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Probabilistic Power Flow Studies Incorporating Correlations of PV Generation for Distribution Networks

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Abstract – This paper presents a probabilistic power flow (PPF) analysis method for distribution network incorporating the randomness and correlation of photovoltaic (PV) generation. Based on the multivariate kernel density estimation theory, the probabilistic model of PV generation is proposed without any assumption of theoretical parametric distribution, which can accurately capture not only the randomness but also the correlation of PV resources at adjacent locations. The PPF method is developed by combining the proposed PV model and Monte Carlo technique to evaluate the influence of the randomness and correlation of PV generation on the performance of distribution networks. The historical power output data of three neighboring PV generators in Oregon, USA, and 34-bus/69-bus radial distribution networks are used to demonstrate the correctness, effectiveness, and application of the proposed PV model and PPF method.

Keywords: Photovoltaic generation, Randomness, Correlation, Nonparametric kernel density estimation, Probabilistic power flow

1. Introduction

In recent years, the development of solar photovoltaic (PV) generation has been steadily increasing around the world; the total global PV capacity reached 100 GW in 2012 [1]. PV generation is becoming an important renewable energy resource. The increasing amount of PV generation significantly affects the performance of distribution networks.

Compared with conventional generation, PV generation is a variable resource with two characteristics 1) Randomness: The power output of PV generator at each time point is random because of uncertain solar irradiation and other related weather conditions. 2) Correlation: PV generation at different sites can be assumed to be independent if they are far away from each other. However, the power outputs of PV generators at adjacent locations may be strongly correlated owing to common effects such as solar irradiation, temperature and other environmental factors. In addition, residential roof-top PV generation is not allowed to provide voltage control. PV generation can potentially cause various power quality issues in distribution networks, such as fluctuation of bus voltage magnitudes, line flows, and voltage violations [2]. Hence, the randomness and correlation of PV generation must be considered and statistically characterized.

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The probabilistic power flow (PPF) analysis, which was first proposed by Borkowska in 1974 [3], is an important and popular approach to evaluate the performance of power systems while considering the uncertainties of load demand and renewable generation [4]. The techniques for solving the PPF can be classified into two categories: analytical solution and Monte Carlo simulation [5]. There are different analytical probabilistic power flow methods such as convolution method [6-9], the point estimation method [10, 11] and the cumulant method [2, 12]. The necessitated unavoidable approximations always exit in the analytical methods [5]. By contrast, Monte Carlo methods can handle various complex conditions without simplification [5]; such methods have been widely used in solving the PPF [4, 13-15].

In recent years, extensive research has been devoted to PPF analysis for power systems with PV generation. The probabilistic model of PV generation is derived from the probabilistic density function of the clearness index, which is used to denote the impact of clouds on solar irradiation [14]. The probabilistic models of global and diffuse irradiation are established in [15]. Based on the functional relationship of PV power output and irradiation, the probabilistic PV model is derived. The Monte Carlo technique is used to analyze the PPF of distribution networks [14, 15]. Note that the correlation of PV generation is not considered in the above PPF methods. A cumulant-based PPF algorithm is presented in [2] to consider the correlation of PV generation. The corresponding probabilistic model of PV generation is acquired by combining the Beta distribution of solar irradiation and the normal distribution of the forecast error of the PV cell temperature. Based on Nataf transformation and Latin Hypercube Sampling, hypercube sampling, a PPF method is proposed in [4] to address correlated power sources while assuming that PV power follows Beta distribution. The assumption of theoretical parametric distribution for PV generation is necessary in all aforementioned PPF methods. However, it is well known from theory and experience that there may be a substantial gap between an assumed theoretical parametric distribution and the physical behavior of actual PV power. In other words, an assumed parametric distribution model may not always achieve satisfactory results [16]. This has been confirmed by recent investigations into considerable PV power data [17]. Consequently, the assumption of unsuitable distribution may lead to biased analysis results of the probability distributions of bus voltage magnitudes, line flows, and so on in the aforementioned PPF methods.

This paper presents a Monte Carlo based PPF method for distribution networks, which can accurately capture the randomness and correlations of PV generators at adjacent locations without any assumption of theoretical parametric distribution and evaluate the integration of PV generation on the performance of distribution network. The main contributions of the paper include the following:

- The probabilistic model of PV generation is established using the nonparametric kernel density estimation, which can accurately capture the randomness and correlation of PV generation without any assumption.
- The actual power data of three PV generators at adjacent locations are used to validate the proposed PV model.
- Combined with the proposed PV model, the PPF method is developed on the basis of Monte Carlo technique to evaluate the influence of the randomness and correlation of PV generation on the performance of distribution networks.
- The 34-bus/69-bus radial distribution network is adopted to demonstrate the effectiveness and validity of the presented PPF method.

The rest of the paper is organized as follows. The probabilistic model of PV generation is proposed in Section II, and the probabilistic load model is presented in Section III. The deterministic power flow is briefly summarized in Section IV. The procedure of the PPF method that considers the correlation of PV generation is proposed in Section V. Case studies are provided in Section VI, followed by conclusions in Section VII.

