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Slant Submanifolds of $(LCS)_n$-manifolds

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Abstract. In this article, we study slant and semi-slant submanifolds of $(LCS)_n$-manifolds. Integrability conditions of distributions involved in definition of semi-slant submanifolds of a $(LCS)_n$-manifold have been obtained.

1. Introduction

The study of slant immersions was initiated by B.Y. Chen [4]. A. Lotta [16], introduced and studied slant submanifolds of an almost contact metric manifold. He also studied the intrinsic geometry of 3-dimensional non-anti-invariant slant submanifolds of K-contact manifolds [17]. In 2000, Cabrerizo *et al.* studied slant submanifolds of a Sasakian manifold and obtained many interesting results. They also gave several examples of slant submanifolds of a Sasakian manifold [6]. The study of semi-slant submanifolds was initiated by Papaghiuc [20]. Semi-slant submanifolds are generalized version of CR-submanifolds. In 1999, Cabrerizo *et al.* [5] studied semi-slant submanifolds of a Sasakian manifold. In [14], authors studied semi-slant submanifolds of a trans-Sasakian manifold. On the otherhand, in 2003 [22], A.A. Shaikh introduced the notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds)and proved its existence by several examples and found its applications to the general relativity and cosmology in [24] and [25]. He also studied some results on $(LCS)_n$-manifolds in [23]. $(LCS)_n$-manifolds are generalization of LP-Sasakian manifolds introduced by Matsumoto [18]. In [11], S.K. Hui and M. Atceken studied contact warped product semi-slant submanifolds of $(LCS)_n$-manifolds. Again, in [1], M. Atceken obtained some interesting results on invariant

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submanifolds of $(LCS)_n$-manifolds.Thus motivated sufficiently, in this paper, we study slant and semi-slant submanifolds of a $(LCS)_n$-manifold. We observe that the metric induced on a submanifold of a $(LCS)_n$-manifold may be either degenerate or non-degenerate. In this article, we focus our attention on non-degenerate submanifolds of $(LCS)_n$-manifolds.

2. Preliminaries

Let $\overline{M}$ be an n-dimensional real differentiable manifold of differentiability class C^∞ endowed with a (1, 1) tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g of type $(0, 2)$ such that for each point $p \in M$, the tensor $g_p : T_p\overline{M} \times T_p\overline{M} \to \mathbb{R}$ is a non-degenerate inner product of signature $(-, +, ..., +)$, where $T_p\overline{M}$ denotes the tangent vector space of $\overline{M}$ at p and $\mathbb{R}$ is the real number space, which satisfies

(2.1) $\eta(\xi) = -1,$

(2.2) $\phi^2 X = X + \eta(X)\xi,$

(2.3) $g(X, \xi) = \eta(X),$

(2.4) $g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y),$

for all vector fields X and Y on $\overline{M}$. Then such a structure $(\phi,\ \xi,\ \eta,\ g)$ is termed as Lorentzian almost paracontact structure and the manifold $\overline{M}$ with a Lorentzian paracontact structure is called a Lorentzian paracontact manifold [18]. Since a Lorentzian metric g is of index 1, Lorentzian manifold has not only spacelike vector fields but also timelike and lightlike vector fields. A non-zero vector $u \in T_p\overline{M}$ is said to be timelike (resp., non-spacelike, null, spacelike) if it satisfies $g_p(u, u) < 0 (resp., \leq 0, = 0, > 0)$.

Definition 2.1. In a Lorentzian manifold $(\overline{M}, g)$, a vector field P defined by $g(X, P) = A(X)$, for any $X \in \chi(M)$, is said to be a concircular vector field if

$$(\nabla_X A)(Y) = \alpha \left\{g(X, Y) + \omega(X)A(Y)\right\},$$

where α is a non-zero scalar and ω is a closed 1-form.

Let $\overline{M}$ admits a unit timelike concircular vector field ξ. Then, on putting $\eta(X) = g(X,\ \xi)$ for any vector field X, we have

(2.5) $(\overline{\nabla}_X\eta)(Y) = \alpha \{g(X, Y) + \eta(X)\eta(Y)\},\ (\alpha \neq 0),$

where $\overline{\nabla}$ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

(2.6) $\overline{\nabla}_X\alpha = (X\alpha) = d\alpha(X) = \rho\eta(X),$

ρ is a certain scalar function given by $\rho = -(\xi\alpha)$.

If we put

(2.7) $\phi X = \frac{1}{\alpha}\overline{\nabla}_X\xi,$

then from (2.5) and (2.7), we have

$$\phi X = X + \eta(X)\xi. \tag{2.8}$$

A Lorentzian manifold $\overline{M}$ together with the unit timelike concircular vector field ξ, its associated 1-form η and a $(1,1)$-tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold) [22]. In particular, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [18]. In a $(LCS)_n$-manifold [22], the following relations hold:

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2.9}$$

$$g(\phi X, Y) = g(X, \phi Y), \tag{2.10}$$

$$(\overline{\nabla}_X \phi)(Y) = \alpha\,\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{2.11}$$

for all vector fields X, Y, Z on $\overline{M}$. Again, if we put $\Phi(X,Y) = g(X, \phi Y)$, where Φ is a symmetric $(0,2)$ tensor field, then we have

$$\Phi(X,Y) = \tfrac{1}{\alpha}(\overline{\nabla}_X \eta)(Y), \tag{2.12}$$

$$(\overline{\nabla}_Z \Phi)(X,Y) = g(X, (\overline{\nabla}_Z \phi)Y) = g((\overline{\nabla}_Z \phi)X, Y), \tag{2.13}$$

$$(\overline{\nabla}_Z \Phi)(X,Y) = \alpha[g(X,Z)\eta(Y) + 2\eta(X)\eta(Y)\eta(Z) + g(Y,Z)\eta(X)], \quad (\alpha \neq 0) \tag{2.14}$$

for any vector fields X, Y, Z, on $\overline{M}$.

