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The Wilcoxon Signed-Rank Fuzzy Test on Rate of Internal Division

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Abstract

We shall consider fuzzy hypotheses test for signed-rank Wilcoxon fuzzy test by fuzzy difference on rate of internal division. Fundamental to these discussion are fuzzy number data and Wilcoxon signed-rank fuzzy test of a fuzzy hypothesis H_{f0} which is based upon a fuzzy statistics whose distribution does not depend upon the specified distribution or any parameters.

Key Words: Wilcoxon signed-rank test, Degree of acceptance and rejection, Fuzzy hypotheses testing, Rate of internal division.

1. Introduction

We test the fuzzy hypothesis under the condition that the distribution of a fuzzy random variable X is unspecified distribution by fuzzy number data.
The parameter happens to be fuzzy quantities of the distribution, and if we work with uncertain function of the order statistics, this method of fuzzy statistical inference is applicable to all distribution.

To use the *Wilcoxon signed-rank fuzzy test*, we need to seek the rank of the fuzzy difference, and compute the fuzzy sums for the fuzzy difference of the rank with adjusted positive range.
The rate of internal division for fuzzy hypotheses membership function with respect to membership function of fuzzy critical region was shown in Kang, and Jung[3], where the results is illustrated by the grade for the judgement of acceptance or rejection for the fuzzy hypotheses[2].

For fuzzy alternative hypotheses testing $H_{f0}: m_\theta = m_{\theta_0}$ there is another procedure that uses the magnitude of fuzzy differences(see [4]).

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Thus we show the *Wilcoxon signed-rank* fuzzy test
by rate of internal division and fuzzy significant level for fuzzy statistics whose distribution does not depend upon specified distribution or any parameters.

In Section 2, we show fuzzy number data for observed fuzzy random sample. signed-rank for fuzzy number data was shown in Section 3. A rate of internal division for fuzzy number by critical region is shown in Section 4. Finally, we illustrate the fuzzy hypothesis in terms of battery's charging time for fuzzy number of fictitious data.

2. Fuzzy data number

Let $K_c(\Re^p)$ be the class of the non-empty compact convex subsets of $\Re^p$. We will consider the *class of fuzzy sets*

$$F_c(\Re^p) = \{U: \Re^p \to [0,1] | U^{(\delta)} \in K_c(\Re^p) \text{ for all } \delta \in [0,1]\} \tag{2.1}$$

where $U^{(\delta)}$ stands for the $\delta-level$ of U(i.e. $U^{(\delta)} = \{x \in \Re^p | U(x) \geq \delta\}$) for all $\delta \in (0,1]$, δ is precision of fuzzy number data in statistical concept. $U^{(0)}$ is the closure of the support of U [1].

Let (Ω, A, P) be the probability space. A mapping

$\boldsymbol{x}: \Omega \to F_c(\Re^p)$ is called a random fuzzy variable(RFV) if the $\delta-level$ functions $\boldsymbol{x}^{(\delta)}: \Omega \to K_c(\Re^p)$, defined by $\boldsymbol{x}^{(\delta)}(\theta) = (\boldsymbol{x}(\theta))^{(\delta)}$ for all $\theta \in \Omega$, are the random sets.

We have random fuzzy data $\boldsymbol{x} = \{\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n\}$ by sample size n. If we observe an object x_i at a time then fuzzy number data is $\widetilde{x_i} = [x_i, x_i, x_i]$, $i = 1,2,\cdots,n$.

When we observe the object x_i twice to get x_{i1}, x_{i2} respectively, we then write fuzzy number data $\widetilde{x_i} = [l, c, r]$ where $l = \min\{x_{i1}, x_{i2}\}$, $c = \dfrac{x_{i1} + x_{i2}}{2}$ and $r = \max\{x_{i1}, x_{i2}\}$, $i = 1,2,\cdots,n$.

