

루에 동적 시스템을 위한 샘플데이터 제어

Sampled-data Control for Lur'e Dynamical Systems

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Abstract - This paper studies the problem of the sampled-data control for Lur'e system with nonlinearities. The nonlinearities are expressed as convex combinations of sector and slope bounds. It is assumed that the sampling periods are arbitrarily varying but bounded. By constructing a new augmented Lyapunov-Krasovskii functional which have an augmented quadratic form with states as well as the nonlinear function, the stabilizing sampled-data controller gains are obtained by solving a set of linear matrix inequalities. The effectiveness of the developed method is demonstrated by numerical simulations.

Key Words : Sampled-data, Time-varying delay, Stability, Lyapunov method, LMI

1. Introduction

All physical systems are nonlinear in nature and there are various kinds of nonlinearities. It has been shown that several nonlinear systems, including neural networks and Chua's circuits, can be represented in the form of Lur'e systems. Sector bounded nonlinearity is commonly encountered in practice such as saturation, quantization, backlash, deadzones, and so on. The existence of sector bounded nonlinearity is a source of degradation or instability of system performance. Thus, the stability analysis of Lur'e systems has been studied by many researchers [1-4]. Using the concept of absolute stability theory, different with sector bounded nonlinearity, there have been presented new stability criteria of sector restricted Lur'e systems in terms of LMIs, by fully exploiting inherent properties of sector restrictions in the time domain [5-9]. However, stabilization problem for the systems with sector bounded nonlinearity only considered by few researchers [10-11]. In [10], the H_∞ control problem of Lur'e systems with sector and slope restricted nonlinearities was considered by using state feedback control, and the authors in [11] considered the robust H_∞ control for uncertain Lur'e systems with sector nonlinearities using PD state feedback.

Because of the rapid growth of the digital hardware technologies, the sampled-data control method, whose the

control signals are kept constant during the sampling period and are allowed to change only at the sampling instant, has been more important than other control approaches. Thus, many important and essential results have been reported in the literature over the past decades[12-15]. Recently, the sampled-data synchronization control problem of chaotic Lur'e systems has been investigated by some researchers [16-18]. To the best of our knowledge, the sampled-data control design problem of Lur'e system has not been investigated in the existing literature.

With this motivation, in this paper, we consider the sampled-control of Lur'e dynamical system with sector restricted nonlinearity. Based on Lyapunov stability theory, the stabilizing sampled-data controller gains are obtained by solving a set of linear matrix inequalities. The main contribution of this paper lies in two aspects. Some new augmented Lyapunov-Krasovskii functional which have not been considered in Lur'e system are introduced. On the other hand, the proposed controller design method is based on a sampled-data control and its gain matrix is derived by solving a set of LMI matrix.

Finally, in order to demonstrate the effective of the proposed method, the Rotational/Translational Actuator (RTAC) benchmark problem is considered as a fourth-order dynamical system involving the sector bounded nonlinear interaction of a translational oscillator and an eccentric rotational proof mass.

Notation: \mathbf{R}^n is the n -dimensional Euclidean space, $\mathbf{R}^{m \times n}$ denotes the set of m by n real matrix. For symmetric matrices X and Y , the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative). I and 0

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denote the identity matrix and zero matrix with appropriate dimensions. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $diag\{\dots\}$ denotes the block diagonal matrix. \star represents the elements below the main diagonal of a symmetric matrix.

2. Problem Statements

Consider the following continuous systems described by the nonlinear differential equation

$$\dot{x}(t) = Ax(t) + Ff(v(t)) + Bu(t), \tag{1}$$

$$v(t) = Dx(t), \tag{2}$$

$$u(t) = Kx(t_k),$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t)$ is a control input, which will be appropriately designed such that the specific control objective is achieved, K are the gain matrix for sampled-data controller, $v(t) \in \mathbf{R}^n$ is the output vector, and A, F, B, D are known matrices of appropriate dimensions.