2. Probabilistic Model of PV Generation Considering Correlation

Assume that n PV generators are geographically close, i.e., the power outputs of the n PV generators are correlated with each other. Let p_i (i = 1, 2 ,…, n) represents the power output of the ith PV generator and $\boldsymbol{P}$ = [p_1, p_2 ,…, p_n]. Additionally, m day samples of PV generators are available and the jth day sample is $P_j = [P_{1j}, P_{2j},\ldots, P_{nj}]$, j = 1, 2 ,.., m. The joint probability density function of $\boldsymbol{P}$ is assumed as $f(\boldsymbol{P})$, which should consider the randomness and correlation of PV power outputs. Nonparametric kernel density estimation theory with multiple variables is introduced to estimate f($\boldsymbol{P}$) [18]. The multivariate kernel density estimation of $f(\boldsymbol{P})$ can be denoted as follows [19]:

$$f_H(\mathbf{P}) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{\det(\mathbf{H})}K\left[\mathbf{H}^{-1}(\mathbf{P}-\mathbf{P}_\mathrm{i})\right] \tag{1}$$

where H is the $n\times n$ symmetric positive definite matrix, and $K(\bullet)$ denotes a multivariate kernel function operating on n arguments. To simplify the calculation, H can be assumed to be a diagonal matrix $H = \mathrm{diag}\,[h_1, h_2 ,\ldots, h_n]$. Thus, $K(\bullet)$ can be expressed in the following form of multiplicative kernel functions:

$$K(u) = k(u_1)k(u_2)\cdots k(u_n) \tag{2}$$

where $k(\bullet)$ is a univariate kernel function. The kernel estimation theory indicates that the type of kernel function has very little effect on the accuracy of kernel density estimation. In this paper, the following Gaussian function, which is widely recommended in mathematics books, is selected as the kernel function.

$$k\left(u\right) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right) \tag{3}$$

The following analytical expression is obtained by substituting (2) and (3) into (1).

$$f_H(\mathbf{P}) = \frac{1}{m}\sum_{i=1}^{m}\left\{\frac{1}{h_1h_2\cdots h_n}k\left(\frac{p_1-P_{1i}}{h_1}\right) k\left(\frac{p_2-P_{2i}}{h_2}\right)\cdots k\left(\frac{p_n-P_{ni}}{h_n}\right)\right\}. \tag{4}$$

Bandwidth vector H is a crucial factor in accurately estimating $f(\boldsymbol{P})$. Different methods can be used to estimate multiple optimum bandwidth vectors in (4). For example, a method of utilizing a multivariate normal distribution as the reference distribution is a popular one. However, this method is only appropriate for the reference probability density function and may fail to consider multimodal densities [18], which may be the case for PV power output. Hence, the Cross-Validation (CV) method [19], which requires no assumption or reference distribution, is applied to select the optimum bandwidth vector whose components are multiple bandwidths.

In the CV method, the estimation error of $f_H(\boldsymbol{P})$ can be represented by the integrated squared error ISE. The bandwidth vector H that minimizes the ISE is the optimum bandwidth.

$$\begin{aligned}\min ISE(\mathbf{H}) &= \int \left[f_H(\mathbf{P}) - f(\mathbf{P}) \right]^2 d\mathbf{P} \\ &= \int f_H^2(\mathbf{P}) d\mathbf{P} - 2\int f_H(\mathbf{P}) f(\mathbf{P}) d\mathbf{P} \quad (5) \\ &+ \int f^2(\mathbf{P}) d\mathbf{P}\end{aligned}$$

In (5), the first term can be easily calculated from the data, and the last term does not depend on H and can be ignored as far as the minimization over H is concerned. Hence, only the second term of (5) is unknown and must be estimated.

$$E(f_H(\mathbf{P})) = \int f_H(\mathbf{P}) f(\mathbf{P}) d\mathbf{P} \tag{6}$$

Here, $E(f_H(\boldsymbol{P}))$, the expected value of $f_H(\boldsymbol{P})$, can be estimated by a leave-one-out estimator, which is also the unbiased estimation of $E(f_H(\boldsymbol{P}))$, i.e., [19]

$$\hat{E}(f_H(\mathbf{P})) = \frac{1}{m} \sum_{i=1}^{m} f_{h,-i}(\mathbf{P}_i) \tag{7}$$

Where

$$f_{h,-i}(\mathbf{P}) = \frac{1}{m-1} \sum_{\substack{j=1 \\ j \neq i}}^{m} \frac{1}{\det(\mathbf{H})} K\left[\mathbf{H}^{-1}(\mathbf{P} - \mathbf{P}_j) \right] \tag{8}$$

By substituting the unbiased estimation of $E(f_H(\boldsymbol{P}))$ in (7) into (5) and ignoring the last term in (5), the following analytical expression can be obtained:

$$\begin{aligned}\min ISE(\mathbf{H}) &= \int f_H^2(\mathbf{P}) d\mathbf{P} - 2\int f_H(\mathbf{P}) f(\mathbf{P}) d\mathbf{P} \\ &= \int f_H^2(\mathbf{P}) d\mathbf{P} - \frac{2}{m(m-1)\det(\mathbf{H})} \\ &\sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}}^{m} K\left[H^{-1}(\mathbf{P}_j - \mathbf{P}_i) \right].\end{aligned} \tag{9}$$