If we observe over three times for an object x_i as $x_{i1}, x_{i2}, x_{i3}, \cdots$, then we have fuzzy number data $\widetilde{x_i} = [x_{il}, x_{ic}, x_{ir}]$, $i = 1,2,\cdots,n$. where

$$mi = \min\{x_{i1}, x_{i2}, x_{i3}, \cdots\}, \tag{2.2.1}$$

$$me = median\{x_{i1}, x_{i2}, x_{i3}, \cdots\}, \tag{2.2.2}$$

$$ma = \max\{x_{i1}, x_{i2}, x_{i3}, \cdots\}, \tag{2.2.3}$$

and

$$x_{il} = mi - (me - mi), \quad x_{ic} = me,$$
$$x_{ir} = ma + (ma - me), \quad i = 1,2,\cdots,n, \tag{2.2.4}$$

The reason why we wrote x_{li} and x_{ri} is that the observation mi and ma are likely to appear by possibility over 0.5.

A $\delta-$level set of a fuzzy number data $\widetilde{x_i}$ is a set of $[\widetilde{x_i}]^{\delta}$ and defined by

$$[\widetilde{x_i}]^{(\delta)} = \{x | m_{x_i}(x) \geqq \delta, 0 \leqq \delta \leqq 1\}, \quad i = 1,2,\cdots,n. \tag{2.3}$$

A $\delta-$level set of fuzzy data number $\widetilde{x_i}$ is a convex fuzzy set which is a normal, closed and bounded interval denoted by $[\widetilde{x_i}]^{(\delta)} = [x_{il}, x_{ic}, x_{ir}]^{(\delta)}$, $i = 1,2,\cdots,n$.

3. Signed-fuzzy rank

Wilcoxon signed-ranks test uses more information than the sign test, it is often a more powerful test. The Wilcoxon signed-rank fuzzy test uses the signed -fuzzy rank. We have the difference degree of fuzzy number $\tilde{A}$ with respect to $\tilde{B}$ define as follows.

Definition 3.1. We define the difference degree of $\tilde{A}$ and $\tilde{B}$ by $\mathrm{d}(\tilde{A} \le \tilde{B})$ as follows;

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{(A_r - A_l) + (B_r - B_l)}{(A_r - A_l) + (B_r - B_l)} = 1, \quad A_r \le B_l, \tag{3.1.1}$$

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{B_r - A_r}{(A_r - A_l) + (B_r - B_l)}, \quad A_l \le B_l \le A_r \le B_r, \tag{3.1.2}$$

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{(B_l - A_l) + (B_r - A_r)}{(A_r - A_l) + (B_r - B_l)} = 0, \quad A_l = B_l,\ A_r = B_r, \tag{3.1.3}$$

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{B_l - A_l}{(A_r - A_l) + (B_r - B_l)}, \quad B_l \le A_l \le B_r \le A_r, \tag{3.1.4}$$

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{(B_l - A_l) + (B_r - A_r)}{(A_r - A_l) + (B_r - B_l)}, \quad A_l \le B_l \le B_r \le A_r \text{ or } B_l \le A_l \le A_r \le B_r, \tag{3.1.5}$$

$$\mathrm{d}(\tilde{A} \le \tilde{B}) = \frac{(A_l - A_r) + (B_l - B_r)}{(A_r - A_l) + (B_r - B_l)} = -1, \quad B_r \le A_l. \tag{3.1.6}$$

To obtain the Wilcoxon signed-ranks fuzzy test statistics, we do the following two procedures.

(i) When an observation fuzzy number data $\widetilde{X_i}$ is equal to the hypothesized median $\widetilde{\theta_0}$, we eliminate it from the calculation and reduce the sample size accordingly.

(ii) We subtract the hypothesized median from each observation; that is, for each observation, we find

$$\widetilde{Z_i} = \widetilde{X_i} \ominus \widetilde{\theta_0}, \quad i = 1,2,\cdots,n \tag{3.2}$$

by Zadeh's extension principle.

We want to know that the rank of the fuzzy differences from the smallest one to the largest one without regard to their signs. In other words, for the rank $|\widetilde{Z_i}|$, the absolute value of the differences is denoted by

$$|\widetilde{Z_i}| = \begin{cases} \widetilde{Z_i}, & Z_{il} > 0 \text{ or } Z_{ir} < 0 \\ \widetilde{Z_i} = [0, \max\{Z_{il}, Z_{ir}\}], & Z_{il} < 0 < Z_{ir}, \end{cases} \tag{3.3}$$

$i = 1,2,\cdots,n$.

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If two or more fuzzy numbers $|\widetilde{Z}_i|$ are equal, we assign each tied value the mean of the rank position occupied by the fuzzy differences that are tied.

