# On the Distributor's EOQ with Inventory-level-dependent Demand Rate in the Presence of Order-size-dependent Delay in Payments

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# 주문량에 종속적인 신용거래 하에 재고 종속형 제품수요를 고려한 경제적 주문량 모형

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#### **Abstract**

본 연구는 공급자(supplier)가 중간 유통자(wholesaler/retailer)의 수요 증대를 목적으로 중간 유통자의 주문량에 따라 차별적으로 외상 기간을 허용한다는 기본적인 가정 하에 중간 유통자의 재고 모형을 다루었다. 문제 분석을 위하여 최종 고객(customer)의 수요는 중간 유통자의 재고 수준에 따라 선형적(linearly)으로 증가한다는 가정하에 모형을 분석하였고, 모형 분석을 통하여 중간 유통자의 이익을 최대화하는 경제적 주문주기와 경제적 주문량결정 방법을 제시하고, 예제를 통하여 그 제시된 해법의 타당성을 보였다.

Keywords: Inventory, EOQ, Credit Period, Order-Size-Dependent Delay in Payments, Inventory-Level-Dependent Demand Rate

#### 1. Introduction

In today's business transaction, it is very common to see that the distributors (wholesalers /retailers) are allowed some grace period before they settle the account with their supplier. Fewings [3] stated that the advantage of trade credit in the supplier's point of view is substantial in terms of influence on the distributor's purchasing and marketing decisions. The supplier usually expects that the increased sales volume can compensate for the capital losses incurred during the credit period. Also, the availability of opportunity to delay in payments effectively reduces the distributor's cost of holding inventories, and thus is likely to

result in larger order quantity. In this regard, Goyal [4] has examined the effects of the credit period on the retailer's EOQ (economic order quantity). Teng [14] extended Goyal's model by considering the difference between unit price and unit cost. Recently, Teng et al. [15] extended Goyal's model to develop an EOQ model in which the supplier permits delay in payments to the retailer, and the retailer also provides the trade credit period to his/her customers. Mahata and Goswami [7], and Tsao and Sheen [16] also extended the inventory model to the case of deteriorating products under trade credit.

All of the research mentioned above held the assumption that the credit period is a certain fixed length that is set by the supplier

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(single-credit period). However, for the sake of better production and inventory control, and lower average production cost per unit, some manufacturers prefer infrequent orders in large lot sizes to frequent orders in smaller lots, if the annual ordering quantities are equal. Thus, rather than giving some discount on unit selling price for larger amount of purchase, they offer a longer credit period (multiple-credit periods). Their policies tend to make distributor's order size large enough to qualify for a certain credit period break and it enables both the supplier and distributor to benefit significantly in a supply chain setting. Based upon the above observation, Shinn and Hwang[13], and Ouyang et al.[9] analyzed the joint pricing and ordering problems in a supply chain under order size dependent delay in payments assuming that the demand rate is a function of the retailer's selling price. Also, Chang et al.[2], and Kreng and Tan[5] evaluated the inventory model under two levels of trade credit policy depending on the order quantity.

For certain commodities in the distribution centers (wholesale market), such as consumer goods, food grains, stationery items etc., the customer's demand rate may depend on the size of the quantity on hand. According to Levin et al. [6], one of the functions of inventories is that of a motivator; they indicated: "At times, the presence of inventory has a motivational effect on the people around it. It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more". Based upon the above observation, Baker and Urban[1] evaluated an inventory assuming the demand rate to be a polynomial functional form of the on-hand inventory level at time. Also, Mandal and Phaujdar [8] discussed an inventory system when the demand rate is a linear function of the on-hand inventory level at that time. Padmanabhan and Vrat[10] analyzed an inventory model for deteriorating products where the demand rate has been assumed to be dependent on the on-hand inventory. Urban[17] also extended his model to the case, in which the terminal condition of zero ending inventory is not imposed on the system. With this type of product, the probability of making a sale would increase as the amount of the product in inventory increases. It is, therefore, likely to have an effect of increasing the size of each order. Moreover, if the distributors are allowed delay in payments before they settle accounts with their supplier, the availability of opportunity to delay the payments effectively reduces the distributor's cost of holding inventories, and thus is likely to result in larger order quantity. In this regard, Shinn[11] presented an inventory system under day-terms supplier credit where the customer's demand rate has been assumed to be a polynomial function of the on-hand inventory level at that time. Also, Shinn[12] extended his model to the case of multiple-credit periods assuming customer's demand rate is a polynomial function of the on-hand inventory level.