By choosing the Gaussian function in (3) as the kernel function and substituting (4) into (9), (9) can be transformed into the following:

$$\begin{aligned}&\min ISE(\mathbf{H}) = \\ &\frac{\det(\mathbf{H})^{-1}}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} \left\{ \prod_{l=1}^{n} \frac{1}{2\sqrt{\pi}} \exp\left[-\frac{1}{4} \left(\frac{P_{lj} - P_{li}}{h_l} \right)^2 \right] \right\} - \\ &\frac{2\det(\mathbf{H})^{-1}}{m(m-1)} \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}}^{m} \left\{ \prod_{l=1}^{n} \frac{1}{2\sqrt{\pi}} \exp\left[-\frac{1}{2} \left(\frac{P_{lj} - P_{li}}{h_l} \right)^2 \right] \right\}.\end{aligned} \tag{10}$$

This equation is a non-constraint optimization problem. In this study, the interior point method is adopted to solve the optimum bandwidth vector H.

3. Probabilistic Model of Load

The normal distribution is the most popular load distribution and is widely used [14]. The probability density function $f(P_L)$ for the normal distribution of load power P_L is given by the following expression:

$$f(P_L) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(P_L - \mu)^2}{2\sigma^2} \right) \tag{11}$$

where μ and σ are the mean and standard deviation of load power P_L, respectively. Thus, an expected value and a standard deviation specify the load at each load point.

4. Probabilistic Power Flow method considering the correlation of PV generation

Combining the probabilistic models of PV generation and load presented in Sections 2 and 3, the PPF analysis method for distribution networks, which considers the correlation of PV generation, is developed based on the Monte Carlo technique. The flow chart is shown in Fig. 1. The procedure is as follows:

1) Input the historical data of PV power output and the electrical and geometrical parameters of the distribution network. Initialize the maximum iteration time $k_{\max}$.
2) Based on the proposed probabilistic PV model in

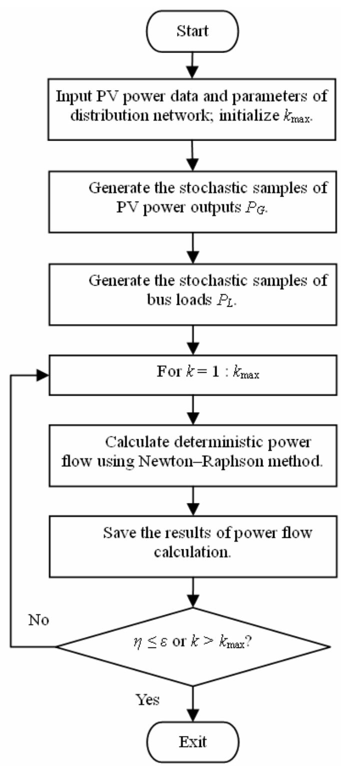

Fig. 1. Flow chart of PPF analysis method

Section 2, generate stochastic samples of PV power outputs P_G using the rejection sampling method [21].
3) Generate stochastic samples of bus loads P_L based on probabilistic load model in Section 3.
4) Use the Newton-Raphson method to solve the power flow with stochastic samples of PV and load powers.
5) Save the result of each power flow calculation, such as bus voltage magnitudes, line flows, and network loss.
6) Take the coefficient of variance η as the convergence criterion. If η is less than the given precision ε, or if k is up to $k_{\max}$, then exit. Otherwise, let $k = k + 1$ and proceed to Step 2.

5. Case Studies

The case studies include two parts: verification of the proposed probabilistic PV model that incorporates the correlation among power outputs of multiple PV generators and the results and analysis of PPF for two distribution networks that incorporates multiple PV generators.

5.1 Data, network, and study conditions

The power output data (2009-2011) of three PV generators at adjacent locations in the Oregon, USA, are used, which is available online [22]. The three PV generators are strongly correlated with one another. This strong correlation enables the testing of the correctness, effectiveness, and adaptability of the proposed PV model against the actual PV power output data. In this paper, P1, P2, and P3 denote the three PV generators.

The proposed PPF algorithm for distribution networks has been implemented using MATLAB and tested on the 34-bus radial distribution network, which consists of 33 lines and 20 loads, and 69-bus radial distribution network, which consists of 68 lines and 48 loads. A full description of the test distribution networks can be found in [23] and [24].

5.2 Verification of the proposed probabilistic model of PV generation

The verification of the proposed PV model includes three aspects. First, the correlation coefficients of PV generation obtained using the PV model should be consistent with those obtained from the original PV power output data. Second, the mean and variance of PV power output obtained using the PV model should be sufficiently close to that obtained from the original PV power output data. Third, the randomness of the power output of a single PV generator should be evaluated correctly.

The stochastic samples are generated by applying the proposed PV model using the historical data of the three PV generators P1, P2, and P3. To analyze the test result conveniently, the stochastic and historical data are both normalized, i.e., the data are divided by their maximum. Hence, all data fall in the interval [0, 1].