It is assumed that $f(v) = [f_1(v_1(t)), f_2(v_2(t)), \dots, f_m(v_m(t))]^T$ is memoryless time-invariant nonlinearities with sector bound and slope restrictions as

$$b_i \leq \frac{f_i(v_i(t))}{v_i(t)} \leq a_i, \tag{3}$$

$$\bar{b}_i \leq \frac{df_i(v_i(t))}{dv_i(t)} \leq \bar{a}_i. \tag{4}$$

The nonlinear function $f(\cdot)$ can be written as a convex combination of the sector bounds such as a_i and b_i :

$$f_i(v_i(t)) = (\lambda_i^l(v_i(t))b_i + \lambda_i^u(v_i(t))a_i)v_i(t) \tag{5}$$

where

$$\lambda_i^l(v_i(t)) = \frac{f_i(v_i(t)) - b_i v_i(t)}{(a_i - b_i)v_i(t)},$$

$$\lambda_i^u(v_i(t)) = \frac{a_i v_i(t) - f_i(v_i(t))}{(a_i - b_i)v_i(t)}.$$

Since $\lambda_i^l(v_i(t)) + \lambda_i^u(v_i(t)) = 1, \lambda_i^l(v_i(t)) \geq 0$ and $\lambda_i^u(v_i(t)) \geq 0$, the nonlinearity $f(\cdot)$ can be rewritten as

$$f_i(v_i(t)) = A_i(v_i(t))v_i(t), \tag{6}$$

where $A_i(v_i(t))$ is an element of a convex hull $\mathcal{C}[b_i, a_i]$. Similarly, the derivative of the nonlinearity can also be expressed as a convex combination of the slope bounds such

$$\dot{f}_i(v_i(t)) = \bar{A}_i(v_i(t))\dot{v}_i(t), \tag{7}$$

where $\bar{A}_i(v_i(t))$ is an element of a convex hull $\mathcal{C}[\bar{b}_i, \bar{a}_i]$.

In this paper, the control signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence of hold times

$$0 \leq t_0 \leq t_1 < \dots < t_k \dots < \lim_{k \rightarrow \infty} t_k = +\infty.$$

Also, the sampling is not required to be periodic, and the only assumption is that the distance between any two consecutive sampling instants is less than a given bound. Specially, it is assumed that

$$t_{k+1} - t_k \leq h.$$

for all $k \geq 0$, where h represents the upper bound of the sampling periods.

Define $t_k = t - (t - d(t))$ with $d(t) = t - t_k$. Then, the system (1) can be represented as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Ff(v(t)) + BKx(t-d(t)), \\ v(t) &= Dx(t), \end{aligned} \tag{8}$$

3. Main Results

In this section, we derive a criterion for sampled-data controller design for Lur'e system with sector nonlinearities. For the simplicity on matrix representation, $e_i \in \mathbf{R}^{7 \times n} (i=1,2,\dots,n)$, e.g., $e_2 = [0_n, I_n, 0_n, 0_n, 0_n, 0_n]$, the augmented vectors are defined as

$$\begin{aligned} x_a^T(t) &= [x^T(t) \ f^T(v(t))], \\ \xi^T(t) &= [x_a^T(t) \ f^T(v(t)) \ x^T(t-h(t)) \ f^T(v(t-h(t))) \\ &\quad x^T(t-h) \ f^T(v(t-h)) \ \dot{x}^T(t) \ \dot{f}^T(v(t))]. \end{aligned}$$