Our research reported in this paper allows for employing multiple—credit periods as stated by Shinn and Hwang[13], and Ouyang et al.[9]. We develop the model for determining the distributor's economic inventory policy for an inventory—level—dependent demand rate items assuming that the customer's demand rate is represented by a linear function of the on—hand inventory. In the next section, we formulate a relevant mathematical model. The properties of an optimal solution are discussed and solution algorithm is given in Section 3. A numerical example is provided in Section 4, which is followed by concluding remarks in Section 5.

#### 2. Model Formulation

The model presented is the continuous and deterministic case of an inventory system under multiple-credit periods in which the customer's demand rate is dependent on the distributor's inventory level. The assumptions of the model are as follows;

(1) Replenishments are instantaneous with a known and constant lead time.

- (2) No shortages are allowed.
- (3) The inventory system involves only one item.
- (4) The customer's demand rate is linearly dependent on the distributor's inventory level.
- (5) The supplier allows a delay in payments for the products supplied where the length of delay is a function of the distributor's total amount of purchase.
- (6) The purchasing cost of the products sold during the credit period is deposited in an interest bearing account with rate I. At the end of the period, the credit is settled and the distributor starts paying the capital opportunity cost for the products in stock with rate R ( $R \ge I$ ).

And in deriving the model, the following notations are used;

C: unit purchase cost.

P: unit selling price.

S: ordering cost.

H: inventory carrying cost, excluding the capital opportunity cost.

R: capital opportunity cost (as a percentage).

I : earned interest rate (as a percentage).

Q: order size.

T: replenishment cycle time.

q(t): inventory level at time t.

D: annual demand rate, as a linear function of the on-hand inventory,  $D = \alpha + \beta q(t)$  where  $\alpha$  and  $\beta$  are non-negative constants.

The objective of this model will be to maximize the distributor's annual net profit from the sales of the products and the analysis will concentrate on the situation in which the customer's demand rate is represented by a linear function of the on-hand inventory. That is, the demand rate at time *t*, will take the form:

$$D = \alpha + \beta q(t), \quad \alpha, \beta \ge 0.$$
 (1)

Then, to determine what the mathematical expression of the inventory level over time is, we can interpret the slope of the curve at any point (the rate of change of inventory level per unit time) as the negative of demand rate

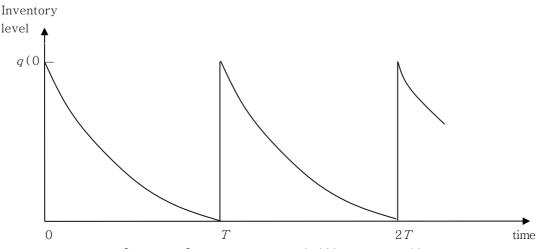
$$\frac{dq(t)}{dt} = -D,\tag{2}$$

$$\frac{dq(t)}{dt} = -\alpha - \beta q(t). \tag{3}$$

Equation (3) is a first order linear differential equation and its solution is equation (4).

$$q(t) = q(0)e^{-\beta t} - \frac{\alpha}{\beta}(1 - e^{-\beta t}).$$
 (4)

Figure 1 illustrates the time behavior of the inventory level. And note that due to the inventory carrying costs, it is clearly optimal to let the inventory level reach zero before reordering, i.e., q(T) = 0. So, given that q(T) = 0,



[Figure 1] Inventory level(q(t)) vs. Time(t).