The correlation coefficients for each pair of variables in the PV power outputs of P1, P2, and P3 are directly estimated from the historical data and given in Table 1. The correlation coefficients calculated using the stochastic samples generated by the proposed PV model are given in Table 2. The percentages in the brackets in Table 2 represent the relative errors of the correlation coefficients obtained using the proposed PV model against those obtained using the historical data of PV power output. It can be observed that the proposed PV model can accurately represent the correlations between the power outputs of multiple PV generators.

The mean and variance values of the power outputs of P1, P2, and P3 are estimated using the historical data and stochastic samples generated by the proposed PV model. The mean values are shown in Table 3, and the variance values are presented in Table 4. The percentages in the brackets in Tables 3 and 4 represent the relative errors of the indices obtained using the proposed PV model against those obtained using the historical data. It can be seen that the results obtained using the proposed PV model match the reality very well. Take the mean values as an example. The maximum error of the proposed PV model is only 4.11%. Hence, the mean and variance of the historical data can be reflected by the proposed PV model with high accuracy.

The discrete probability distributions of the power outputs of the three PV generators P1, P2, and P3 are acquired using the historical data and stochastic samples generated by the proposed PV model respectively. The histograms of the three discrete distributions of P1, P2, and P3 are shown in Figs 2-4. As revealed in the figures, the theoretical parametric distribution (such as the beta and

Table 1. Original correlation matrix of PV power outputs

	P1	P2	P3
P1	1.0000	0.4950	0.5706
P2	0.4950	1.0000	0.4257
P3	0.5706	0.4257	1.0000

Table 2. Calculated correlation matrix of PV power outputs

	P1	P2	P3
P1	1.0000(0.00%)	0.5026(1.54%)	0.5565(2.47%)
P2	0.5026 (1.54%)	1.0000(0.00%)	0.4332(1.76%)
P3	0.5565 (2.47%)	0.4332(1.76%)	1.0000(0.00%)

Table 3. Mean values of PV power outputs

	P1	P2	P3
Actual	0.3655	0.3507	0.3310
Sampled	0.3785(3.56%)	0.3421(2.45%)	0.3174(4.11%)

Table 4. Variance values of PV power outputs

	P1	P2	P3
Actual	0.1041	0.09369	0.08628
Sampled	0.1067(2.50%)	0.09930(5.99%)	0.08981(4.09%)

normal distribution) cannot be used to establish the probabilistic model of PV power outputs because the histograms of P1, P2, and P3 have approximately two peaks. Furthermore, the probability distributions of the PV power outputs obtained using the proposed PV model basically match those estimated using the original historical data. This condition is attributable to the fact that the proposed PV model based on nonparametric kernel density estimation theory does not need any assumption of theoretical parametric distribution. Consequently, the probability distributions of the PV power outputs, regardless of whether they can be described by certain parametric distribution, can be accurately reflected by the proposed PV model.

The test results reveal that the randomness (represented by mean, variance, and probability distribution) of PV generation and the correlation between power outputs of multiple PV generators can be accurately captured by the proposed PV model.

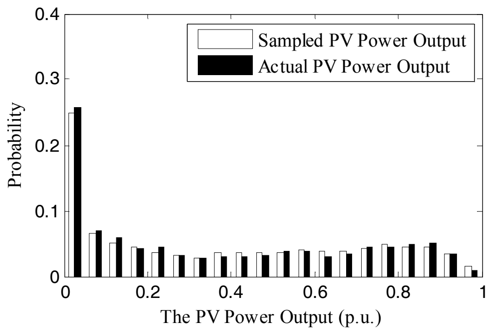

Fig. 2. Probability distribution of power output of P1

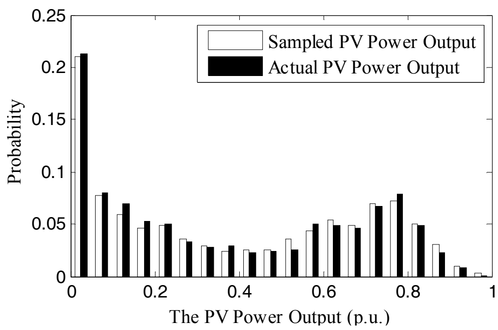

Fig. 3. Probability distribution of power output of P2

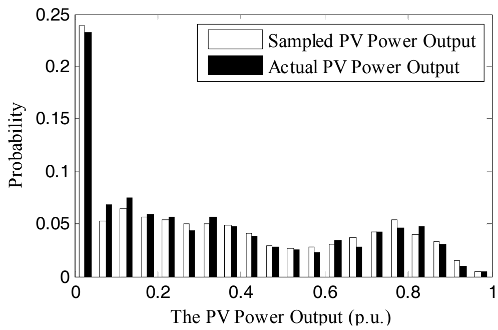

Fig. 4. Probability distribution of power output of P3

5.3 Probabilistic power flow analysis for 34-bus distribution network containing PV generation

The PPF studies for distribution networks that consider the correlations of PV generation are performed using the 34-bus test system incorporating the three PV generators. The 34-bus radial distribution network is shown in Fig. 5. The numbers with underline denote the load bus numbers and the numbers behind 'L' denote the line numbers.

