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<Original Research Paper>

An integrated Single Vendor-Single Buyer Production Inventory System Incorporating Warehouse Sizing Decisions

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창고 크기의사결정을 포함한 단일 공급자구매자 생산재고 통합관리 시스템

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This study considers warehouse sizing decisions in an integrated single vendor-single buyer production inventory system by incorporating new decision variables and constraints associated with warehouse size into the formulations. Two typical inventory control policies proposed in the literature (i.e., Identical Delivery Quantity and Deliver What is Produced) have been investigated with consideration of warehouse investment costs. The numerical study shows that Deliver What is Produced is less flexible than Identical Delivery Quantity, resulting in the conclusion that the latter would be preferable when considering warehouse investment costs.

Keywords: Inventory, Integrated Vendor-buyer Model, Warehouse Sizing, Decisions, Inventory Control Policy

1. Introduction

A major trend of inventory management research studies has been the integration of logistics activities and inventory control, as well as the collaborative inventory management between sellers and buyers, considering deterministic and stochastic demands (Williams and Tokar, 2008). However, even though a coordinated approach between the strategic and tactical levels in supply chains is required to achieve more optimal management, only a few studies have dealt with such an integrated perspective, in particular, considering the warehouse capacity (strategic level) in the inventory control problem (tactical level) (White and Francis, 1971; Lowe *et al*., 1979; Cormier and Gunn, 1996; Goh *et al*., 2001; Petinis *et al*., 2005; Lee and Elsayed, 2005, and Lee and Wang, 2008).

White and Francis (1971) investigated the strategy of owned and leased warehouses from the buyer's perspective under deterministic and probabilistic demands. Lowe *et al*. (1979) also proposed a similar model. In 1996, Cormier and Gunn addressed the warehouse sizing problem, considering the ordering policy from the vendor's perspective. Goh *et al*. (2001) studied the warehouse sizing problem from the vendor's perspective, dealing with multiple stock-keeping units (SKUs). For one SKU, they provided a closed form solution by using discrete choices of warehouse sizes and a piecewise linear cost function, while, for multi SKUs, they developed a heuristic to solve the problem. Similarly, Petinis *et al*. (2005)

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dealt with the multi product EOQ problem, considering additional decision variables for warehouse sizing.

Lee and Elsayed (2005), suggested an optimization model for the warehouse storage capacity from the vendor's perspective under a storage policy based on full turnover. They also considered the total cost of owned and leased storage spaces. Lee and Wang (2008), dealt with managing the level of the consigned inventory with a buyer's warehouse capacity constraint. In their model, the warehouse was provided by the buyer for the vendor to stock, so they assumed that the inventory holding cost for the vendor was greater than that of the buyer, since all inventory costs would be paid by the buyer. They did not consider the warehouse capacity as a decision variable, but instead as some given values under which the available amount of stock is restricted. To our knowledge, no one has yet dealt with an integrated production-inventory system while also considering the warehouse sizing problem.

Work towards a solution for the production-inventory policy problem for the integrated vendor-buyer system started with Goyal (1976). Monahan (1984) and Banerjee (1986) addressed the pricing model in the integrated system. Lu (1995) provided an integrated model with a finite production rate. A generalized shipment policy was suggested by Hill (1997). Viswanathan (1998) compared the two extreme cases of shipment policy, Identical Delivery Quantity (IDQ) and Deliver What is Produced (DWP). Hill (1999), provided a globally optimal shipment policy in the integrated system. Hogue and Goyal (2000), considered the capacity constraints of the transport equipment.

For given production and demand rates in the production-inventory policy problem for the integrated vendor-buyer system, the warehouse size can be determined by the maximum inventory level during the time horizon. If the warehouse size is unrestricted, the optimal integrated inventory management policy will follow Hill's result from 1999. However, if the warehouse size is finite, more constraints, as well as the cost of building a new warehouse, must be considered in the model.

Until now, most research studies have dealt with the warehouse sizing problem only from the buyer's perspective, or the integrated production-inventory policy problem for the single-vendor and the single-buyer with a fixed warehouse capacity constraint. Therefore, the first objective of this study is to develop an integrated model for strategic level decision making for warehouse sizing, and for tactical decision making for inventory management policy. The second objective is to analyze the relationship between the shipment policy and warehouse sizes by comparing the two extreme cases of shipment policy for inventory control (i.e., IDQ and DWP) with respect to different warehouse size decisions.

2. Problem Statement

In the system considered, one vendor and one buyer are the actors in the system. They coordinate their production and inventory policies to minimize the total cost of the integrated system. The vendor will produce a large amount of products in one production cycle with a finite production rate. The products will then be delivered to the buyer through multiple shipments during one production cycle. During those operations, the amount of inventory stored in each warehouse of the vendor and the buyer must be less than the capacity of each warehouse.