$$q(0) = \frac{\alpha}{\beta} (e^{\beta T} - 1). \tag{5}$$

And then,

$$q(t) = \frac{\alpha}{\beta} (e^{\beta(T-t)} - 1), \ 0 \le t \le T.$$
 (6)

Also, from the fact that q(0) = Q, the quantity ordered per cycle is

$$Q = \frac{\alpha}{\beta} (e^{\beta T} - 1). \tag{7}$$

The purpose of this paper is to find the distributor's economic replenishment cycle time which maximizes his annual net profit,  $\Pi(T)$ . For the formulation of  $\Pi(T)$ , we consider the inequalities,  $v_{j-1} \leq CQ < v_j$  for  $j=1,2,\cdots,m$ . Using equation (7), the inequalities can be reduced to

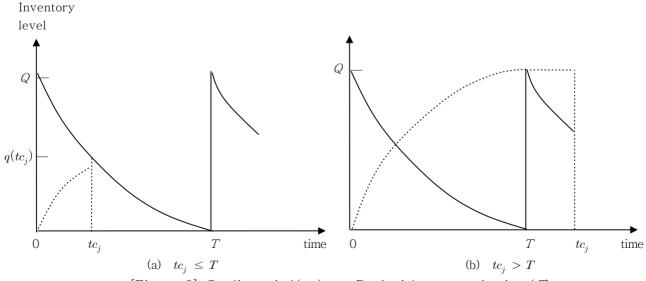
$$\begin{split} l_{j-1} & \leq T < l_j, \ j=1,2,\, \cdots, m \\ \text{where } l_j & = \frac{1}{\beta} ln(\frac{\beta}{C\alpha} v_j + 1)\,. \end{split} \tag{8}$$

Then the distributor's annual net profit consists of the following five elements.

- (1) Annual sales revenue =  $\frac{PQ}{T} = \frac{P\alpha(e^{\beta T} 1)}{\beta T}$ .
- (2) Annual purchasing cost =  $\frac{CQ}{T}$  =  $\frac{C\alpha(e^{\beta T}-1)}{\beta T}$ .
- (3) Annual inventory carrying cost =  $\frac{H}{T} \int_{0}^{T} q(t)dt$

- $= \frac{H\alpha (e^{\beta T} \beta T 1)}{\beta^2 T}.$
- (4) Annual ordering cost =  $\frac{S}{T}$ .
- (5) Annual capital opportunity cost for  $l_{j-1} \leq T < l_j$
- (i) Case  $1(tc_j \leq T)$ : (see Figure 2. (a)) As products are sold, the sales revenue is used to earn interest with annual rate I during the credit period  $tc_j$ . And the average number of products in stock earning interest during time  $(0,\,tc_j)$  is  $(Qtc_j-\int_0^{tc_j}q(t)dt)/tc_j$  and the interest earned per order becomes  $CI(Qtc_j-\int_0^{tc_j}q(t)dt)$ . When the credit is settled, the products still in stock have to be financed with annual rate R. Since the average number of products during time  $(tc_j,\,T)$  becomes  $\int_{tc_j}^Tq(t)dt/(T-tc_j)$  and the interest payable per order can be expressed as  $CR\int_{tc_j}^Tq(t)dt$ . Therefore, the annual capital opportunity cost

$$\begin{split} &=\frac{\mathit{CR}\!\int_{tc_j}^T\!\!q(t)dt-\mathit{CI}(\mathit{Qtc}_j\!-\!\int_0^{tc_j}\!\!q(t)dt)}{T}\\ &=\frac{\alpha\mathit{C}(\mathit{R}(e^{\beta(T-tc_j)}\!-\!\beta(T\!-\!tc_j)\!-\!1)\!-\!\mathit{I}e^{\beta T}(e^{-\beta tc_j}\!+\!\beta tc_j\!-\!1))}{\beta^2\,T}. \end{split}$$
 (ii) Case  $2(tc_i>T)$ : (see Figure 2. (b)) For



[Figure 2] Credit period( $tc_i$ ) vs. Replenishment cycle time(T).