and define the matrices

$$\begin{aligned} \hat{\Xi}_1 &= [e_1 \ e_2] \hat{P} [e_7 \ e_8]^T + [e_7 \ e_8] \hat{P} [e_1 \ e_2]^T, \\ \hat{\Xi}_2 &= [e_1 \ e_2] \hat{Q} [e_1 \ e_2]^T + [e_3 \ e_6] \hat{Q} [e_3 \ e_6]^T, \\ \hat{\Xi}_3 &= h^2 [e_7 \ e_8] \hat{R} [e_7 \ e_8]^T - [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6] \\ &\quad \begin{bmatrix} \hat{R} \hat{S} \\ \hat{S} \hat{R} \end{bmatrix} [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6]^T, \\ \Phi &= [AG \ FG \ BT \ 0 \ 0 \ 0 \ -G \ 0], \\ T &= KG, \\ A &= diag\{A_1(v_1(t)), A_2(v_2(t)), \dots, A_m(v_m(t))\}, \\ \bar{A} &= diag\{\bar{A}_1(v_1(t)), \bar{A}_2(v_2(t)), \dots, \bar{A}_m(v_m(t))\}, \\ \Delta_1 &= diag\{b_1, b_2, \dots, b_m\}, \Delta_2 = diag\{a_1, a_2, \dots, a_m\}, \\ \bar{\Delta}_1 &= diag\{\bar{b}_1, \bar{b}_2, \dots, \bar{b}_m\}, \bar{\Delta}_2 = diag\{\bar{a}_1, \bar{a}_2, \dots, \bar{a}_m\}. \end{aligned}$$

Then, the nonlinearities $f(v(t))$ and $\dot{f}(v(t))$ can be expressed as

$$f(v(t)) = Av(t), \dot{f}(v(t)) = \bar{A}\dot{v}(t), \tag{9}$$

and the parameters belong to the following set

$$\Phi := \{(\Lambda, \bar{\Lambda}) | \Lambda \in \text{Co}\{\Delta_1, \Delta_2\}, \bar{\Lambda} \in \text{Co}\{\bar{\Lambda}_1, \bar{\Lambda}_2\}\}. \quad (10)$$

Now, we have the following theorem.

Theorem 1. For given positive scalars h and δ , the system (1) with the sampled-data controller Eq.(2) is stable, if there exist positive definite matrices $\mathbf{P} \in \mathbb{R}^{2n \times 2n}$, $\mathbf{Q}, \mathbf{R} \in \mathbb{R}^{2n \times 2n}$, any matrices $\mathbf{S} \in \mathbb{R}^{2n \times 2n}$, symmetric matrices $\mathbf{G} \in \mathbb{R}^{n \times n}$ and appropriate dimension matrix T satisfying the following LMIs

$$\sum_{i=1}^4 \hat{\Xi}_i - 2e_2 G^{-1} e_2^T + e_2 G^{-1} \bar{\Lambda} D e_1^T + e_1 D^T \bar{\Lambda}^T G^{-1} e_2^T \quad (11)$$

$$\begin{aligned} & - 2e_8 G^{-1} e_8^T + e_8 G^{-1} \bar{\Lambda} D e_7^T + e_7 D^T \bar{\Lambda}^T G^{-1} e_8^T \\ & + (e_1 + \delta e_7) G^{-1} \Phi G^{-1} + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)^T < 0, \end{aligned} \quad (12)$$

$$\begin{bmatrix} \hat{R} & \hat{S} \\ \hat{S} & \hat{R} \end{bmatrix} \geq 0.$$

Further, the sampled-data controller gain matrix in (2) are given by

$$K = TG^{-1}. \quad (13)$$

Proof. Consider the following L-K functional candidate as

$$V = V_1 + V_2 + V_3, \quad (14)$$

where

$$V_1 = x_a^T(t) P x_a(t),$$

$$V_2 = \int_{t-h}^t x_a^T(s) Q x_a(s) ds,$$

$$V_3 = h \int_{t-h}^t \int_{t+s}^t \dot{x}_a^T(u) Q_1 \dot{x}_a(u) du ds.$$

The time-derivative of V_1 can be obtained as

$$\dot{V}_1 = \dot{x}_a^T(t) P x_a(t) + x_a^T(t) P \dot{x}_a(t) = \xi^T(t) \Xi_1 \xi(t), \quad (15)$$

where

$$\Xi_1 = [e_1 \ e_2] P [e_7 \ e_8]^T + [e_7 \ e_8] P [e_1 \ e_2]^T.$$

By calculating the time-derivative of V_2 , we have

$$\dot{V}_2 = x_a^T(t) Q x_a(t) - x_a^T(t-h) Q x_a(t-h) = \xi^T(t) \Xi_2 \xi(t), \quad (16)$$

where

$$\Xi_2 = [e_1 \ e_2] Q [e_1 \ e_2]^T - [e_5 \ e_6] Q [e_5 \ e_6]^T.$$

The time-derivative of V_3 is

$$\dot{V}_3 = h^2 \dot{x}_a^T(t) R \dot{x}_a(t) - h \int_{t-h}^t \dot{x}_a^T(s) R \dot{x}_a(s) ds. \quad (17)$$