Assign to each rank the sign of the fuzzy difference.

In order to seek the rank of fuzzy number $|\widetilde{Z}_i|$, we have the following definition by definition 3.1 and obtain the ranks with $|\widetilde{Z}_i|$ as following definition.

Definition 3.2. Instead of the rank $|\widetilde{Z}_i|$, actually, we use S_i by

$$S_i = \sum_{j=1}^{n} I_j D(|\widetilde{Z}_i| \le |\widetilde{Z}_j|) \tag{3.4}$$

where $I_j = \begin{cases} 0, & D(|\widetilde{Z}_i| \le |\widetilde{Z}_j|) \le 0 \\ 1, & D(|\widetilde{Z}_i| \le |\widetilde{Z}_j|) > 0, \ i = 1,2,\cdots,n. \end{cases}$

Obtain the sum of the rank with positive sign T^+, we practically use $S_i, \ i=1,2,\cdots,n$ by Definition 3.2.

$R(S_i)$ denotes the rank of $|\widetilde{Z}_i|$ among $\{S_1, S_2, \cdots, S_n\}$, where the rankings are from low to high for $i = 1,2,\cdots,n$.

From Definition 3.1 and Definition 3.2, we have signed-rank Wilcoxon fuzzy test statistic by

$$T^+ = \sum_{i=1}^{n} \psi(\widetilde{Z}_i) R(|\widetilde{Z}_i|) \tag{3.5}$$

where $\psi(\widetilde{Z}_i) = \begin{cases} 0, & Z_{ir} < 0 \\ \dfrac{Z_{ir}}{Z_{ir} - Z_{lr}}, & Z_{lr} < 0 < Z_{ir} \\ 1, & 0 < Z_{il}, \end{cases}$

$i = 1,2,\cdots,n$.

We reject $H_{f0} : m_\theta = m_{\theta_0}$ at the $\tilde{\alpha}$ level of fuzzy significance in favor of H_{f1} if T^+ is less than $t^+(\alpha, n)$ for n. Alternatively we compare our calculated value of T^+ with the tabulated values of $t^+(\alpha, n)$ to see whether the probability α associated with T^+ is less than our stated level of fuzzy significance.

4. Weighted rate of internal division

Let the fuzzy number T be a fuzzy test statistics by fuzzy random sample from sample space Ω.

Let $\{P_\theta, \theta \in \Omega\}$ be a family of fuzzy probability distribution, where θ is a parameter vector of Ω.

Choose a fuzzy number T whose value is likely appears to best reflect the plausibility of the fuzzy hypothesis being tested. Let us consider a fuzzy number C of critical region, we have a rejection or acceptance degree index of rate for internal division of T which regard to C by $\delta-level$ [5].

Definition 4.1. If we have a fuzzy number T in $\Re$ then we consider a rate of internal division $\mathbb{R}_{ID}$ by $\delta-level$ as;

$$\mathbb{R}_{ID} = \frac{T_r - k}{T_r - T_l} \quad \text{for all } k \in (T_l, T_r) \tag{4.1}$$

for the fuzzy number $[T]^{(\delta)} = \{x | m_T(x) \geqq \delta, 0 \leqq \delta \leqq 1\}$ $= [T_l, \ T_c, \ T_r]^{(\delta)}$.

Definition 4.2. We define real-valued function $\mathbb{R}(\cdot)^{(\delta)}$ by supremum grade of rejection or acceptance degree by rate of internal division by $\delta-$level as;

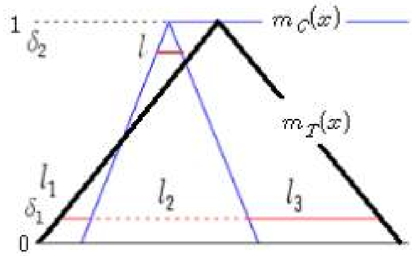
$$\mathbb{R}(0)^{(\delta)} = sup_\delta \left\{ \frac{(C_r^{(\delta)} - C_l^{(\delta)}) \times \frac{1}{2} + (T_r^{(\delta)} - C_r^{(\delta)}) \times 1}{T_r^{(\delta)} - T_l^{(\delta)}} \right\} \tag{4.2}$$

where the fuzzy number $[C]^{(\delta)} = \{x | m_C(x) \geqq \delta, 0 \leqq \delta \leqq 1\} = [C_l, \ C_c, \ C_r]^{(\delta)}$ for $T_l^{(\delta)} < C_l^{(\delta)} < C_r^{(\delta)} < T_r^{(\delta)}$,

$$\mathbb{R}(1)^{(\delta)} = 1 - \mathbb{R}(0)^{(\delta)} , \tag{4.3}$$

for the fuzzy hypothesis testing, respectively as Figure 4.1.