the case of  $tc_j > T$ , all the sales revenue is used to earn interest with annual rate I during the credit period  $tc_j$ . The average number of products in stock earning interest during time  $(0,\,tc_j)$  is  $(Qtc_j - \int_0^T q(t)dt)/tc_j$ . Therefore, the annual capital opportunity cost

$$\begin{split} &= \frac{-\operatorname{CI}(\operatorname{Qtc}_j - \int_0^T q(t)dt)}{T} \\ &= \frac{\alpha\operatorname{CI}(e^{\beta T}(1-\beta tc_j) - \beta(T-tc_j) - 1)}{\beta^2 T}. \end{split}$$

The annual net profit  $\Pi(T)$  can be expressed as

 $\Pi(T) = \text{Sales revenue} - \text{Purchasing cost} - \text{Inventory carrying cost} - \text{Ordering cost} - \text{Capital opportunity cost.}$ 

Depending on the relative size of  $tc_j$  to T,  $\Pi(T)$  has two different expressions as follows:

1. Case 1 
$$(tc_j \le T)$$

$$\Pi_{1,j}(T) = \frac{\alpha(P - C)(e^{\beta T} - 1)}{\beta T} - \frac{\alpha H(e^{\beta T} - \beta T - 1)}{\beta^2 T} - \frac{S}{T}$$

$$-\frac{\alpha C(R(e^{\beta(T-tc_{j})}-\beta(T-tc_{j})-1)-Ie^{\beta T}(e^{-\beta tc_{j}}+\beta tc_{j}-1))}{\beta^{2}T}, \quad j=1,2,\cdots,m,$$
 (9)

2. Case 2 
$$(tc_j > T)$$

$$\Pi_{2,j}(T) = \frac{\alpha(P - C)(e^{\beta T} - 1)}{\beta T} - \frac{\alpha H(e^{\beta T} - \beta T - 1)}{\beta^2 T} - \frac{S}{T}$$

$$- \frac{\alpha CI(e^{\beta T}(1 - \beta tc_j) - \beta(T - tc_j) - 1)}{\beta^2 T}$$

$$, j = 1, 2, \dots, m.$$
 (10)

# 3. Determination of Optimal Policy

The problem is to find an optimal replenishment cycle time  $T^*$  which maximizes  $\Pi(T)$ . Also, an optimal lot size  $Q^*$  can be obtained by equation (7). Then, we can consider the necessary and sufficient conditions for maximizing  $\Pi(T)$  with respect to T. But, while the objective function can be differentiated, the resulting equation is mathematically intractable;

that is, it is difficult to solve the first derivative (set equal to zero) for *T*. Therefore, a solution to the general model will not be obtained in explicit form. Fortunately, the model can be solved approximately by using a truncated Taylor series expansion for the exponential function, i.e.,

$$e^{\beta T} \approx 1 + \beta T + \frac{1}{2}\beta^2 T^2, \tag{11}$$

which is a valid approximation for smaller values of  $\beta T$ . Thus, using the Taylor series approximation the annual net profit function can be rewritten as:

$$\begin{split} & \varPi_{1,j}(T) \ = \ \alpha(P - C(1 - (R + I\beta tc_j/2)tc_j)) \ - \ \frac{S + \alpha C(R - I)tc_j^2/2}{T} \\ & - \ \frac{\alpha T(H - P\beta + C\beta + CR - CI\beta^2 tc_j^2/2)}{2}, \ \ j = 1, 2, \, \cdots, m, \end{split} \tag{12} \\ & \varPi_{2,j}(T) \ = \ \alpha(P - C(1 - c_j)) \ - \ \frac{S}{T} \\ & - \ \frac{\alpha T(H - P\beta + C\beta + CI - CI\beta tc_j)}{2}, \qquad j = 1, 2, \, \cdots, m. \end{aligned} \tag{13}$$

Note that from equations (12) and (13), the annual net profit at  $T=tc_j$  has the following relation:

$$\Pi_{1,j}(tc_j) > \Pi_{2,j}(tc_j), \ j = 1, 2, \dots, m.$$
 (14)