Even though the system considered could be regarded a little bit simple in comparison with real business circumstances, an example relative to that can be found in the petrochemical industry : a petrochemical company tends to make a long-time (i.e., 1 year or more) contract with a feedstock (i.e., naphtha) supplier to ensure consistent, quality feedstock procurement; multiple shipments of feedstock occur during one production cycle of the latter; each shipment size is restricted normally by the size of storage tanks in the former. Even though a petrochemical company could make contracts with multiple feedstock suppliers, the number of suppliers is usually very small because of managerial issues and this study may be able to provide basic and essential insight for such cases.

In the classical model proposed by Hill (1997), the objective function was to minimize the joint average annual cost, which comprises the setup cost of a vendor, the ordering cost of a buyer, and the inventory holding costs of both vendor and buyer.

If we build or lease a warehouse to store products, the investment for the warehouse incurs an additional cost to the system, regardless of the actual amount of inventory. Since the warehouse capacity must be greater than the maximum amount of inventory, the shipment policy will significantly affect the required sizes of the warehouses for both the vendor and the buyer. In order to investigate the impact of the investment cost for warehouses on the production-inventory strategy, this study employs new strategic decision factors for the warehouse sizes for both the vendor and the buyer. This kind of problem can occur when a company, which produces products, needs to make a decision for building a new warehouse to store their products, or when a highly cooperating buyer has a plan to build a new warehouse.

The notation used in this paper is given as follows.

A_1 : Set up cost per production batch.
A_2 : Shipment cost.
b_1 : Variable cost for building a warehouse per unit of product for the vendor.
b_2 : Variable cost for building a warehouse per unit of product for the buyer.
$C1^0$: Total annual cost of the integrated system under IDQ without warehouse building costs.
$C1$: Total annual cost of the integrated system under IDQ with warehouse building costs.
$C^s_{Q,n}$: Total annual cost of the integrated system under IDQ with warehouse building costs for given Q and n.
$C2$: Total annual cost of the integrated system under DWP with warehouse building costs.
D : Total yearly demand rate.
F_1 : Fixed cost for building a warehouse for the vendor.
F_2 : Fixed cost for building a warehouse for the buyer.
h_1 : Stock holding cost per unit time for the vendor.
h_2 : Stock holding cost per unit time for the buyer.
$I_1^{\max}$: Maximum inventory level for the vendor.
$I_2^{\max}$: Maximum inventory level for the vendor.
m_1 : Annual variable cost for building a warehouse per unit of product for the vendor $(= b_1 r)$.
m_2 : Annual variable cost for building a warehouse per unit of product for the buyer $(= b_2 r)$.
n : The number of total shipments during one production cycle.
P : Yearly production rate.
Q : Lot size.
q_i : ith shipment size under DWP.
q : Shipment size under IDQ ($q = q_1$).
r : Discount rate (interest).
t_k : Time when kth shipment is delivered.
t_p : Time to process a batch to produce Q amount $(= Q/P)$.
T_1^0 : Shipment cycle time when the warehouse building cost is not considered in IDQ $(= q/P)$.
T_1 : Shipment cycle time when the warehouse building cost is considered in IDQ ($\leq T_1^0$).
u_1^0 : Warehouse size of the vendor without considering warehouse building costs.
u_1 : Warehouse size of the vendor with consideration of the warehouse building costs.
u_2^0 : Warehouse size of the buyer without considering warehouse building costs.
u_2 : Warehouse size of the buyer with consideration of the warehouse building costs.
z : Total stock in the system when the production of a batch starts.
λ : Shipment increase factor regarding the size of successive shipments within one production cycle ($\lambda = 1$ for IDQ, $\lambda = P/D$ for DWP).

To construct the problem, we employed several assumptions.

1. The demand rate for the product is deterministic and constant over time.
2. A fixed transportation cost is incurred for each shipment.
3. Shortages are not allowed.
4. Set-up and transportation times are ignored.
5. The manufacturing set-up cost, unit inventory holding cost for the vendor and the buyer, cost of a shipment from the vendor to the buyer and the transportation capacity are known.
6. The time horizon is infinite.
7. The inventory holding cost of the buyer is greater than or equal to that of the vendor.
8. Successive shipment sizes in one production cycle increase in accordance with the factor λ.
9. The maximum inventory level must be less than the warehouse capacity.

The decision variables are the production lot size Q, the number of shipments in one production cycle n, the shipment cycle time T_1, and the warehouse sizes for the vendor and the buyer (i.e., u_1 and u_2). Note that, for the two special shipment policies (i.e., IDQ and DWP), the i^{th} shipment size, q_i, can be determined by Q and n. The constraints that the maximum inventory level is less than the warehouse capacity for each of the vendor and the buyer should also be incorporated into the model.

3. Identical Delivery Quantity

3.1 Formulation

(1) Warehouse building cost

The Capital Recovery Factor (CFR) converges to r as the time horizon goes to infinity. The total warehouse building cost converted to each annual period is as follows:

$$(F_1 + F_2)r + (b_1 u_1 + b_2 u_2)r = (F_1 + F_2)r + m_1 u_1 + m_2 u_2$$

where $(F_1 + F_2)r$ is the annual fixed building cost and $(b_1 u_1 + b_2 u_2)r$ is the annual variable building cost with respect to the warehouse size ($m_1 = b_1 r$ and $m_2 = b_2 r$).

