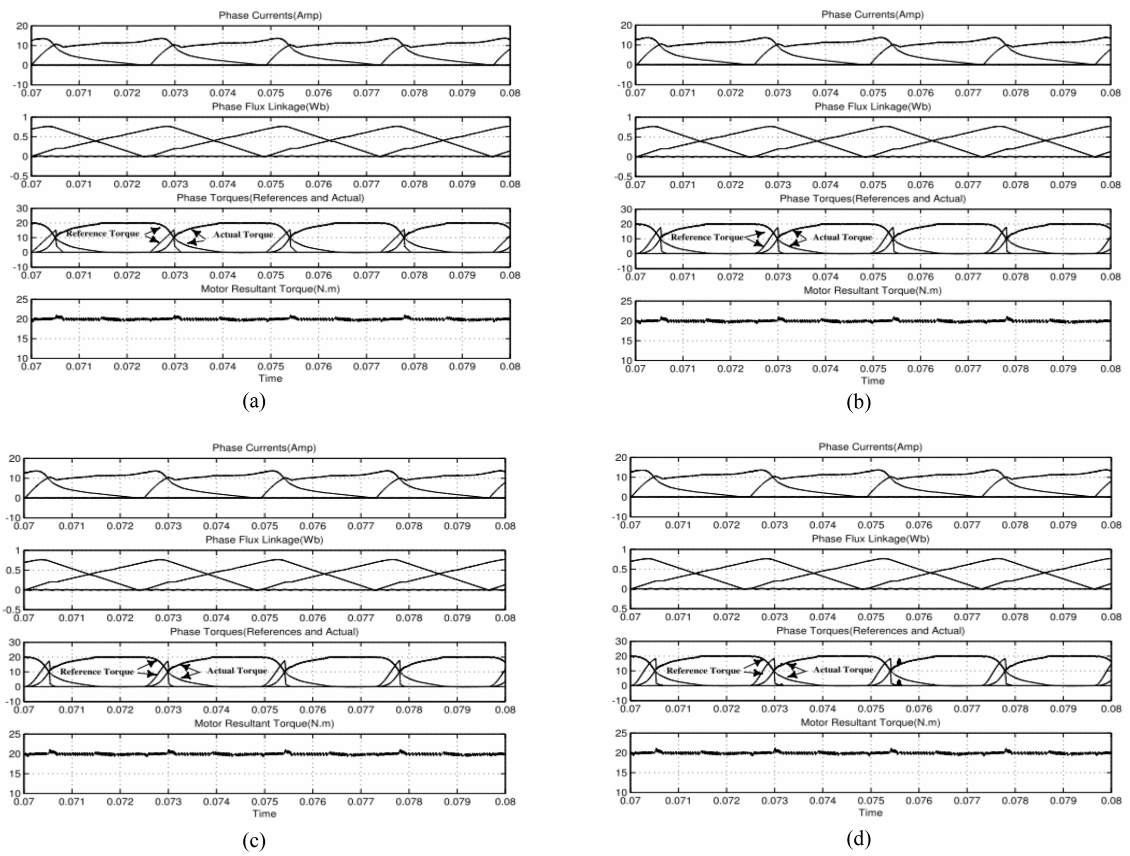


Fig. 11. Simulation results at 1000 rpm and load torque of 20N.m, a) linear b) cosine c) cubic and d) exponential TSFs using the proposed modification scheme.

TSFs and the proposed method. In this figure, average torque is reduced from the reference value above the ω_{TRF} where represents the maximum speed with torque ripple-free operation. As it can be seen in Fig. 12 ω_{TRF} has the less value in linear TSF and has the greatest value in cosine and cubic TSFs, respectively. Considering fig. 10 for the flux linkage derivative profile, it can be obtained that P in linear TSF has the maximum value compared than the other TSFs. This yields that when the speed increased, the voltage constraint would be unsatisfied earlier at the starting of conducting phase in the linear TSF.

Due to the fact that starting effect has the major impression on motor resultant torque, average torque will be reduced earlier in this type of TSF. In order to show the modifications more clearly on incoming and outgoing reference phase torques, Fig. 12 represents the expanded conduction region of two adjacent phases for linear TSF. In this figure, dash lines display the turn-on and turn-off angles which considered 35 and 54 degree respectively. In Fig. 13, the reduction slopes of average torque versus speed in the proposed modification method are greater than the conventional TSFs.