Now, we investigate the characteristics of  $\Pi_{i,j}(T)$ , i=1,2 and  $j=1,2,\cdots,m$ . The first order condition with respect to T is:

$$\frac{d\Pi_{1,j}(T)}{dT} = -\frac{\alpha(H - P\beta + C\beta + CR - CI\beta^2 t c_j^2/2)}{2}$$

$$+ \frac{S + \alpha C(R - I)t c_j^2/2}{T^2}, \qquad (15)$$

$$\frac{d\Pi_{2,j}(T)}{dT} = -\frac{\alpha(H - P\beta + C\beta + CI - CI\beta t c_j)}{2} + \frac{S}{T^2}. \quad (16)$$

Similarly the second order condition with respect to T is:

$$\frac{d^{2}\Pi_{1,j}(T)}{dT^{2}} = -\frac{2(S + \alpha C(R - I)tc_{j}^{2}/2)}{T^{8}},$$
(17)

$$\frac{d^2 \Pi_{2,j}(T)}{dT^2} = -\frac{2S}{T^3}. (18)$$

Therefore,  $\Pi_{i,j}(T)$  is a concave function of T for the normal condition  $(R \ge I)$  as stated by Goyal [4]. And so, there exists a unique value  $T_{i,j}$ , which maximizes  $\Pi_{i,j}(T)$  and they are:

$$\begin{split} T_{1,j} &= \sqrt{\frac{2S_1}{\alpha H_1}} \\ \text{where } S_1 &= S + \alpha \, C(R-I)tc_j^2/2 \text{ and} \\ H_1 &= H - (P - C)\beta + CR - CI\beta^2tc_j^2/2, \quad (19) \\ T_{2,j} &= \sqrt{\frac{2S}{\alpha H_2}} \\ \text{where } H_2 &= H - (P - C)\beta + CI(1 - \beta tc_j). \quad (20) \end{split}$$

From equations (19) and (20), we have the following relation for  $T_{i,j}$ .

$$T_{i,j-1} < T_{i,j}$$
 for  $i = 1, 2$  and  $j = 1, 2, \dots, m$ . (21)

And  $T_{i,j}$  and  $\Pi_{i,j}(T)$  can be shown to have the following properties.

**Property 1.** There exists at least one  $T_{i,j} \in [l_{j-1}, l_j)$  for i=1,2 and  $j=1,2,\cdots,m$ . **Proof.** From equation (21), if there were no  $T_{i,j} \in [l_{j-1}, l_j)$  for i=1,2 and  $j=1,2,\cdots,m$ , either  $T_{i,j} \geq l_j$  or  $T_{i,j} < l_{j-1}$  for every i and j. Hence, either  $T_{i,m} \geq l_m$  or  $T_{i,1} < l_0$  holds, which contradicts the feasibility of T, i.e.,  $0 < T < \infty$ .

**Property 2.** For any T,  $T \ge tc_j$ ,  $\Pi_{1,j-1}(T) < \Pi_{1,j}(T)$ ,  $j = 1, 2, \dots, m$ .

*Proof.* Because  $\frac{tc_j}{T} \le 1$ , we can show that  $\frac{d\Pi_{1,j}(T)}{dtc_j} > 0$ ,  $j = 1, 2, \cdots, m$ , and it implies that the annual net profit function is an increasing

Property 3. For any T,  $\Pi_{2,j-1}(T) < \Pi_{2,j}(T)$ ,  $j=1,2,\cdots,m$ .

Proof. From equation (13), we can also show that  $\frac{d\Pi_{2,j}(T)}{dtc_j} > 0$ ,  $j = 1, 2, \cdots, m$ , and so the annual net profit function is an increasing function of  $tc_j$ . Therefore  $\Pi_{2,j-1}(T) < \Pi_{2,j}(T)$  holds since  $tc_{j-1} < tc_j$ . Q.E.D.

