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A Study of Inventory Models for Imperfect Manufacturing Setup Considering Work-in-Process Inventory

Misbah Ullah · Chang W. Kang† · Shehereyar Mohsin Qureshi

Department of Industrial and Management Engineering, ERICA Campus, Hanyang University

재공품 재고를 고려한 제조 시스템에서의 재고 관리 모델 연구

미스바 울라 · 강창욱† · 쉐리야르 모신 코레시

한양대학교 산업경영공학과

Optimum lot size calculation for real world manufacturing environment has been focused since last few decades. Several extensions have been made to the basic economic order and production order quantity models to realize the possible practical situations in industry. However, focus on work-in-process inventory has been ignored relatively. This paper provides a comprehensive review of the models developed for group technology based manufacturing environment focusing on work-in-process inventory. Models have been extended from a perfect manufacturing conditions to an imperfect manufacturing situation considering rework, rejection and inspection. Optimum lot size has been evaluated using a simple algebraic optimization approach. Significant parameters are highlighted using sensitivity analysis for the developed models. Numerical example is used to illustrate the utilization of such models in day-to-day production setups and the impact of significant factors' variation on total cost and optimum lot size.

Keywords : Work-In-Process, Imperfect, Lot Size, Economic Order Quantity

1. Introduction

Inventory models have been widely used in todays' manufacturing and service industries in order to obtain optimum lot size with average cost minimization. Harris [1] introduced economic order quantity model for the first time in Feb, 1913. The model was later extended to production order quantity model in 1918. Since then, an extensive research has been carried out to develop models for manufacturing environments keeping average cost minimization and demands fulfillment on priority.

One major reason for these extensions is the assumption of the aforementioned models regarding processes perfection. Economic and production order quantity models assume perfect product quality. As processes are affected by a number of factors including tool wear and tear, material internal defects, machines failure and breakdowns, and environmental conditions etc. Therefore several extensions to these basic models have been made during past few decades. Imperfect production units can be reworked and its ultimate impact on lot size was initially studied by Gupta and Chakraborti [2]. Lee and Rosenblatt [3] gave the idea of increased investment in quality control techniques. They concluded that this will help organization to reduce defect rate and other inventory cost including setup cost and carrying cost. Cheng [4] focused imperfect production setup and developed a model for economical quantity order based on demand dependency for a unit production cost. Ben-Daya and Harija [5]

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† Corresponding Author : cwkang57@hanyang.ac.kr

assumed that deviation of production process from an ideal conditions is a random process. These deviations results non-conforming products. Economic order quantity model was extended for electronic units manufacturing by Salameh and Jaber [6]. They assumed that all incoming products are screened before they are processed in a manufacturing setup. It was further added that non-reworkable products are sold in a single batch at low price. Goyal and Cardenas-Barron [7] introduced simple algebraic optimization technique to calculate optimum economic order quantity. The results obtained via this approach were similar as obtained through other methods of optimization.

Biswas and Sarker [8] introduced the idea of lean manufacturing into the lot sizing inventory problems. They assumed that scrap items are recognized 'before', 'during' and 'after' the rework operation. Buffer stations idea was also incorporated to meet the customer demands and avoid shortages. Jamal et al. [9] developed model based on the assumption that reworking process can be executed either within the cycle or at the end of all cycles. Other who added in this particular direction include are [10-17, 26, 27]. Wee et al. [18] calculated lot size based on total average profit in order to optimize economic order quantity. They used renewal reward theorem (RRT) in addition to screening constraints and imperfect manufacturing processes. Researchers established that reworking an imperfect items is of significance for the manufacturing company. [19, 20, 21]. Khan et al. [22] presented a brief review of economic order quantity model for imperfect items. They mainly focused model developed by Salameh and Jaber. Their review categorized different extended models based on quality, imperfect items produced, shortages and backordering, and supply chain perspective etc.