(2) Inventory profile change with consideration of the warehouse building cost

Prior studies (Lu, 1995; Hill, 1997) proposed the following formula for the annual cost incurred by the integrated system without warehouse building costs :

$$C1^0 = \frac{(A_1 + nA_2)D}{Q} + h_1\left\{\frac{QD}{np} + \frac{Q}{2}\left(1 - \frac{D}{P}\right)\right\} + \frac{Q}{2n}(h_2 - h_1) \quad (1)$$

Since all inventory must be stored in the warehouses (see assumption 9), new constraints should be considered to deal with the relationship between the warehouse capacities and maximum inventory levels : the maximum inventory level for the vendor, $I_1^{\max}$, must be less than or equal to u_1; and the maximum inventory level for the buyer, $I_2^{\max}$, must be less than or equal to u_2.

Now, we describe how the system could be affected by the warehouse building costs. If we consider the warehouse building costs, it is obvious that u_1 and u_2 must be equal to $I_1^{\max}$ and $I_2^{\max}$, respectively. Let's suppose that the annual warehouse building cost for the vendor, m_1, is more expensive than that for the buyer, m_2. In order to reduce the total warehouse building cost, we may need to reduce the vendor's warehouse size. If we deliver stock to the buyer earlier than the original cycle time, T_1^0(i.e., q/D), $I_1^{\max}$ will decrease and $I_2^{\max}$ will increase so that the total warehouse building cost could decrease. However, from the reduction of the shipment cycle time, the total inventory holding cost will increase. Therefore, we need to investigate this trade-off between the warehouse building cost and the inventory holding cost. Note that $I_1^{\max}$ is dependent on the batch production amount, Q, and the number of shipments in one production cycle, n.

On the other hand, if the warehouse building cost for the buyer, m_2, is more expensive than that of the vendor, m_1, reducing the buyer's warehouse size would be helpful to reduce the total warehouse building cost. Under the IDQ policy without consideration of warehouse building costs, the maximum amount of inventory in the buyer's warehouse is based on the number of shipments in one production cycle, n. Therefore, we can reduce the buyer's warehouse size by increasing n. However, since this will increase the shipping cost, we must consider the trade-off between the warehouse building cost and the shipping cost as well.

For the moment, we shall fix the Q and n, and focus on inventory profile changes. If we reduce the shipment cycle time to be less than T_1^0, the warehouse size for the buyer, u_2, will be greater than or equal to u_2^0(i.e., q). Accordingly, the warehouse size for the buyer, u_2, will be less than or equal to u_2^0. Note that the smallest possible value of u_1 is the shipment size, q, and the new shipment cycle time, T_1, is dependent on the maximum inventory level for the buyer, $I_2^{\max}$. <Figure 1> depicts the new inventory profile when the new shipment cycle time $T_1(\leq T_1^0)$ is considered. In this case, while the inventory holding cost may increase, the warehouse building cost may decrease.

(3) Annual inventory cost with consideration of the warehouse building cost

<Figure 2> describes how to calculate the buyer's inventory amount. When the shipment cycle time is T_1^0, the inventory amount is $nqT_1^0/2$. If the shipment cycle time is reduced from T_1^0 to T_1, the inventory amount increases by the parallelogram area in <Figure 2>, $q(T_1^0 - T_1)$ times $n-1$ for each n^{th} shipment. Therefore, the total inventory amount during

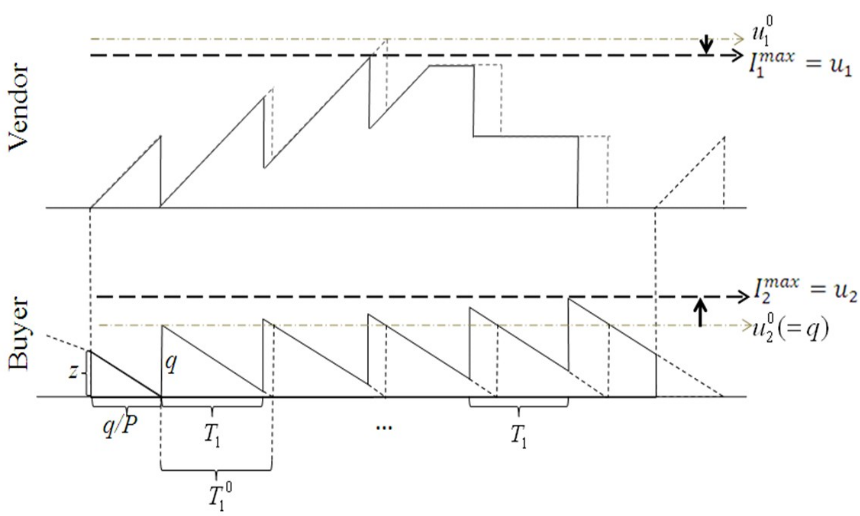

Figure 1. Inventory profile with consideration of the warehouse building cost, for given Q and n

one production cycle is

$$nT_1^0\frac{q}{2}+q\left(T_1^0-T_1\right)\frac{n(n-1)}{2} \tag{2}$$

Even when the shipment cycle time is reduced, the total system inventory amount is not changed because only the stock location is changed from the vendor to the buyer. As derived by Hill (1997), the total system inventory amount during one production cycle is

$$\frac{qQ}{p}+\frac{Q^2}{2}\left(\frac{1}{D}-\frac{1}{P}\right) \tag{3}$$

Therefore, the vendor's inventory amount is the total system inventory minus the buyer's inventory :

$$\frac{qQ}{p}+\frac{Q^2}{2}\left(\frac{1}{D}-\frac{1}{P}\right)-\frac{nqT_1^0}{2}-\frac{qn(n-1)}{2}\left(T_1^0-T_1\right) \tag{4}$$