Since $\begin{bmatrix} R & S \\ S & R \end{bmatrix} \geq 0$, by employing Jensen's inequality and the reciprocally convex combination technique [19], one can obtain

$$\begin{aligned} & -h \int_{t-h}^t \dot{x}_a^T(s) R \dot{x}_a(s) ds \\ & \leq \begin{bmatrix} \int_{t-h}^t \dot{x}_a(s) ds \\ \int_{t-h}^t \dot{x}_a(s) ds \end{bmatrix}^T \begin{bmatrix} R & S \\ S & R \end{bmatrix} \begin{bmatrix} \int_{t-h}^t \dot{x}_a(s) ds \\ \int_{t-h}^t \dot{x}_a(s) ds \end{bmatrix}. \end{aligned} \quad (18)$$

Hence, from Eqs. (17) and (18), we have

$$\dot{V}_3 \leq \xi^T(t) \Xi_3 \xi(t), \quad (19)$$

where

$$\begin{aligned} \Xi_3 &= h^2 [e_7 \ e_8] R [e_7 \ e_8]^T - [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6] \\ & \begin{bmatrix} R & S \\ S & R \end{bmatrix} [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6]^T. \end{aligned}$$

From Eqs. (8) and (9), for any symmetric matrices G , the following equations are satisfied

$$2f^T(v(t)) G^{-1} [AD - I \ 0 \ 0 \ 0 \ 0 \ 0] \xi(t) = 0, \quad (20)$$

$$2\bar{f}^T(v(t)) G^{-1} [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \bar{\Lambda} D - I] \xi(t) = 0, \quad (21)$$

$$\begin{aligned} & 2[x^T(t) G^{-1} + \delta \bar{x}^T(t) G^{-1}] [A \ F \ BK \ 0 \ 0 \ 0 - I] \xi(t) \\ & = \xi^T(t) ((e_1 + \delta e_7) G^{-1} \Phi G^{-1} + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)) \xi(t) = 0. \end{aligned} \quad (22)$$

An upper bound of the difference of $V(t)$ is

$$\begin{aligned} \dot{V}(t) & \leq \xi^T(t) \left(\sum_{i=1}^3 \hat{\Xi}_i - 2e_2 G^{-1} e_2^T + e_2 G^{-1} \bar{\Lambda} D e_1^T + e_1 D^T \bar{\Lambda}^T G^{-1} e_2^T \right. \\ & \quad \left. - 2e_8 G^{-1} e_8^T + e_8 G^{-1} \bar{\Lambda} D e_7^T + e_7 D^T \bar{\Lambda}^T G^{-1} e_8^T \right. \\ & \quad \left. + (e_1 + \delta e_7) G^{-1} \Phi G^{-1} + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)^T \right) \xi(t). \end{aligned} \quad (23)$$

Let us define

$$\begin{aligned} \hat{P} &= \text{diag}\{G, G\}^T P \text{diag}\{G, G\}, \\ \hat{Q} &= \text{diag}\{G, G\}^T Q \text{diag}\{G, G\}, \\ \hat{R} &= \text{diag}\{G, G, G, G\}^T R \text{diag}\{G, G, G, G\}, \\ \hat{S} &= \text{diag}\{G, G\}^T S \text{diag}\{G, G\}, \end{aligned}$$

then pre and post multiplying the matrix $\text{diag}\{G, G, G, G, G, G, G\}^T$ and $\text{diag}\{G, G, G, G, G, G, G\}$ in Eq.(23) leads to LMI (11). This completes the proof. ■

Remark 3.1 In this paper, a new Lyapunov functional (14) is constructed based on augmented vector $x_a(t)$, which is considered in [10,11,16-18], is handled by the

reciprocally convex combination technique [19], which is less conservative than Jensen inequality, and involves fewer decision variables than free weighting matrix.