Models considered above mainly focused two kinds of inventories i.e. raw material inventory and finish goods inventory. However, the third kind of inventory i.e. *work-in-process (WIP)* inventory has been ignored relatively. The role of *work-in-process* inventory in group technology work environment, in particular, has been significant. Lot sizing models are required to incorporate *work-in-process* inventory cost while optimizing the total cost function. The role of *work-in-process* inventory can be more substantial when the manufacturing processes are imperfect. This paper has presented a brief re-view of models developed for optimum lot size calculation focusing *work-in-process* inventory. Paper highlight significant parameters in each model with numerical examples using sensitivity analysis. The remaining sections of this paper are ordered as follows. Next section gives an overview of the inventory models developed for work-in-process inventory with assumptions and notation used. Section 3 gives illustration of the developed models through numerical examples in addition to sensitivity analysis. Conclusion and possible future extensions have been discussed in the last section.

2. Group Technology Order Quantity Model

Boucher [23] introduced the concept of *work-in-process* inventory for group technology manufacturing environment. The traditional economic order quantity model was extended by incorporating work-in-process cost into the total cost function with these assumptions. 1) Shortages are not allowed. 2). Demand is known and constant. 3). Manufacturing processes are perfect. 4). Inspection takes no time. 5). All other parameters including demand rate, set up and manufacturing time etc. are fixed and pre-determined.

Following notation have been used to develop the model.

Parameters

D demand rate of good quality product (unit product per unit time)

s setup time per cycle (unit time per setup)

m machining time per unit for lot size Q (unit time per unit product)

T_c cycle time

T_p total processing time

T average manufacturing time for each product item

C_M raw material cost per unit ($/unit of product)

C_p cost of purchase per unit of time ($/unit of time)

C_s setup cost per unit of time ($/unit of time)

C_{wip} work in process holding cost per unit of time ($/unit of time)

C_H carrying cost per unit of time ($/unit of time)

C_{total} cost (total) per unit of time ($/unit of time)

I average storage inventory

W average monetary value of the *WIP* inventory ($)

i inventory holding cost per unit of time ($/unit of time)

c average unit value of each product cost (unit of money ($) per unit)

R rate charged per unit of cell production time including all overheads, moving cost, loading/unloading cost, etc. (unit of money ($) per unit of time)

Decision Variable

Q production lot size per cycle (number of product units per cycle)

The total cost function include setup cost, finish components carrying cost and work-in-process (WIP) inventory carrying cost. *Work-in-process* carrying cost is associated with products carried during its manufacturing phase till their production.

GTOQ model, total cost function is given by

$$C_{total} = \frac{AD}{Q} + \frac{iQ}{2}\left(C_M + \frac{s}{Q}R + mR\right) + iD(C_M + \frac{s}{2Q}R + \frac{m}{2}R)(s + mQ) \quad (1)$$

where A is the setup cost and is equal to the setup time (s) multiplied by rate charged per unit of the cell production *(R).*

Algebraic optimization method can be used to obtain the optimum lot size [24] without using differentiation method of optimization.

Equation (1) can be rearranged as,

$$C_{total} = \frac{AD}{Q} + \frac{iQC_M}{2} + \frac{isR}{2} + \frac{iQmR}{2} + iDsC_M + \frac{iDs^2R}{2Q} + iDmsR + iDmQC_M + \frac{iDm^2RQ}{2}$$

Factorizing above Equation,

$$C_{total} = \left(\frac{iC_M}{2} + \frac{imR}{2} + iDmC_M + \frac{iDm^2R}{2}\right) Q + \left(AD + \frac{iDs^2R}{2}\right)\frac{1}{Q} + \frac{isR}{2} + iDsC_M + iDmsR\Big\} \quad (2)$$

Equation (2) is of the form $y(x) = a_1x + \frac{a_2}{x} + a_3$. Therefore using algebraic optimization method to obtain the optimum lot size as follow,

$$Q^* = x = \sqrt{\frac{a_2}{a_1}} = \sqrt{\frac{AD + \frac{iDs^2R}{2}}{\frac{iC_M}{2} + \frac{imR}{2} + iDmC_M + \frac{iDm^2R}{2}}}$$

$$Q^* = \sqrt{\frac{(AD + \frac{iDs^2R}{2})}{i\left(\frac{C_M + mR}{2}\right) + Dmi\left(C_M + \frac{mR}{2}\right)}} \quad (3)$$