This is caused by adding the torque error in incoming phase to the reference torque in outgoing phase which can be lead to entering the tail current to the negative torque region. This can be prevented by receding the turn off

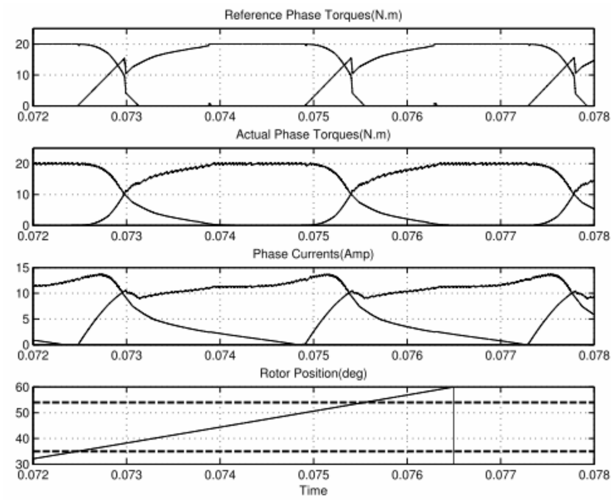


Fig. 12. Expanded simulation results for modified linear TSF, reference, actual phase torques, phase currents and rotor position from top to bottom, respectively

angle. In order to increase ω_{TRF} in conventional TSF it should be reduce the turn-on and turn-off angles to have sufficient time to reach to the desired torque values. For an instance, if it's desired to have 20 Nm under torque ripple-free condition in 1000 rpm using linear TSF, switching on

Table 3. Comparison of conventional TSFs with proposed modifications at 1000 rpm and 20Nm load torque

Type	Min. Flux Derivative	Max. Flux Derivative	*TRF%*	T_{ave}
Linear	-2.06	3.07	5	20
Cosine	-2.59	3.15	5	20
Cubic	-2.63	3.14	5	20
Exponential	-2.62	3.17	5	20

Table 4. Comparison of linear TSF with proposed modification based on experimental results

Type	Peak phase current(A)	*TRF%*	T_{ave}
Linear	12	25.7	9.7
Modified Linear	10	5	10

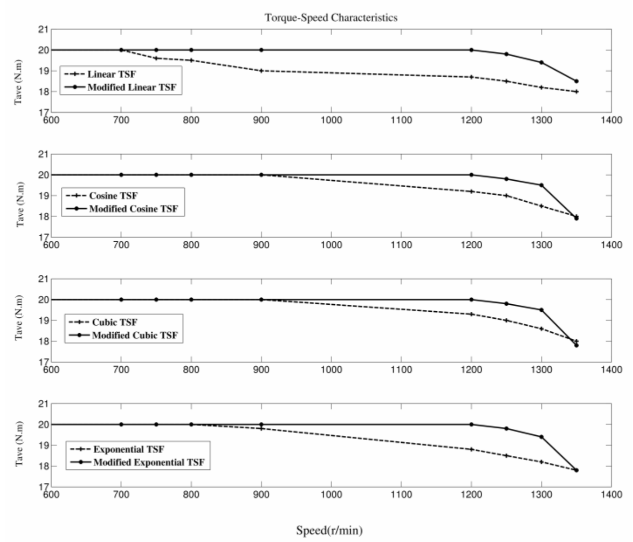


Fig. 13. Comparison of torque-speed characteristics of four-conventional TSFs using the proposed method for 20N.m load torque.

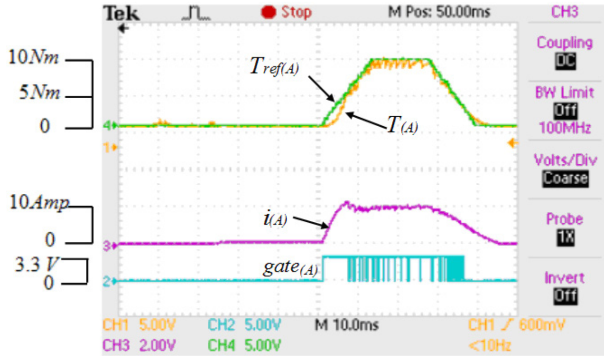


Fig. 14. Experimental results in linear TSF at the speed of 200r/min and load torque of 10Nm. Reference phase torque, estimated phase torque, phase current and switching gate pulse from top to bottom respectively.