To verify the proposed PPF method and analyze the impacts of the randomness and correlation of PV generation on the PPF, the following three cases are studied. The randomness of PV generation is considered in both cases 1 and 2 but the correlation of PV generation is only considered in case 1. No PV generator is included in case 3.

Case 1: Three PV generators P1 (45 kW), P2 (18 kW), and P3 (60 kW) that are correlated with one another are added to buses 26, 27, and 34, respectively. The total capacity of the added PV power is 123 kW. The PV generation penetration is 29.7%, which is obtained by dividing the capacity of integrated PV generators with the mean value of total network active loads. The proposed probabilistic PV model is used to model the randomness and correlation of PV generation.

Case 2: The connection location and capacity of the PV generators are the same as those in Case 1. The probabilistic model of the PV generation in 0 is used to model the power outputs of P1, P2, and P3 respectively, that is, only the

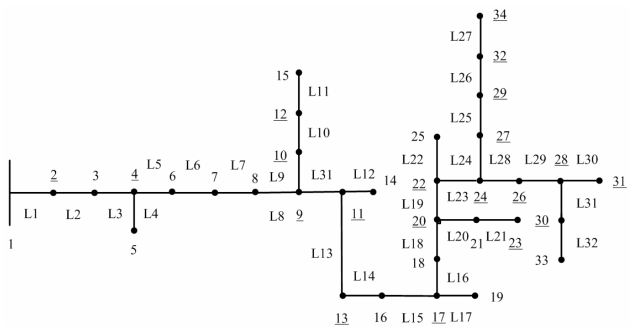

Fig. 5. 34-bus distribution network

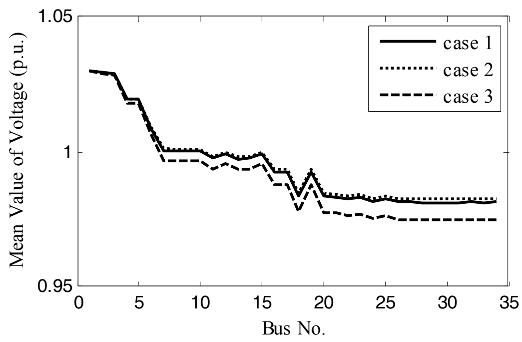

Fig. 6. Mean values of bus voltage magnitudes

randomness of PV generation is considered, and the power outputs of the different PV generators are regarded as independent of one another.

Case 3: No PV generator is connected to the distribution network.

The mean and variance values of the voltage magnitudes/angles at each bus are obtained based on the proposed PPF analysis method under three different cases and the results are given in Figs. 6-9 respectively. Fig. 10 shows the probability density curves of the voltage magnitude at bus 26 under three different cases. The values in the horizontal and vertical axis denote the voltage magnitude and probability density respectively in Fig. 10. The following observations can be made:

The voltage level of the distribution network is improved by the connection of the PV generators, especially the PV connection bus. Take bus 26 as an example. The probability of voltage magnitude in the interval [0.98, 1] becomes extremely bigger than that in the cases without a connected PV generator.

The fluctuation of bus voltage increases because the variance of bus voltage increases with the connection of PV generators. This increase is caused by PV generation being a variable resource whose production is influenced by ever-changing weather conditions.

Neglecting the correlation of PV generation leads to a biased estimation of bus voltage magnitudes and angles. The correlation has little impact on the mean values of the bus voltage magnitudes and angles, but significantly influences the variance values or probability distributions of bus voltage magnitudes and angles.