Then, the annual vendor's inventory cost is

$$\frac{\left\{\frac{qQ}{p}+\frac{Q^2}{2}\left(\frac{1}{D}-\frac{1}{P}\right)-\frac{nqT_1^0}{2}-\frac{qn(n-1)}{2}\left(T_1^0-T_1\right)\right\}}{Q/D}h_1 \tag{5}$$

and the annual buyer's inventory cost is

$$\frac{\left\{\frac{nqT_1^0}{2}+\frac{qn(n-1)}{2}\left(T_1^0-T_1\right)\right\}}{Q/D}h_2 \tag{6}$$

The total annual inventory cost is the sum of the annual vendor's inventory cost plus the annual buyer's inventory cost :

$$\left\{\frac{QD}{nP}+\frac{Q}{2}\left(1-\frac{D}{P}\right)\right\}h_1+\left(\frac{Q}{2n}+\frac{Q(n-1)}{2n}\left(1-\frac{nDT_1}{Q}\right)\right)(h_2-h_1) \tag{7}$$

As T_1 becomes close to T_1^0 which is $\frac{Q}{nD}$, the term $\frac{Q(n-1)}{2n}\left(1-\frac{nDT_1}{Q}\right)$ becomes close to zero.

(4) Formulation

Now, we propose the final form of the cost function and the associated constraints as follows.

Problem *P1* :

Minimize *C1*

$$\begin{aligned} C1(Q, n, u_1, u_2\, T_1) = & \frac{(A_1+nA_2)D}{Q} \\ & +h_1\left\{\frac{QD}{np}+\frac{Q}{2}\left(1-\frac{D}{P}\right)\right\}+\frac{Q}{2n}(h_2-h_1) \\ & +\frac{Q(n-1)}{2n}\left(1-\frac{nDT_1}{Q}\right)(h_2-h_1)+m_1u_1+m_2u_2 \end{aligned} \tag{8}$$

s.t.

$$I_1^{\max}-u_1=0 \tag{9}$$

$$I_2^{\max}-u_2=0 \tag{10}$$

$$Q\geq 0 \tag{11}$$

$$n : \text{integer} \tag{12}$$

$$u_i\geq 0,\ i=1,2 \tag{13}$$

$$T_1\geq 0 \tag{14}$$

The annual fixed building cost, $(F_1+F_2)r$, is omitted because it is a constant.

Solving $P1$ is not trivial because the cost function, C, involves more decision variables compared to that in prior models (i.e., C^0 in Eq. (1)), and constraints 9 and 10 have

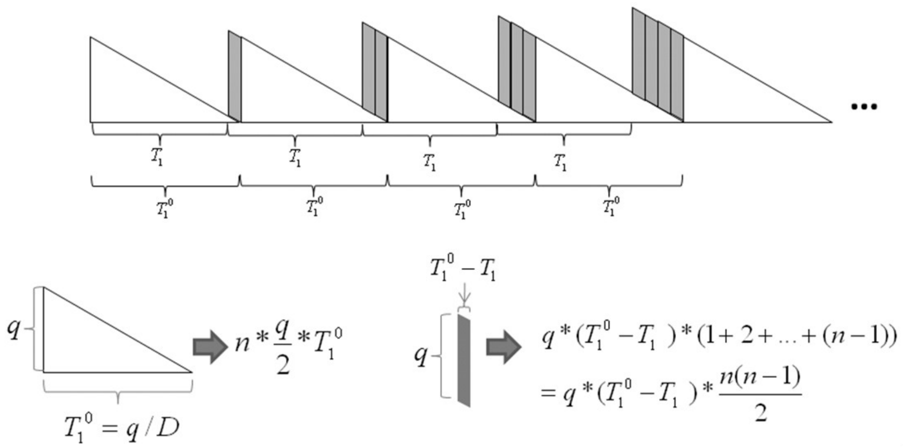

Figure 2. The buyer's inventory profile when the shipment cycle time is reduced from T_1^0 to T_1

been added. In order to find an efficient method to solve $P1$, Q and n will be momentarily fixed. Since the first three terms in the cost function C1 are constants, we can then omit those terms as well. The simplified formulation, given Q and n, is as follows.

Subproblem *S1* :
Minimize $C^s_{Q,n}$

$$C^s_{Q,n}(u_1, u_2, T_1) = \frac{Q(n-1)}{2n}\left(1-\frac{nDT_1}{Q}\right)(h_2-h_1) + m_1u_1 + m_2u_2 \quad (15)$$

s.t.