2.1 Group Technology Order Quantity Model Considering Rework (GTOQR)

The GTOQ model focused perfect production processes and it was assumed that no rework or rejection will be produced by the manufacturing cell. However, in practical situations, the concept of rework is common especially in manufacturing cells. Barzoki et al. [25] extended the GTOQ model by incorporating the rework operation into the manufacturing lot focusing work-in-process inventory. This new extended model, known as group technology order quantity model with rework consideration (GTOQR), assumed that process may produce imperfect products which are either reworkable or non-reworkable. The demand for non-reworkable products always exist and can be sold out at reduced price. Few assumptions were made in order to derive this model in addition to GTOQ model assumptions. 1). No imperfect products are produced during the rework phase 2). Inspection takes no time. 3). Process can be stopped for rework operation at any stage of manufacturing a lot. 4). Rework-able and non-reworkable imperfect products' proportion is known and constant.

Several notation are added to develop the model.

Parameters

p_1 reworkable products percentage (%)

p_2 non-reworkable products percentage (%)

C_p purchase cost per unit of time ($/unit of time)

The model also added material purchase cost and inspection cost to calculate the total cost function. Therefore, total cost function comprises purchase cost, setup cost, inspection cost, work-in-process carrying cost and inventory holding cost for finish goods.

Therefore,

$$C_{total} = \frac{DC_M}{(1-p_2)} + \frac{AD}{Q(1-p_2)} + \frac{ID}{(1-p_2)} + \frac{i}{2}\left(\left(\frac{D}{1-p_2}\right)(s + mQ(1+p_1))(2C_M + \frac{Rs}{Q} + mR(1+p_1)\right) + \frac{i}{2}(1-p_2)Q\left(C_M + \frac{Rs}{Q} + mR(1+p_1)\right)$$

Factorizing above Equation

$$C_{total} = \left(\frac{i(1-p_2)}{2}(C_M + mR(1+p_1)) + \frac{Dim}{2(1-p_2)}(2C_M + mr(1+p_1)\right) \tag{4}$$
$$Q + \left(\frac{AD}{(1-p_2)} + \frac{Dis^2R}{2(1-p_2)}\right)\frac{1}{Q} + \frac{D}{(1-p_2)}$$
$$(C_M + I + i(C_M s + R_{ms} + R_{msp_1}) + \frac{iRs}{2}(1-p_2)$$

Equation (4) is of the form $y(x) = a_1 x + \frac{a_2}{x} + a_3$. Therefore, using algebraic optimization method to obtain the optimum lot size as follow :

$$Q^* = x = \sqrt{\frac{a_2}{a_2}}$$

$$\sqrt{\frac{\frac{AD}{(1-p_2)} + \frac{Dis^2R}{2(1-p_2)}}{\frac{i(1-p_2)}{2}(C_M + mR(1+p_1)) + \frac{Dim}{2(1-p_2)}(2C_M + mr(1+p_1)}}$$

Rearranging above Equation

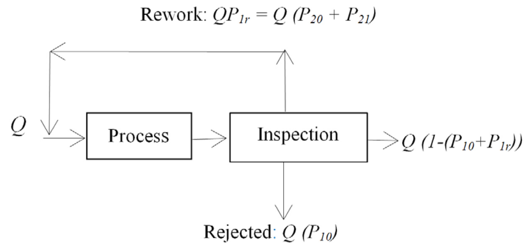
$$Q^* = \sqrt{\frac{2AD + Dis^2R}{i(1-p_2)^2(C_M + mR(1+p_1)) + Dim(2C_M + mr(1+p_1)}} \tag{5}$$

The model highlighted the impact of reworkable and non-reworkable products produced during the manufacturing process. The model is considered as extension of the basic GTOQ model as the optimum lot size calculated via GTOQR model is the same if we consider the manufacturing process as perfect. The resultant lot size will be the same as the one calculated by GTOQ model. The proof can be obtained easily if we assume that imperfection doesn't exist in manufacturing process i.e. $p_1 = p_2 = 0$.