The mean and variance values of line flows are shown in

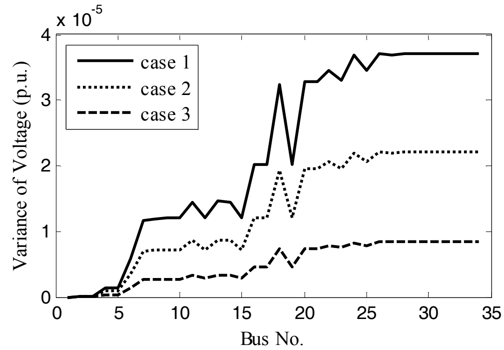

Fig. 7. Variance values of bus voltage magnitudes

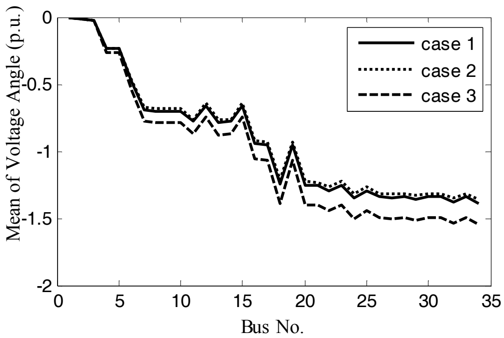

Fig. 8. Mean values of bus voltage angles

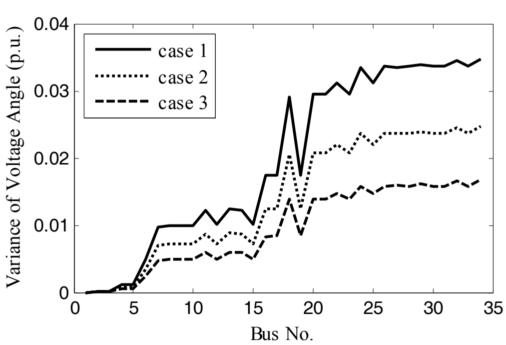

Fig. 9. Variance values of bus voltage angles

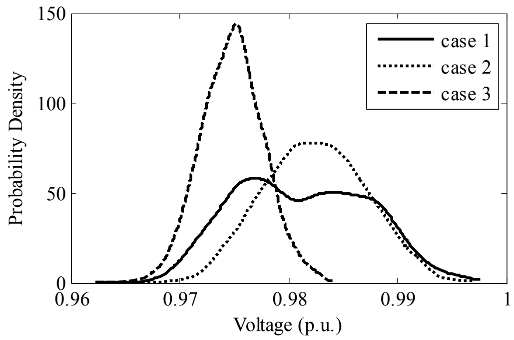

Fig. 10. Probability density curves of voltage magnitude at bus 26

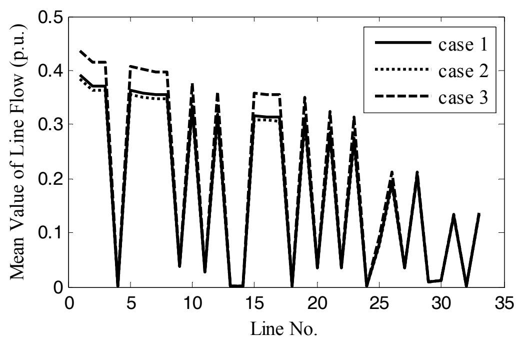

Fig. 11. Mean values of line flows

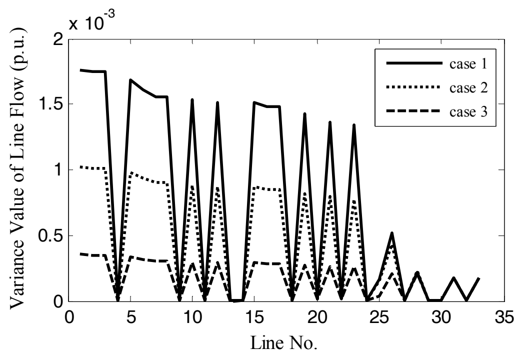

Fig. 12. Variance values of line flows

Figs. 11 and 12, respectively. The probability density and cumulative probability curves of line flow through line 23 under three different cases are given in Figs. 13 and 14

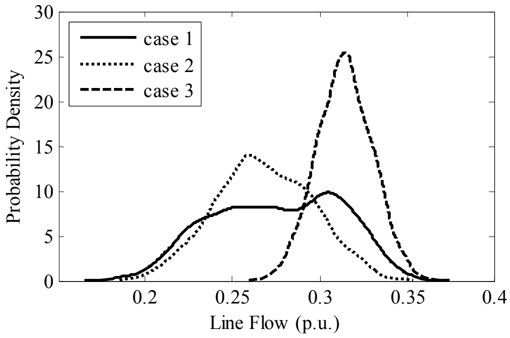

Fig. 13. Probability density curves of line flow through line 23

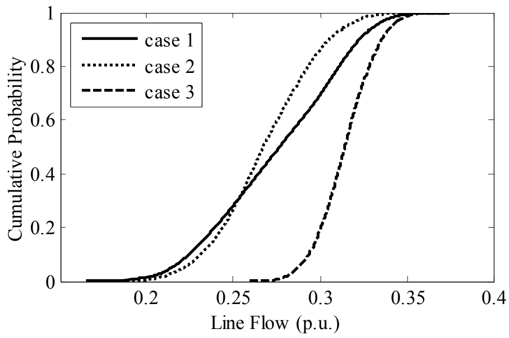

Fig. 14. Cumulative probability curves of line flow through line 23

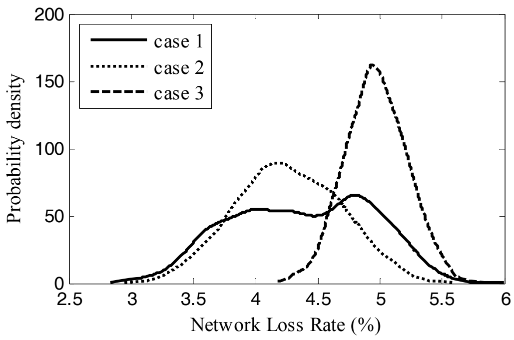

Fig. 15. Probability density curves of network loss rate

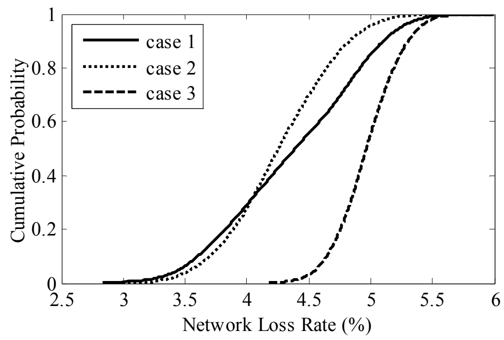

Fig. 16. Cumulative probability curves of network loss rate

respectively. The values in the horizontal axis denote the line flow and the values in the vertical axis denote the probability density and cumulative probability respectively in Figs. 13 and 14. As shown in Figs. 11 and 12, the mean values of line flows are decreased, whereas the variance values are increased by the connection of PV generation. The correlation of PV generation has little impact on the mean values of line flow. However, neglecting the correlation of PV generation has great impact on the underestimation of line flow variance. Figs. 13 and 14 show that the probability distribution of line flow through line 23 that are connected to three correlated PV generators is significantly influenced by the correlation of PV generation. That is, ignoring the correlation leads to a biased analysis of line flows.