(9), (10), (13) and (14).

3.2 Analysis

Property 1 : For given Q and n, at optimal values, T_1 is a piecewise linear function of u_1.

<Figure 3> shows graphically the maximum inventory level of the vendor : it is I_{t_k} or I_{t_p}, where I_{t_k} and I_{t_p} are the vendor's inventory levels when the k^{th} shipment is delivered and when the production of Q products is completed, respectively. If the time between t_k and t_p is less than q/P, I_{t_k} will be greater than I_{t_p} :

$$I_1^{\max} = \begin{cases} I_{t_k}, \ If \ t_p - t_k \le q/P \\ I_{t_p}, \ otherwise \end{cases} \quad (16)$$

where $I_{t_k} = P\{(k-1)T_1 + q/P\} - (k-1)q = P(k-1)T + (2-k)q$ and $I_{t_p} = Q - kq$.

Accordingly, from Eq. (9) (i.e., $I_1^{\max} - u_1 = 0$),

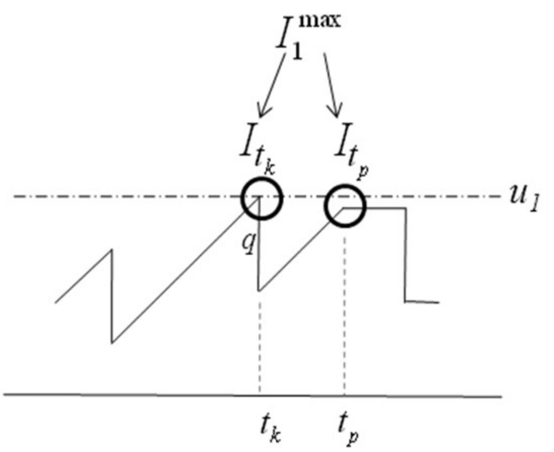

Figure 3. Maximum inventory level of the vendor

$$u_1 = \begin{cases} P(k-1)T_1 + (2-k)q, \ If \ t_p - t_k \le q/P \\ Q - kq \qquad , otherwise \end{cases} \quad (17)$$

For given Q and n, if we reduce the cycle time, T_1, $I_1^{\max}$ will decrease down to q so that the smallest available warehouse size for the vendor, $u_1^{\min}$, is q. On the other hand, when T_1 increases up to the maximum cycle time, T_1^0, $I_1^{\max}$ will increase up to the maximum value, $u_1^{\max}$, which can be calculated using Eq. (17). From Eq. (17), we can derive T_1 as a function of u_1 :

$$T_1 = \frac{u_1-(2-k)q}{(k-1)P} = \frac{1}{(k-1)P}u_1 + \frac{(k-2)q}{(k-1)P} = f(u_1) \quad (18)$$

In addition, as u_1 decreases, the number of shipments before time t_p, k increases (see Appendix).

<Figure 4> charts T_1 as a function of u_1. The intercept point of the T_1 axis is q/P when u_1 is at the minimum value, q. While this intercept point is regardless of k, the slope of Eq. (18), $\frac{1}{(k-1)P}$, decreases as k increases. Since k step w is increases as u_1 decreases, and vice versa, the slope of Eq. (18) stepwise decreases as u_1 decreases. Therefore, T_1 is a piecewise linear function of u_1.

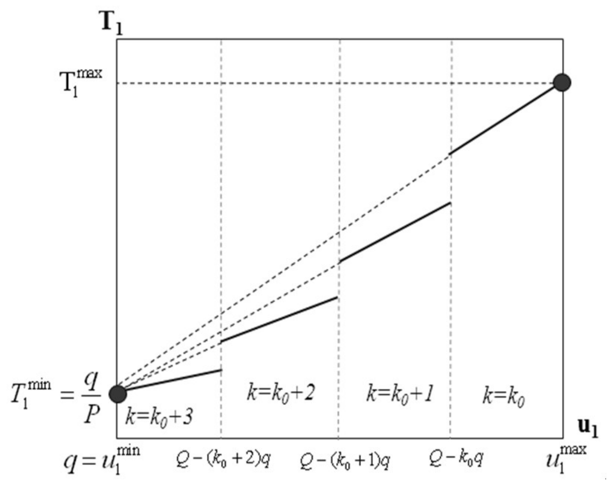

Figure 4. The relationship between T_1 and u_1 at optimal values

Property 2 : Given Q and n, at optimal values, u_2 is a piecewise linear function of u_1.

<Figure 5> shows that the buyer maximum inventory level, $I_2^{\max}$, occurs at time t_n which is the time when n^{th} shipment arrives.