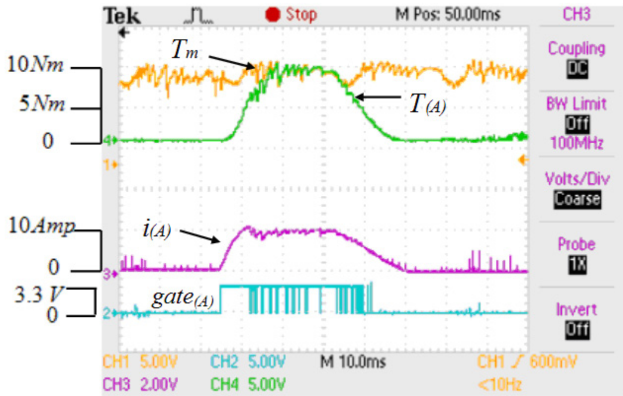


Fig. 15. Experimental results in linear TSF at the speed of 200r/min and load torque of 10Nm. Motor resultant torque, estimated phase torque, phase current and gate pulse from top to bottom respectively.

angle should be decreased to 33 and phase excitation have to be switched off in 56 degree. This will lead to extend the overlap interval between incoming and outgoing phases and reduce the P and D values in flux linkage derivative profile. The phase peak current in this condition will be 15(A). However, using the proposed modification method the peak current would be reduced to 13.5(A). Moreover, the power loss would be decreased due to regulating motor torque based on compensating reference torque utilizing the most energy efficient phase for each two adjacent phases under commutation. The introduced modification method implemented on SRM rig. Table 3 represents the results using the modification scheme applied on conventional TSFs for 1000 rpm. Since the proposed modification is adapted with motor speed, results are shown for one specific speed. The main controller for implementing the proposed scheme is designed by TMS320F28335 from Texas Instruments. Phase currents were used as feedback signals as well as 10 bits digital signals from the absolute encoder by 1024ppr.

Look up tables and modification rules were program into the DSP while phase torques are considered as feedback in outer control loop and switching frequency is considered 10kHz. In order to perceive the lagging effect of sampling time, predictions were investigated to calculate the next rotor position based on motor speed. The predicted rotor position is used to obtain next reference torque regardless of modification. Fig. 14 shows the results at 200r/min and the load torque of 10 Nm using linear TSF to verify the superiority of the proposed method. In order to show the effectiveness of the proposed method, phase voltage and overlap angle is reduced to generate the torque error at this speed.

In this case, turn-on and off angles are considered 34 and 50 degree respectively. Torque error at the starting of phase conduction in incoming phase will lead to rise some dips in resultant torque as it's shown in Fig. 15.

The dip value would be greater than 2.5 Nm which can

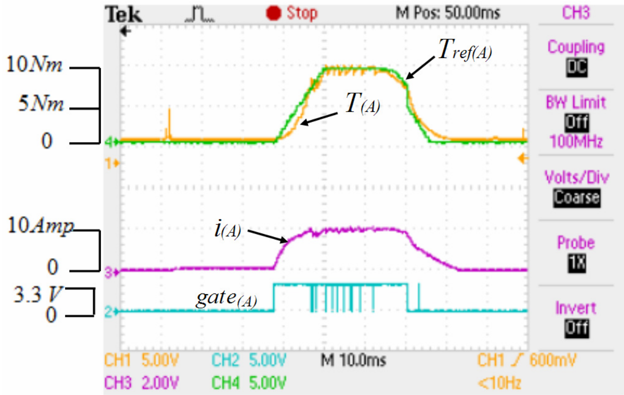


Fig. 16. Results in proposed linear TSF at the speed of 200r/min and load torque of 10Nm. Reference phase torque, estimated phase torque, phase current and gate pulse from top to bottom respectively.