The network loss rate is also calculated by the proposed PPF method and the probability density and cumulative probability curves of loss rate under three PV connection cases are given in Figs. 15 and 16 respectively. The values in the horizontal axis denote the network loss rate and the values in the vertical axis denote the probability density and cumulative probability respectively in Figs. 15 and 16. It can be observed that the loss rate is decreased by the PV generation. The probability that the loss rate falls in interval [4%, 4.5%] under case 3 is distinct from that under cases 1 and 2. Neglecting the correlation of PV generation has significant impact on the probability distribution of loss rate. The correlation is related to the synchronism of the PV power output values and affects the total power output of the multiple PV generators and network loss in each simulation. Hence, the probabilistic distribution of network loss is associated with the correlation of PV generation.

5.4 Probabilistic power flow analysis for 69-bus distribution network containing PV generation

The proposed PPF method is also tested by the 69-bus distribution network incorporating three PV generators. Fig. 17 shows the 69-bus radial distribution network. The numbers with underline denote the load bus numbers.

The following three cases are developed to analyze the impacts of the randomness and correlation of PV generation on the performance of distribution network. The

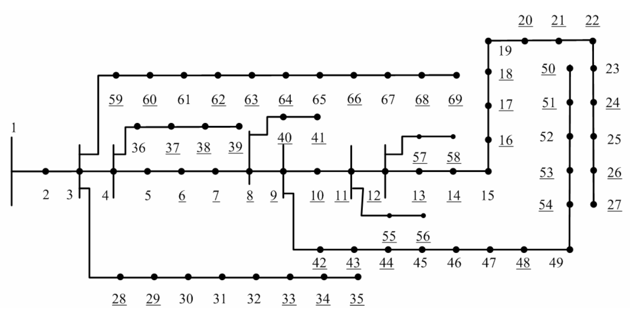

Fig. 17. 69-bus distribution network

randomness of PV generation is considered in both cases 4 and 5 but the correlation of PV generation is only considered in case 4. No PV generator is included in case 6.

Case 4: Three PV generators P4 (300 kW), P5 (300 kW), and P6 (200 kW) that are correlated with one another are added to buses 38, 39, and 53, respectively. The total capacity of the added PV power is 800 kW. The PV generation penetration is 21.1%. The proposed probabilistic PV model is used to model the randomness and correlation of PV generation.

Case 5: The connection location and capacity of the PV generators are the same as those in Case 4. The probabilistic model of the PV generation in [17] is used to model the power outputs of P4, P5, and P6 respectively, that is, only the randomness of PV generation is considered, and the power outputs of the different PV generators are regarded as independent of one another.

Case 6: No PV generator is connected to the distribution network.