$I_2^{\max}$ can be calculated from the sum of stock at time zero, z, and the delivered amount minus the consumed amount until time t_n :

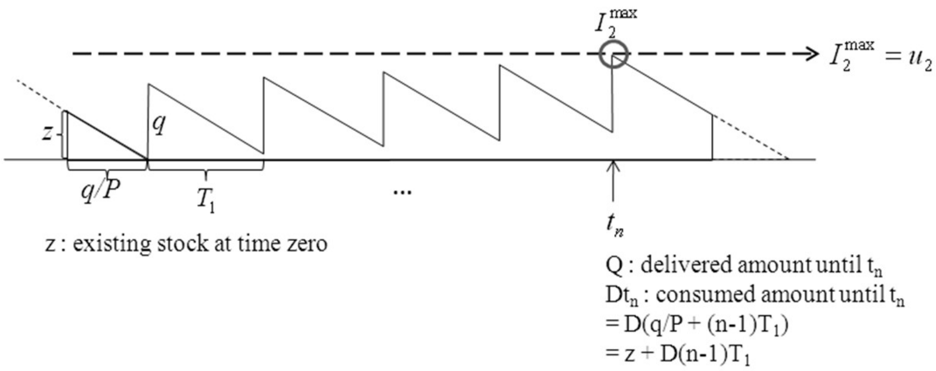

Figure 5. The buyer's maximum inventory level

$$
\begin{aligned}
I_2^{\max} &= z + Q - \{z + D(n-1)T_1\} \quad (19)\\
&= Q - D(n-1)T_1\\
&= Q - D(n-1)\left(\frac{1}{(k-1)P}u_1 + \frac{(k-2)q}{(k-1)P}\right)\\
&= Q - \frac{(n-1)D}{(k-1)P}u_1 - \frac{(n-1)(k-2)qD}{(k-1)P}.
\end{aligned}
$$

Using Eq. (10) and Eq. (19), u_2 is a function of u_1:

$$u_2 = I_2^{\max} = Q - \frac{(n-1)D}{(k-1)P}u_1 - \frac{(n-1)(k-2)qD}{(k-1)P} = g(u_1) \quad (20)$$

The intercept point of $g(u_1)$ on the u_2 axis is $Q-(n-1)qD/P$ when u_1 is at the minimum value, q. This intercept point is regardless of k. Also, k stepwise increases as u_1 decreases so that the slope of Eq. (20) stepwise decreases as u_1 decreases. Therefore, u_2 is a piecewise linear function of u_1 (see <Figure 6>).

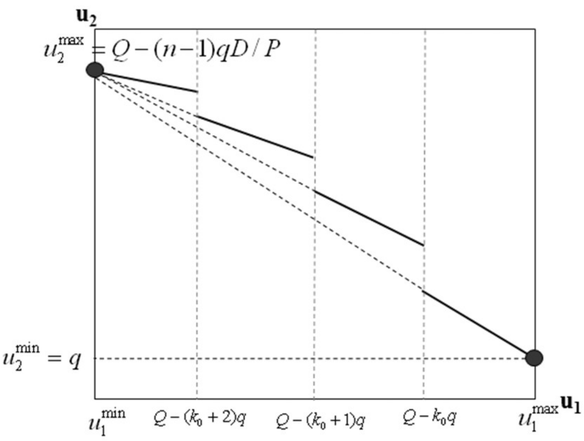

Figure 6. The Relationship between u_1 and u_2

Property 3 : C^s is at a mimimum when u_1 is at a maximum, and accordingly u_2 is at a minimum, and vice versa.

Eq. (15) shows that $C^s_{Q,n}$ is a linear combination of $-T_1$, u_1, and u_2. Here, in order to facilitate a clear description, we use $-T_1$ instead of T_1 so that all the coefficients of $-T_1$, u_1, u_2 are non-negative. Since $-T_1$ and u_2 are functions of u_1 at optimal values, these variables can be replaced with functions of u_1 which are proposed in Properties 1 and 2. <Figure 7> charts the overall relationship among $-T_1$, u_2 and u_1. Even though $-T_1$ is not a concave function of u_1, all piecewise lines are above the line extended from the lowest line; u_2 shows the same pattern as well. $C^s_{Q,n}$ will also be a piecewise linear function such that all piecewise lines are above the line extended from the lowest line. Therefore, $C^s_{Q,n}$ will be at the lowest value(i.e., the optimal value) when u_1 is equal to $u_1^{\min}$ or $u_1^{\max}$. Note that $-T_1$ and u_2 are at their maximum values when u_1 is minimum, and vice versa.

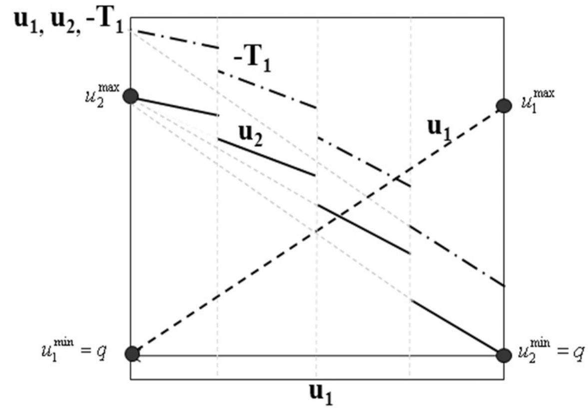

Figure 7. The relationship aming $-T_1$, and u_2 and u_1

3.3 Solution Procedure

By using property 1, 2 and 3, we can rewrite the problem P1 in the following way.