Based on the above properties, we develop the following solution procedure to determine an optimal replenishment cycle time  $T^*$  for the approximate model. For ease of understanding the solution algorithm, we present Figures 3, which shows the shape of the annual net profit of the example problem introduced in Section 4. Note that  $\Pi_{i,j}(T)$  for i=1,2 and  $j=1,2,\cdots,m$ , satisfy the above properties.

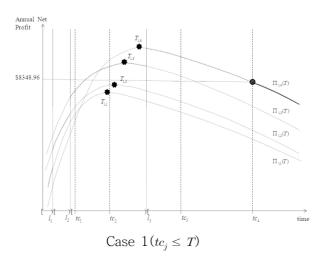
#### Solution algorithm

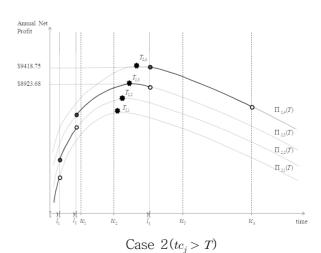
Step 1.Compute  $T_{1,j}$  by equation (19) and find the largest index a such that  $T_{1,j} \in [l_{j-1}, l_j)$  for  $j = 1, 2, \dots, m$ .

Step 2. (for the indices j > a)

In case

2.1  $tc_j < l_{j-1}$ , then compute the annual net profit with equation (12) for  $T = l_{j-1}$ .





[Figure. 3]  $\Pi_{i,j}(T)$  of an example problem in Section 4.

- 2.2  $l_{j-1} \le tc_j < l_j$ , then compute the annual net profit with equation (12) for  $T = tc_i$ .
- 2.3  $l_j \leq tc_j$ , then there exists no candidate value of the maximum annual net profit for  $T \in [l_{j-1}, l_j)$ .
- Step 3. (for the index j = a)

In case

- 3.1  $tc_j < T_{1,j}$ , then compute the annual net profit with equation (12) for  $T = T_{1,j}$  and go to Step 5.
- 3.2  $T_{1,j} \le tc_j < l_j$ , then compute the annual net profit with equation (12) for  $T = tc_j$  and go to Step 4.
- 3.3  $l_i \leq tc_i$ , go to Step 4.
- Step 4. (for the indices j < a)

In case

- 4.1  $tc_j < l_j$  and  $l_j \le T_{1,j}$ , then compute the annual net profit with equation (12) for  $T = l_j \epsilon$  and go to Step 5.
- 4.2  $tc_j < l_j$  and  $\max[tc_j, l_{j-1}] \le T_{1,j} < l_j$ , then compute the annual net profit with equation (12) for  $T = T_{1,j}$  and go to Step 5.
- 4.3  $tc_j < l_j$  and  $T_{1,j} < \max[tc_j, l_{j-1}]$ , then compute the annual net profit with equation (12) for  $T = \max[tc_i, l_{j-1}]$ .
- 4.4  $l_j \leq tc_j$ , then there exists no candidate value of the maximum annual net profit for  $T \equiv [l_{j-1}, l_j)$ .
- Step 5.Compute  $T_{2,j}$  by equation (20) and find the largest index b such that  $T_{2,j} = [l_{j-1}, l_j)$  for  $j = 1, 2, \dots, m$ .
- Step 6.If  $tc_b > T_{2,b}$ , then compute the annual net profit with equation (13) for  $T = T_{2,b}$  and go to Step 7. Otherwise, go to Step 7.
- Step 7. (for the indices j > b)

  If  $te_j \ge l_{j-1}$ , then compute the annual net profit with equation (13) for  $T = l_{j-1}$ .

  Otherwise, there exists no candidate value of the maximum annual net profit for  $T \in [l_{j-1}, l_j)$ .
- Step 8. Select the replenishment cycle  $time(T^*)$  with the maximum annual net profit value evaluated in previous steps.

### 4. Numerical Example

For the purpose of illustrating the proposed model and the solution algorithm, the following example problem is considered.