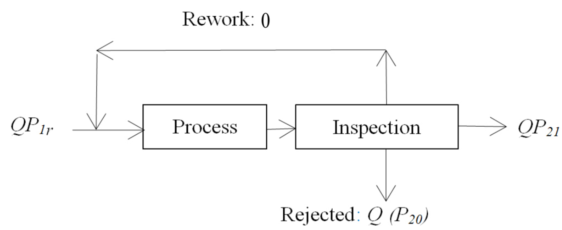
2.2 Group Technology Order Quantity Model Considering Rework and Inspection

The GTOQR is based on several assumptions including no inspection time, rework operation is a perfect process, and reworkable products qualification without inspection etc. Ullah and Kang [26] extended this model by incorporating 1) inspection time into the production cycle. 2) Lot manufacturing was divided into two phases, in second phase only reworkable products were processed and they were classified into either accepted or rejected. 3) Inspection for reworkable products is assumed to be mandatory as they were relatively more sensitive regarding qualification. 4) Previous WIP based GTOQR model assumed that manufacturing station can be stopped at any time for rework operation. This assumption was considered against the general practice in industry as usually rework is done when complete lot is passed through the manufacturing process. This extended model was named as group technology order quantity model considering inspection and rework (GTOQIR).

This model assume that machining process is followed by an inspection process which consume time and resources. Inspection process results in either rejected units, reworkable units, or accepted products as shown in <Figure 1>. Reworkable products are processed again through the machining process followed by an inspection process. Inspection station classify unit products as either qualified or rejected as shown in <Figure 2>.



<Figure 1> Phase 1



<Figure 2> Phase 2

Parameters

- m_1 machining time per unit for the lot size Q in phase 1. (unit time per unit product)
- m_r machining time per unit for rework products (unit time per unit product)
- P_1 percentage of products rejected as poor quality in phase 1
- P_{11} percentage of products qualified as good quality in phase 1
- P_{21} percentage of products qualified as good quality in phase 2

P_{20} percentage of products rejected as poor quality in phase 2

P_{1r} percentage of products rework able in phase 1, $QP_{1r} = Q(P_{20}+P_{21})$

P_b percentage of poor quality products at the end of cycle

P_g percentage of good quality products at the end of cycle

The total cost function include purchase cost, setup cost, inspection cost, work-in-process inventory carrying cost and finish goods inventory holding cost.
Therefore, total cost function per unit time is given by

$$C_{total} = C_p + C_s + C_i + C_H + C_{wip} \tag{6}$$

Ullah and Kang [25] developed mathematical expressions for the above mentioned costs. Purchase cost is calculated for raw materials purchased in each cycle. Setups cost is calculated only once for lot size Q during each cycle.

The cost associated with *WIP* composed of carrying cost for products waiting for operation, cost of carrying rejected products, and cost of carrying good quality products during the cycle time. Inspection cost is calculated for lot manufacturing during regular time and for reworked products. Therefore, the total cost function is,

$$\begin{aligned} C_{total} = {} & \frac{C_M D}{(1-P_b)} + \frac{AD}{Q\{1-P_b\}} + \left\{\frac{1D}{(1-P_b)}\right\}(1+P_{1r})\Bigg\} \\ & + \frac{1}{2}i\left(\left\{C_M + R\left(\frac{s+Qm_1+IQ+Qm_rP_{1r}+IQP_{1r}}{Q}\right)\right\}\right. \\ & \left. + Q(1-P_b)\right) + \frac{1}{2}\left[\frac{Di}{(1-P_b)}\right]\left(2C_M + \frac{Rs}{Q} + Rm_1 + Rm_rP_{1r}\right. \\ & \left. + IR + IRP_{1r}\right) \times (s + Qm_1 + QP_{1r}m_r + QI + QIP_{1r}) \end{aligned} \tag{7}$$