Minimize C2

$$
\begin{aligned}
C2(Q,n) = &\frac{(A_1+nA_2)D}{Q}\\
&+ h_1\left\{\frac{QD}{np} + \frac{Q}{2}\left(1-\frac{D}{P}\right)\right\} + \frac{Q}{2n}(h_2-h_1)
\end{aligned} \quad (21)
$$

$$+Min\{C^s_{Q,n}(u_1^{\max},g(u_1^{\max}),f(u_1^{\max})),\; C^{\min}_{Q,n}(u_1,g(u_1^{\min}),f(u_1^{\min}))\}$$

s.t.

(11).

Note that the constraints in Eq. (13) associated with u_1 and u_2 have disappeared. Now, since it is trivial to solve the problem P1 for given Q and n, his study employ a grid search on Q and n to find the optimal solution of P1.

4. Deliver What is Produced (DWP)

4.1 Formulation

4.1.1 Inventory profile

Under the DWP strategy, the inventory profile after adding new decision variables about warehouse sizes (i.e., u_1 and u_2) is the same as the original one because the maximum inventory levels of both sides cannot be changed. The maximum inventory levels for both the vendor and the buyer are the same at $\lambda^{n-1}-q$. Under the DWP strategy, the shipment increase factor λ is a fixed value of P/D. Therefore, for given q and n, the DWP strategy does not allow the manipulation of the inventory profile to reduce the total system cost.

4.1.2 Formulation

As derived by Hill (1997), the annual cost incurred by the integrated system without warehouse building costs is

$$(A_1+nA_2)\frac{D(\lambda-1)}{q(\lambda^n-1)}+(h_1+\lambda h_2)\frac{q(\lambda^{n-1})}{2\lambda(\lambda-1)}. \quad (22)$$

Since the inventory profile does not change after adding new decision variables about warehouse sizes (i.e., u_1 and u_2) under the DWP strategy, the annual cost with warehouse building costs included is just

$$(A_1+nA_2)\frac{D(\lambda-1)}{q(\lambda^n-1)}+(h_1+\lambda h_2)\frac{q(\lambda^{n-1})}{2\lambda(\lambda-1)}q + m_1u_1+m_2u_2. \quad (23)$$

Since $u_1=I_1^{\max}=\lambda^{n-1}q$ and $u_2=I_2^{\max}=\lambda^{n-1}q$. we can replace u_1 and u_2 with $\lambda^{n-1}q$. The final form of the formulation is as follows. The final form of the formulation is as follows.

Minimize $C2$

$$C2(n,q)=(A_1+nA_2)\frac{D(\lambda-1)}{(\lambda^n-1)q}+\left\{(h_1+\lambda h_2)\frac{(\lambda^n+1)}{2\lambda(\lambda+1)}+(m_1+m_2)\lambda^{n-1}\right\}q \quad (24)$$

s.t.

$$I_1^{\max}-\lambda^{n-1}q=0 \quad (25)$$

$$I_2^{\max}-\lambda^{n-1}q=0 \quad (26)$$

$$q\geq 0 \quad (27)$$

and (11).

4.2 Solution Procedure

In Eq. (24), let α be $(A_1+nA_2)\frac{D(\lambda-1)}{(\lambda^n-1)}$ and β be $(h_1+\lambda h_2)\frac{(\lambda^n-1)}{2\lambda(\lambda-1)}+(m_1+m_2)\lambda^{n-1}$. Then, $C2(n,\,q)$ is $\alpha/q+\beta q$, which is a convex function of q for a given n. Therefore, the optimal solution value of q for a given n is

$$q^*=(\alpha/\beta)^{1/2} \quad (29)$$

Replacing q in Eq. (24) with q^* gives the following result,

$$C2(n,\,q^*)=\alpha(\alpha/\beta)^{-\frac{1}{2}}+\beta(\alpha/\beta)^{\frac{1}{2}}=2(\alpha\beta)^{\frac{1}{2}} \quad (30)$$

$$=2\left\{\frac{(A_1+nA_2)(h_1+\lambda h_2)D}{2}\frac{(\lambda-1)(\lambda^n+1)}{\lambda(\lambda+1)(\lambda^n-1)} + (m_1+m_2)\lambda^{n-1}\right\}^{1/2}$$

Eq. (30) is a concave function of n. The proof is trivial and similar to that of Hill (1997). Therefore, it is straight forward to find the optimal number of shipments, n^*, and the optimal objective value, $C2(n^*,q^*)$.

5. Results and Discussion

This study considers the examples given by Hill (1999) for the numerical test :

[Example 1] $A_1=400$, $A_2=25$, $h_1=4$, $h_2=5$, $D=1000$, $P=3200$;

[Example 2] $A_1=400$, $A_2=25$, $h_1=4$, $h_2=7$, $D=1000$, $P=3200$.