- (1) C = \$20, P = \$23, = \$100, H = \$5, R = 0.15( = 15%), I = 0.1( = 10%),  $\alpha = 3,200$  and  $\beta = 0.3$ .
- (2) Supplier's credit schedule:

Total amount of purchase(order size)	Credit period
$0 \le CQ < \$1,000 (0 \le Q < 50)$	$tc_1 = 0.05$
$1,000 \le CQ < 3,000(50 \le Q < 150)$	$tc_2 = 0.1$
$3,000 \le CQ < 10,000(150 \le Q < 500)$	$tc_3=0.2$
$\$10,000 \le CQ < \infty (500 \le Q < \infty)$	$tc_4=0.3$

Figure 3 is a plot of the annual net profit  $\Pi_{i,j}(T)$  for i=1,2 and  $j=1,2,\cdots,m$ . The solution procedure generates an optimal solution,  $T^*$  for the approximate model through the following steps and an optimal lot size,  $Q^*$  can be obtained by equation (7).

- Step 1. Since  $T_{1,4}(=0.1466) < l_3(=0.1527)$  and  $l_2(=0.0465) \le T_{1,3}(=0.1202), \ a=3.$
- Step 2. Since  $l_3(=0.1527) \le tc_4 \le l_4(=\infty)$ , compute  $\Pi_{1,4}(tc_4)$  by equation (12) and go to Step 3.
- Step 3. Since  $l_3(=0.1527) \leq tc_3(=0.2)$ , go to Step 4.
- Step 4. Since  $l_2(=0.0465) \le tc_2(=0.1)$  and  $l_1(=0.0156) \le tc_1(=0.05)$ , go to Step 5.
- Step 5. Since  $T_{2,4}(=0.1027) < l_3(=0.1527)$  and  $l_2(=0.0465) \le T_{2,3}(=0.1022), \ b=3.$
- Step 6. Since  $T_{2,3}(=0.1022) < tc_3(=0.2)$ , compute  $\Pi_{2,3}(T_{2,3})$  by equation (13) and go to Step 7.
- Step 7. Since  $l_3(=0.1527) \leq tc_4(=0.3)$ , compute  $\varPi_{2,4}(l_3)$  by equation (13) and go to Step 8
- Step 8 Since  $\Pi_{1,4}(tc_3) = \$8348.96$ ,  $\Pi_{2,3}(T_{2,3}) = \$8923.68$  and  $\Pi_{2,4}(l_3) = \$9418.75$ , an optimal replenishment cycle time becomes  $l_3(=0.1527)$  with its maximum annual net profit \$9418.75.

#### Conclusions

Trade credit affects the conduct of business significantly for many reasons. For a supplier who offers trade credit, it is an effective means of price discrimination as well as efficient method to stimulate the demand of his products. The length of credit period is considered as a supplier's dominant strategy against competitive suppliers. Among a number of factors that determine the length of the credit period in a given line of business, credit risk, the size of account, customer type, and competition are known to be more important.

In this regard, this paper dealt with the distributor's optimal ordering quantity determination problem when the supplier allows a delay in payments and the length of delay in payments is a function of the distributor's order size. Recognizing that for certain commodities, the customer's demand rate may depend on the size of the quantity on hand, we expressed the customer's demand rate of the product with a linear function of the on-hand inventory. The on-hand inventory level is one of the important factors related to the variation of the customer's demand rate and it is, therefore, likely to have an effect of increasing the size of each order. For the system presented, a mathematical model was developed. Recognizing that the model has a very complicated structure, a truncated Taylor series expansion is utilized to find a solution procedure which leads to a distributor's ordering quantity for the model developed. To illustrate the validity of the solution procedure, an example problem was chosen and solved.

There are several interesting opportunities for future research in this subject. As stated in Urban[17], it may be desirable to order large quantities, resulting in remaining at the end of the cycle, due to the potential profits resulting from the increased demand. Hence, the assumption of zero ending inventory (i.e., q(T) = 0) of this model can be extended to the case, in which the terminal condition of zero ending

inventory is not imposed on the system. And also, the model can be easily extended to the case of deteriorating product.

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## 저 자 소 개

#### 신 성 환



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