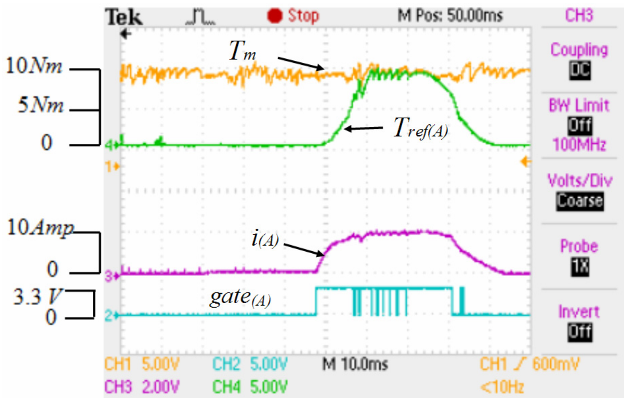


Fig. 17. Experimental results in proposed linear TSF at the speed of 200 r/min and load torque of 10Nm. Motor resultant torque, estimated phase torque, phase current and gate pulse from top to bottom respectively.

be represented as 25% of load torque and peak phase current is 12A. Fig. 16 illustrates the resultant reference torque using proposed modification scheme. In this figure, it's shown that torque error in incoming phase has been added to the reference torque in outgoing phase to retain the total torque at constant level.

Fig. 17 shows the total motor torque in which dip shapes have been eliminated properly compared than conventional linear TSF. Moreover, peak phase current reduced to 10 A as it can be seen from this figure. Table IV also shows the comparison of the conventional and modified linear TSF based on experimental results.

5. Conclusions

In this paper, a modification method introduced which has the ability to be applied on any conventional torque sharing functions. The proposed scheme considered voltage saturation and limitation of SRM converter. Due to low capability of torque production at the starting of incoming phase and converter voltage constraints within the commutation area, incoming phase torque control failed to operate properly. This yields dip ripples on the resultant motor torque and deteriorates torque average as well as ripple factor dramatically. Similarly, torque error in outgoing phase torque controller may cause some tips on total motor torque. In the proposed modification method, torque error in incoming phase added to the reference torque of outgoing phase to regulate total torque during the starting of phase conduction. Moreover, phase torque error of outgoing phase added to the reference torque of incoming phase as a negative value to decrease the required torque and voltage requirements as well. The introduced approach caused to extend torque-speed characteristics with constant torque and reduced peak phase current while switching angles were considered fixed. Simulations and experimental results demonstrated the effectiveness and superiority of the proposed modification scheme compared than conventional TSFs.

Appendix

Nameplate data for 4 phase SR test motor:

Nominal current:	9 Amp
Supply frequency:	50/60 Hz
Nominal Speed:	1500rpm
Rated power:	4Kw/5.5hp
Rated voltage:	380/415 V

Acknowledgment

This work was supported by the Ministry of Education, (MOE) through BK21 Plus.

References

[1] S.K. Sahoo, S. Dasgupta, S.K. Panda, J.X. Xu, "A Lyapunov Function Based Robust Direct torque Controller for Switched Reluctance Motor Drive System", *IEEE Trans. On Power Electronics*, vol. 27, no. 2, pp. 555-564, Feb. 2012.

[2] R. Krishnan, Switched Reluctance Motor Drives: Modeling, Simulation, Analysis, Design and Applications. Boca Rotan, FL: CRC Press, 2001.

[3] Y. K. Choi, H.S. Yoon, C. S. Koh, "Pole-shaped optimization of a switched reluctance motor for torque ripple reduction", *IEEE Trans. Magnetics*, vol. 43, no. 4, pp. 1797-1800, Apr.2007.

[4] S. I. Nebeta, I. E. Chabu, L. Lebensztajn, D. A. P. Correa, W. M. Dasilva, K. Hameyer, "Mitigation of the torque ripple of a switched reluctance motor

through a multi objective optimization", *IEEE Trans. Magnetics,* vol. 44, no. 6, pp. 1018-1021, Jun. 2008.

[5] I. Husain, "Minimization of torque ripples in SRM drives", *IEEE Trans. Industrial Electronics,* vol. 49, no.1, pp. 28-39, Feb. 2002.