In order to check the effect of the warehouse building costs, several combinations of warehouse building costs are tested based upon examples 1 and 2 :

$$(m_1, m_2) : (0, 0), (1, 1), (5, 1), (1, 5).$$

The first case, (m_1, m_2) = (0, 0), means that warehouse building costs for both sides are zero so that the model is exactly the same as the classical models. The second case, (m_1, m_2) = (1, 1), represents the case that the warehouse building costs of both sides are the same, and are \$1. The third and fourth cases are considered to check the impact of different warehouse building costs for the vendor and the buyer.

<Table 1> provides the total cost, batch size (Q), number of shipments (n) and each shipment size for each case in examples 1 and 2. When (m_1, m_2) = (0, 0), DWP is better than IDQ for example 1 and IDQ is better than DWP for example 2. This result corresponds with the result of Viswana than (1998) such that no one strategy outperforms another for all possible problem parameters.

However, when the system considers nonzero warehouse building costs, (i.e., (m_1, m_2) > (0, 0)), IDQ outperforms DWP for all cases. In addition, as warehouse building costs increase, the increasing rate of the total system cost under DWP is much higher than that under IDQ. This is because the inventory profile after considering warehouse building costs under the DWP strategy cannot be changed for given q and n. On the other hand, warehouse building costs under IDQ can be adjusted to reduce the total system cost. In this respect, DWP may not be a good choice since it appears to have less flexibility than IDQ. This result provides a different viewpoint compared to what previous studies have shown (e.g., Viswanathan (1998)).

6. Conclusions

This study has addressed new decision factors associated with warehouse sizes in the integrated vendor-buyer production inventory system, and identified the impact of warehouse building costs on two typical inventory policies (i.e., IDQ and DWP). A few new decision variables and constraints associated with warehouse sizes have been incorporated into the formulations for IDQ and DWP. The formulations have been simplified through the analysis of the relationship between the cost functions and warehouse sizes so that we could find optimal solutions using relatively straightforward solution methods. The numerical study showed that the IDQ

Table 1. Numerical results with several warehouse building costs under the IDQ and DWP policies

Example	Policy	Warehouse building cost (\$)		Total cost (\$)	Q	n	Shipment size
		m_1	m_2				
1	IDQ	0	0	1903.3	551.7	5	110.3, 110.3, 110.3, 110.3, 110.3
		1	**1**	**2320.9**	452.4	5	90.5, 90.5, 90.5, 90.5, 90.5
		5	**1**	**2782.4**	413.3	7	59.0, 59.0, 59.0, 59.0, 59.0, 59.0, 59.0
		1	**5**	**2635.3**	455.3	8	56.9, 56.9, 56.9, 56.9, 56.9, 56.9, 56.9, 56.9
	DWP	**0**	**0**	**1818.2**	522.5	3	36.2, 115.8, 370.5
		1	1	3453.9	275.1	3	19.1, 61.0, 195.1
		5	1	5401.4	175.9	3	12.2, 39.0, 124.7
		1	5	5401.4	175.9	3	12.2, 39.0, 124.7
2	IDQ	**0**	**0**	**2008.3**	547.7	6	91.3, 91.3, 91.3, 91.3, 91.3, 91.3
		1	**1**	**2409.7**	435.7	5	87.2, 87.2, 87.2, 87.2, 87.2
		5	**1**	**3063.4**	359.1	6	59.9, 59.9, 59.9, 59.9, 59.9, 59.9
		1	**5**	**2691.7**	445.8	8	55.7, 55.7, 55.7, 55.7, 55.7, 55.7, 55.7, 55.7
	DWP	0	0	2089.0	454.8	3	31.5, 100.8, 322.5
		1	1	3603.7	263.6	3	18.3, 58.4, 186.9
		5	1	5498.5	172.8	3	12.0, 38.3, 122.5
		1	5	5498.5	172.8	3	12.0, 38.3, 122.5

strategy outperforms DWP under the consideration of warehouse building costs, implying that DWP may be less flexible than IDQ.

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<Appendix>

Proposition : k is stepwise increasing as u_1 decreases.

proof : suppose $k = k_0$ for some value of u_1. Since $t_p = Q/P$, t_p is independent of u_1, t_{k_0} decreases as u_1 decreases because $t_{k_0} = (k_0 - 1)\,T_1 + q/P$ and T_1 decreases as u_1 decreases. From this relationship, the interval $t_p - t_{k_0}$ will increase as u_1 decreases. When $t_p - t_{k_0} = T_1$, u_1 is equal to $Q - k_0 q$. Thus,

$$t_p = t_{k_0} + T_1 = (k_0 - 1)\,T_1 + \frac{q}{P} + T_1 = k_0 T_1 + \frac{q}{P} \tag{A.1}$$

In order to reduce u_1 further below $Q - k_0 q$, T_1 should decrease further. However, since t_p is a fixed value for given Q and n, if we want to reduce T_1 further, then any other vlaue should be changed in Eq. (A.1). Since q and P are given parameters, the number of shipments beforetime t_p, k can be increase from k_0 to $k_0 + 1$. This increasing pattern for k occurs again whenever T_1 decreases down to the value of $t_p - t_k$. Therefore, k is stepwise increasing as T_1 decreases. Since T_1 decreases as u_1 decreases, k is stepwise increasing as u_1 decreases. ■