Assuming

$$K_1 = ((C_M + Rm_1 + IR) + (Rm_r + IR)P_{1r})$$
$$K_2 = ((m_1 + m_rP_{1r})(2C_M + Rm_1 + Rm_rP_{1r}))$$
$$K_3 = (I(1+P_{1r})(2C_M + IR + 2Rm_1 + 2Rm_rP_{1r} + IRP_{1r}),$$

then, the total cost function can be rearranged in factorized form as follows :

$$\begin{aligned} C_{total} = {} & \left(\frac{i}{2}(1-P_b)K_1 + \left(\frac{Di}{2(1-P_b)}\right)(K_2+K_3)\right)Q + \\ & \left(\frac{D_i}{2(1-P_b)}(Rs^2) + \frac{AD}{(1-P_b)}\right)\frac{1}{Q} + \left(\frac{C_M D}{(1-P_b)} + \left(\frac{ID}{(1-P_b)}\right)\right. \\ & (1+P_{1r}) + \frac{Di}{2(1-P_b)} \\ & (Rsm_1 + Rsm_rP_{1r} + RsI + RsIP_{1r}) \end{aligned} \tag{8}$$

In order to obtain the optimum lot size, algebraic optimization technique has been used. Equation (8) is of the form of the Equation $y(x) = a_1x + \frac{a_2}{x} + a_3$, therefore, following Equation is obtained for the optimum lot size Q

$$Q^* = x = \sqrt{\frac{a_2}{a_1}}$$

$$= \sqrt{\frac{\frac{AD}{(1-P_b)} + \frac{Di}{2(1-P_b)}(Rs^2)}{\left(\frac{i}{2}(1-P_b)K_1 + \left(\frac{Di}{2(1-P_b)}\right)(K_2+K_3)\right)}}$$

Rearranging,

$$Q^* = \sqrt{\frac{2AD + iDR(s)^2}{i(1-P_b)^2K_1 + Di(K_2+K_3)}} \tag{9}$$

Equation (9) give the closed form optimum solution for the lot size *(Q)*.

3. Numerical Computation and Sensitivity Analysis

For sensitivity analysis all above mentioned models are analyzed using following example. Different parameters of the aforementioned models are varied from -50% to +50% to realize the changes in the parameter value and its ultimate effect on the optimum lot size. The results are summarized in tabular form.

A numerical example from GTOQR model [24] has been used for this purpose. The data assume include A =11.9 (\$/unit), D = 14000 (units/year), i = 35%, C_M = 1(\$/unit), s = 0.0017(years/unit), m_1 = 0.12 (mints/unit), m_r = 5% (m_1), R = 7000 (\$/year). It is further added that rework rate has been taken as 5% of the regular production time and rejection rate as 20% during calculation.

Six different parameters including setup cost, demand rate, manufacturing time, rework percentage, rejection percentage, and inspection time have been analyzed against each model and their responses have been shown in following Tables.

Group Technology Order Quantity (GTOQ) and its onward extensions till date (GTOQR and GTOQIR models) analysis has been shown in <Table 1>, <Table 2> and <Table 3> respectively. It can be observed that the change in lot size with respect to setup cost variation is significant. Lot size increase with increases in setup cost and vice versa. GTOQ Model results in lowest lot size value (Q^* = 678) whereas GTOQR model response to the setup cost produce maximum value (Q^* = 1455) under the given conditions. GTOQR and GTOQIR value is increased due to the processes imperfection.