4. Numerical Examples

In this section, a numerical example is given to show the effectiveness of the proposed sampled-data controller design.

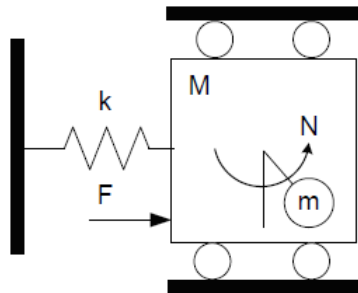


그림 1 RTAC 시스템.
Fig. 1 RTAC system.

Example 4.1 To illustrate the effectiveness of the proposed method, consider the RTAC(Rotational and translational actuator) benchmark problem [20] as shown in Figure 1. For simplicity, the following transformed state equation is employed [21]

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= -x_1(t) + 0.2\sin(x_3(t)), \\ \dot{x}_3(t) &= x_3(t), \\ \dot{x}_4(t) &= u(t). \end{aligned} \tag{24}$$

It can be found that RTAC can be represented in Lur'e form with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$f_3(x_3(t)) = x_3(t) - 0.2\sin(x_3(t)),$$

$$f_1(x_1(t)) = f_2(x_2(t)) = f_4(x_4(t)) = 0.$$

Applying Theorem 1 with $\delta=0.7$, we can obtain the maximum values of the upper bound h is 0.47. The corresponding gain matrix are

$$K = [0.7794 \ 0.3880 \ 1.3337 \ 1.9171].$$

Figure 1 and Figure 2 show the state response and input

of the RTAC system with the above controller gain with the sector condition $0.8 \leq \frac{f(v)}{v} \leq 1.0434$ and the initial condition $x(0) = [0.2 \ 0.4 \ 0.1 \ 0.2]^T$, respectively. It is clear that the state converges to zero asymptotically.

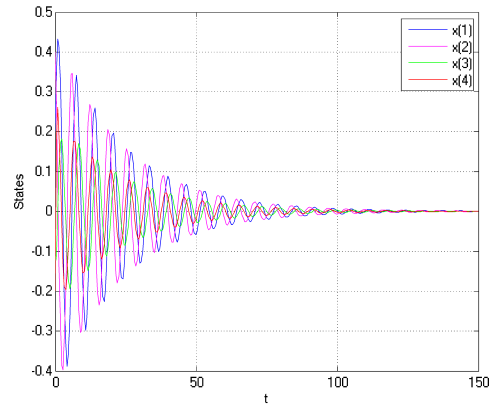


그림 2 초기조건 에서의 $x(0)=[0.2, 0.4, -0.1, -0.2]$ 제어상태 응답.

Fig. 2 State response under $x(0)=[0.2, 0.4, -0.1, -0.2]$ with control.

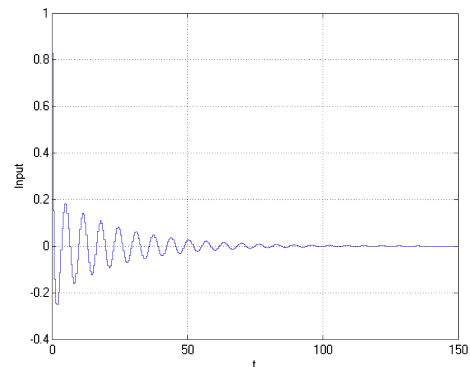


그림 3 제어입력.

Fig. 3 Control Input.

5. Conclusions

In this paper, the design of sampled-data controller for stabilization of Lur'e systems has been studied. The properties of a nonlinear function that was restricted by sector and slope bounded nonlinearity is represented by using equality constraints and convex representations. Based on LMIs, a novel criterion was presented to design the sampled-data controller, which guarantees the asymptotic stability of the closed-loop system. Furthermore, RTAC model is given to illustrate the effectiveness of the proposed control scheme.

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