[6] R. S. Wallace, D.G. Taylor, "A balanced commutator for switched reluctance motors to reduce torque ripple", *IEEE Trans. Power Electronics*, vol. 7, no. 4, pp. 617-626, Oct. 1992.

[7] K. Russa, I. Husain, M. Elbuluk, "Torque-Ripple minimization of switched reluctance machines over a wide speed range", *IEEE Trans. On Industry Applications*, vol. 34, no. 5, pp. 1105-1112, September/ October 1998.

[8] C. Choi, S. Kim, Y. Kim, K. Park, "A new torque control method of a switched reluctance motor using a torque-sharing function", *IEEE Trans. On Magnetics*, vol. 38, no. 5, pp 3288-3290, September 2002.

[9] V. P. Vujicic, "Minimization of torque ripple and copper losses in switched reluctance drive", *IEEE Trans. Power Electronics,* vol. 27, no. 1, Jan. 2012.

[10] D.H., Lee , S.Y., Ahn, J.W., Ahn, "A simple negative torque compensation scheme for a high speed switched reluctance motor", *journal of power electronics,* vol.12, no. 1, pp. 58-66, January 2012.

[11] D. H. Lee, J. Liang, Z. G. Lee, J. W. Ahn, "A simple nonlinear logical torque sharing function for low toruqe ripple SR drives", *IEEE Trans. Industrial Electronics*, vol. 56, no. 8, pp. 3021-3028, Aug. 2009.

[12] X. D. Xue, K. W. E. Cheng, S. L. Ho, "Optimization and evaluation of torque-sharing functions for torque ripple minimization in switched reluctance motor drives," *IEEE Trans. Power Electronics*, vol. 24, no. 9, pp. 2076-2090, Sep. 2009.

[13] J.W. Ahn, "Torque control strategy for high performance SR drives", *Journal of Electrical Engineering & Technology*, vol. 3, no. 4, pp 538-545, 2008.

[14] M.N.F. Nashed, K. Obyama, K. Aso, H. Fujii, H. Uebara, " Automatic turn-off angle control for high speed SRM drives", *Journal of Power Electronics*, vol. 7, no. 1, pp. 81-88, January 2007.

[15] I.H. Al-Bahadly, "Examination of a sensorless rotor-position-measurement method for switched reluctance drive", IEEE Trans. Industrial Elec., vol. 55, no. 1, pp. 288-295, Jan. 2008.

Milad Dowlatshahi was born in Isfahan, Iran, in 1984. He received his B.S. and M.S in electrical engineering from Isfahan University of Technology, Isfahan, Iran, in 2006 and 2009, respectively. He is studying toward Ph.D degree in electrical engineering and his research interests are electric motor drives, power electronics and switched reluctance motor design and control. He was a visiting researcher in smart mechatronics lab in Kyungsung University, Busan, Korea from July 2012 to January 2013.

Sayed Mortaza Saghaiannejad was born in Isfahan, Iran, in 1952. He received his B.S., M.S. and Ph.D. in Electrical Engineering from the University of Kentucky, USA in 1977, 1979 and 1989, respectively. Since 1979, he has been a faculty member of Isfahan University of Technology, where he is currently a Professor in electrical and computer engineering department. He is the author or coauthor of various journal and conference papers. His current research interests are in the areas of electrical machines, power electronics and advanced motor drive systems.

Jin-Woo Ahn was born in Busan, Korea in 1958. He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Pusan National University, Pusan, Korea in 1984, 1986, and 1992, respectively. He has been with Kyungsung University, Busan, Korea as a Professor in the Department of Mechatronics Engineering since 1992. He is the author of five books including SRM and more than 120 research papers, and has more than 20 patents. His current research interests are advanced motor drive systems and electric vehicle drives. He is now serving as an Editor-in-Chief of JICEMS (Journal of International Conference on Electric Machines and Systems) and General Chairman of ICEMS 2013 and IEEE/ICIT 2014.

Mehdi Moallem received the Ph.D. degree in electrical engineering from Purdue University, West Lafayette, IN, in 1989. He is currently a Full Professor at the Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran. He is the author or coauthor of various journal and conference papers. His research interests include design and optimization of electromagnetic devices, application of advance numerical techniques and expert systems to analysis and design of electrical machines, and power quality. Prof. Moallem was a recipient of many international and national awards.