The mean and variance values of bus voltage magnitudes/

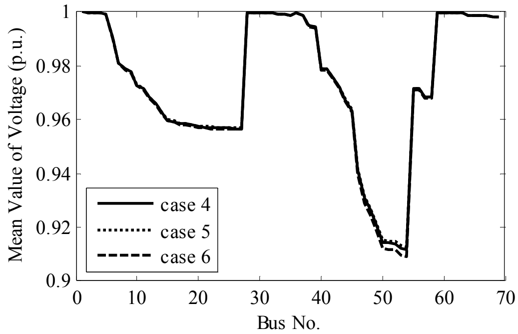

Fig. 18. Mean values of bus voltage magnitudes

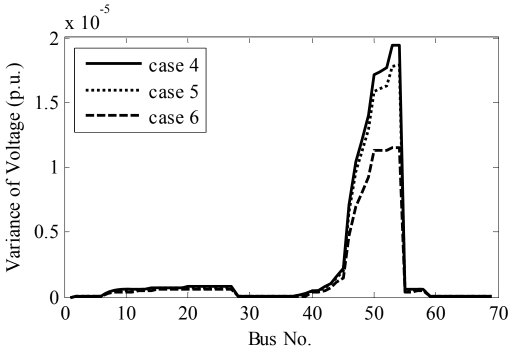

Fig. 19. Variance values of bus voltage magnitudes

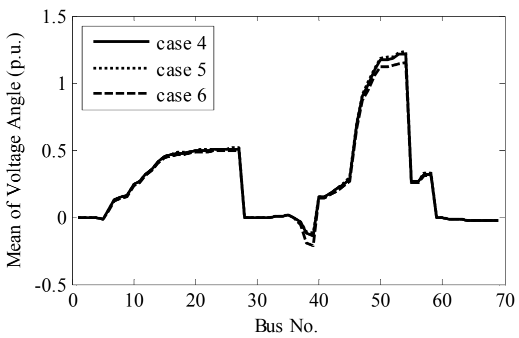

Fig. 20. Mean values of bus voltage angles

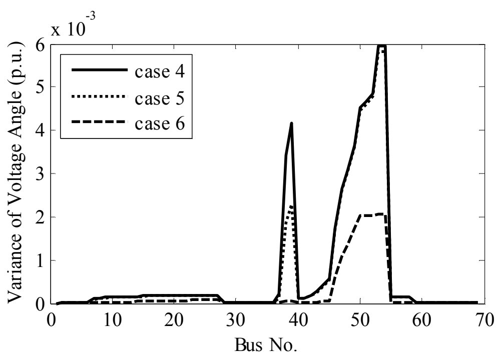

Fig. 21. Variance values of bus voltage angles

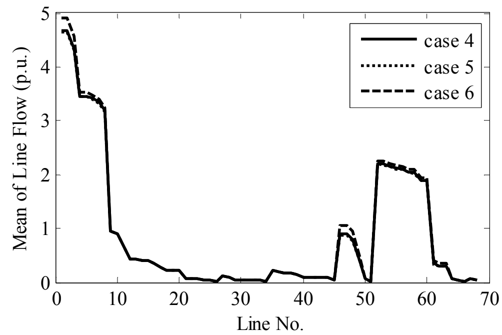

Fig. 22. Mean values of line flows

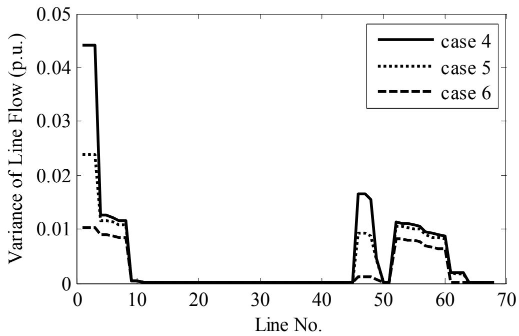

Fig. 23. Variance values of line flows

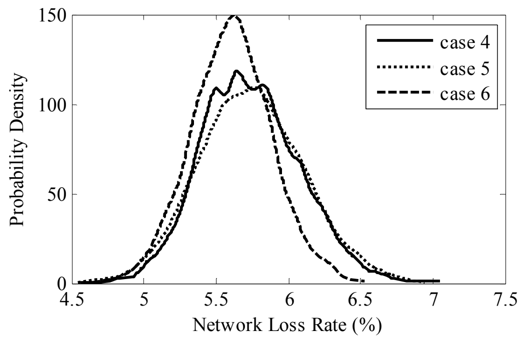

Fig. 24. Probability density curves of network loss rate

angle are shown in Figs. 18-21 respectively. The mean and variance values of line flows are given in Figs. 22 and 23 respectively. Fig. 24 is probability density curves of network loss rate under three different PV integration cases. It can be observed that the fluctuations of bus voltage magnitudes/angles and line flows are increased, but the line flows and network loss are decreased with the connection of PV generation. Moreover, neglecting the correlations of PV generation has little impact on the mean values of bus voltage magnitudes/angles and line flows but significantly influences the variance values or probability distributions of bus voltage magnitudes/angles, line flows, and network loss.

6. Conclusions

This paper presents a PPF analysis method for distribution networks that incorporates the correlation of PV generation. Based on nonparametric kernel density estimation theory, the probabilistic model of PV generation is proposed without any assumption of theoretical parametric distribution. Combined with the proposed PV model, the PPF method is developed based on the Monte Carlo technique to evaluate the influence of the randomness and correlation of PV generation on the performance of distribution networks.

The historical power output data of three neighboring PV generators located in Oregon, USA, and 34-bus/69-bus radial distribution network are used to demonstrate the correctness, effectiveness, and application of the proposed PV model and PPF method. Based on the simulation results and the discussion in previous sections, the following conclusions can be drawn:

The randomness and correlation of PV generation can be captured by the proposed PV model with high accuracy.

The PPF method can efficiently calculate the impact of the randomness and correlation of PV generation on distribution networks in terms of bus voltage, line flows, and network loss.

The line flows and network loss are decreased since the PV generation is close to loads. Meanwhile, the fluctuations of bus voltage magnitudes/angles, line flows and network loss are increased with the connection of PV generation.

The correlations of PV generation must be considered in the analysis of PPF for distribution networks. Neglecting or underestimating the correlations of PV generation has little impact on the mean values of bus voltage magnitudes/ angles and line flows but significantly influences the variance values or probability distributions of bus voltage magnitudes/angles, line flows, and network loss.

The correlation of PV generation has an important impact on the PPF analysis for distribution networks with PV generation. Accurately modeling of PV generation correlations will become highly important as the PV generation penetration level increases in the future.

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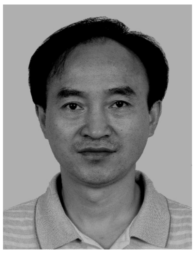

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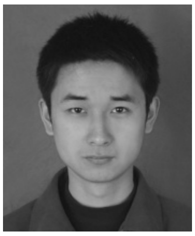

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