<Table 1> Sensitivity Analysis (GTOQ Model)

S N	Parameter	Changes(%)	Q*
01	Setup cost (A)	-50% -25% +25% +50%	678 831 1072 1175
02	Demand rate (D)	-50% -25% +25% +50%	683 833 1069 1167
03	Manufacturing time (m_I)	-50% -25% +25% +50%	967 963 955 951
04	Rework (%)	-50% -25% +25% +50%	959 959 959 959
05	Rejection (%)	-50% -25% +25% +50%	959 959 959 959
06	Inspection time (I)	-50% -25% +25% +50%	959 959 959 959

<Table 2> Sensitivity Analysis (GTOQR Model)

S N	Parameter	Changes(%)	Q*
01	Setup cost (A)	-50% -25% +25% +50%	840 1029 1329 1455
02	Demand rate (D)	-50% -25% +25% +50%	850 1035 1322 1440
03	Manufacturing time (m_I)	-50% -25% +25% +50%	1204 1196 1181 1174
04	Rework (%)	-50% -25% +25% +50%	1189 1189 1188 1188
05	Rejection (%)	-50% -25% +25% +50%	1061 1121 1264 1349
06	Inspection time (I)	-50% -25% +25% +50%	1188 1188 1188 1188

<Table 3> Sensitivity Analysis (GTOQIR Model)

S N	Parameter	Changes(%)	Q*
01	Setup cost (A)	-50% -25% +25% +50%	821 1005 1297 1421
02	Demand rate (D)	-50% -25% +25% +50%	838 1015 1284 1393
03	Manufacturing time (m_I)	-50% -25% +25% +50%	1174 1167 1154 1147
04	Rework (%)	-50% -25% +25% +50%	1161 1161 1160 1160
05	Rejection (%)	-50% -25% +25% +50%	1041 1097 1231 1310
06	Inspection time (I)	-50% -25% +25% +50%	1175 1168 1153 1147

Demand rate has also been varied from -50% to +50% against the optimum lot size and variation can be observed for all the three models. It is shown that demand rate effect on lot size is also considerable. Lot size increases with increase in demand rate per year. Higher demand rate results in higher values of lot size for GTOQR model. The values of GTOQIR are relatively reduced due to the concept of rework operation involved.

Manufacturing time affect cycle time. Manufacturing time increasing affect results in decreasing the lot size in a manufacturing cell. As time for manufacturing decreases, lot size for group technology manufacturing environment increases. The GTOQR model give highest value (Q^* = 1204) in comparison to Q^* = 967 and Q^* = 1174 for GTOQ and GTOQIR models respectively.

Rework operation on optimum lot for various values of rework percentage has been shown in all the three Tables. There is no change in the optimum lot size for GTOQ model as it doesn't take into consideration processes imperfection. GTOQR and GTOQIR model values are changed less significantly when rework operation is varied from 2.5% to 7.5% of the lot size. This impact can be significant for higher values of rework percentage in both of these models.

Change in rejection percentage in a cycle time and its impact on lot size can be observed in GTOQR and GTOQIR models. GTOQ model don't consider rejection in manufacturing processes. The lot size increases as the rejection percentage increases from 10% to 30%. Optimum lot size calculated by GTOQIR model is relatively low as compared to GTOQR model. The change in lot size for both the models is significant as shown in <Table 2> and <Table 3> respectively.

At the end the impact of inspection time required for unit product qualification can be analyzed. It can be observed that optimum lot size for GTOQ and GTOQR model remains unchanged for different values of inspection time as shown in <Table 1> and <Table 2>. However this effect can be observed in <Table 3> for GTOQIR model. Lot size has been reduced for higher values of inspection time per unit product.

4. Conclusion

The paper summarize inventory models developed for group technology work environment focusing work-in-process inventory. Models are derived in a chronological order i.e. from a perfect manufacturing setup to an imperfect manufacturing situation considering rework, rejection and inspection process. A simple algebraic optimization approach has been used to get the optimum lot size for the developed models. Mathematical models for optimum lot size under different working conditions have been illustrated with numerical example. Sensitivity analysis for several parameters has been highlighted via numerical examples. The impact of demand rate, setup cost, manufacturing time, rejection and inspection time remain significant for calculation of optimum lot size in an imperfect manufacturing setup. The paper provide an insight to manufacturing engineers to calculate optimum lot size under different imperfect manufacturing conditions with average cost function minimization. Further extension are possible using probabilistic approach for imperfect manufacturing setup focusing *work-in-process* inventories.

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