The weight "$\frac{1}{2}$" and "1" in equation (4.2) are ambiguity decision and exactly decision value within fuzzy rejection region.



<Fig. 1. Rate of internal division for T with C>

For example, let $l = T_r^{(\delta_1)} - T_l^{(\delta_1)}$, $l_1 = C_r^{(\delta_1)} - T_l^{(\delta_1)}$, $l_2 = T_r^{(\delta_1)} - C_l^{(\delta_1)}$ and $l_3 = T_r^{(\delta_1)} - C_r^{(\delta_1)}$ then

$$\mathbb{R}(0)^{(\delta_1)} = sup_\delta \left\{ \frac{l_1 \times 0 + l_2 \times \frac{1}{2} + l_3 \times 1}{l} \right\} \tag{4.4}$$

by $\delta = \delta_1$.

If $l = l_1$, then $\mathbb{R}(0)^{(\delta_2)} = 0$ by $\delta = \delta_2$ as Figure 4.1.

5. Illustration

We illustrate the signed-rank Wilcoxon fuzzy test by rate of internal division for fuzzy number data. We have battery's charging time by fuzzy number data for fictitious data 10 piece of smart phone's battery as Table 5.1. We want to predict the fuzzy charging time are less than $\theta_0 = [1.7, 1.8, 1.9]$.

Consider fuzzy hypothesis testing for

$$H_{f0} : m_\theta = m_{\theta_0} \text{ versus } H_{f1} : m_\theta < m_{\theta_0}, \quad (5.1)$$

There is another procedure that uses the magnitude of fuzzy differences when these are fuzzy variable.

To use this procedure, we know that

$$P_0\{T^+ < t^+(\tilde{\alpha}, n)\} = \tilde{\alpha} \quad (5.2)$$

by tabulated values.

If we have fuzzy significance level $\tilde{\alpha} = [0.05, 0.75, 0.10]$, and $T^+ = 12.845$ by equation (3.5) and fuzzy data of Table 5.1, we know that $P_0\{T^+ \geq 12.845\} = 0.937$ by tabulated values for n and $\tilde{\alpha}$. By using interpolation, we have $P_0\{T^+ \leq 12.845\} = 0.079$.

Consequently, we know that rejection degree $\mathbb{R}(0)^{(\delta=0)} = 0.58$ for $0.078 > \tilde{\alpha}$ on the fuzzy hypothesis by Definition 4.2 and Table 5.2.

[Table 1. Fictitious data of battery's charging time by fuzzy number, $i = 1, 2, \cdots, 10$]

x_{li}	x_{il}	x_{ci}	x_{ir}	x_{ri}
1.3	1.4	1.5	1.55	1.6
2	2.1	2.2	2.3	2.4
0.8	0.85	0.9	1	1.1
1.2	1.25	1.3	1.4	1.5
1.8	1.9	2	2.1	2.2
1.4	1.5	1.6	1.65	1.7
1.4	1.45	1.5	1.55	1.6
1.8	1.9	2	2.15	2.3
1.1	1.15	1.2	1.3	1.4
1.5	1.6	1.7	1.8	1.9

[Table 2. Adjusted of $|\tilde{Z}_i|$ and rank of S_i $i = 1, 2, \cdots, 10$]

$\lvert Z_{li}\rvert$	$\lvert Z_{ci}\rvert$	$\lvert Z_{ri}\rvert$	$R(S_i)$
0.1	0.3	0.6	6
0.1	0.4	0.7	7
0.6	0.9	1.1	10
0.2	0.5	0.7	8
0.1	0.2	0.5	2.5
0	0.2	0.5	2.5
0.1	0.3	0.5	5
0.1	0.2	0.6	4
0.3	0.6	0.8	9
0.2	0.1	0.4	1

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