Now, let M be a non-degenerate submanifold immersed in $\overline{M}$. We denote the Riemannian metric induced on M by same symbol g. Let TM and $T^\perp M$ be the Lie algebra of vector fields tangential to M and normal to M respectively. Then Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{2.15}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^\perp_X N, \tag{2.16}$$

for each $X, Y \in TM$ and $N \in T^\perp M$, where ∇ is the Levi-Civita connection on M, $\nabla^\perp$ is the connection on the normal bundle $T^\perp M$, h is the second fundamental form of M and A_N is the shape operator with respect to the normal section N, which are related by

$$g(A_N X, Y) = g(h(X,Y), N). \tag{2.17}$$

For any $X \in TM$ and $N \in T^\perp M$, we put

$$\phi X = PX + FX, \tag{2.18}$$

where PX (resp. FX) is the tangential component (resp. normal component) of ϕX. Similarly, for $N \in T^\perp M$, we put

$$\phi N = tN + fN, \tag{2.19}$$

where tN (resp. fN) is the tangential component (resp. normal component) of ϕN.

From (2.10) and (2.15), it follows that

$$g(PX,Y) = g(X,PY), \tag{2.20}$$

and therefore $g(P^2X,Y) = g(X,P^2Y)$. Thus P^2 which is denoted by Q, is self adjoint. We define the covariant derivatives of Q, P and F as

$$(\nabla_X Q)Y = \nabla_X(QY) - Q\nabla_X Y, \tag{2.21}$$

$$(\nabla_X P)Y = \nabla_X(PY) - P\nabla_X Y, \tag{2.22}$$

$$(\nabla_X F)Y = \nabla^{\perp}_X(FY) - F\nabla_X Y, \tag{2.23}$$

for any $X, Y \in TM$.

Using equations (2.15), (2.16), (2.17), (2.18), (2.19), (2.22) and (2.23) in (2.11), we get

$$\begin{aligned}(\nabla_X P)Y = {} & A_{FY}X + th(X,Y) \\ & +\alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\},\end{aligned} \tag{2.24}$$

$$(\nabla_X F)Y = -h(X,PY) + fh(X,Y). \tag{2.25}$$

3. Slant Submanifolds

A non-degenerate submanifold M of a $(LCS)_n$-manifold $\overline{M}$ is said to be slant if for any $x \in M$ and any $X \in T_xM$, linearly independent on ξ, the angle between ϕX and T_xM is a constant $\theta \in [0, \frac{\pi}{2}]$, called the slant angle of M in $\overline{M}$. The invariant and anti-invariant submanifolds of $\overline{M}$ are slant submanifolds with slant angle $\theta = 0, \frac{\pi}{2}$. If the slant angle $\theta \neq 0, \frac{\pi}{2}$, then the slant submanifold is called a proper slant submanifold. We suppose that the structure vector field ξ is tangent to M. If we denote by D the distribution orthogonal to ξ in TM, we have the orthogonal direct decomposition

$$TM = D \oplus \langle\xi\rangle.$$

For a proper slant submanifold M of a $(LCS)_n$-manifold $\overline{M}$ with slant angle θ, we have

$$QX = -\cos^2\theta(X - \eta(X)\xi),$$

for any $X \in TM$.

Now, we have following results which characterize non-degenerate slant submanifolds of a $(LCS)_n$-manifold.

Theorem 3.1. *Let M be a submanifold of a $(LCS)_n$-manifold $\overline{M}$ such that $\xi \in TM$. Then, M is slant if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$Q = -\lambda(I - \eta \otimes \xi). \tag{3.1}$$

Furthermore, in such case, if θ is the slant angle of M, it satisfies $\lambda = \cos^2\theta$.

Theorem 3.2. *Let M be a slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then at each point $x \in M$, $Q|_D$ has only one eigenvalue λ, where $\lambda = \cos^2\theta$, θ being the slant angle of M.*

The proof of above theorems follow by using similar steps as in Theorem [2.2] and Lemma [4.2], in [6] respectively.
Now, we have

Theorem 3.3. *Let M be a slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then $\nabla Q \neq 0$, i.e. Q is not parallel.*

Proof. Let M be a slant submanifold of $(LCS)_n$-manifold $\overline{M}$ and θ be the slant angle of M. Then for any X, Y in TM, by equation (3.1), we get

$$Q(\nabla_X Y) = \cos^2\theta(-\nabla_X Y + \eta(\nabla_X Y)\xi), \tag{3.2}$$

$$QY = \cos^2\theta(-Y + \eta(Y)\xi). \tag{3.3}$$

Differentiating (3.3) covariantly with respect to X, we get

$$\begin{aligned}(\nabla_X Q)Y + Q(\nabla_X Y) = {} & \cos^2\theta(-\nabla_X Y + \eta(\nabla_X Y)\xi) \\ & + \cos^2\theta(g(Y, \alpha\phi X)\xi + \eta(Y)\alpha\phi X).\end{aligned} \tag{3.4}$$

Using equations (2.7) and (3.2) in (3.4), we obtain

$$(\nabla_X Q)Y = \alpha\cos^2\theta(g(Y, X)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X). \tag{3.5}$$

From (3.5), it is clear that $\nabla Q = 0$, if and only if $\theta = \frac{\pi}{2}$. In view of Theorem 3.1 [9], the result follows. □

Theorem 3.4. *Let M be a submanifold of $(LCS)_n$-manifold $\overline{M}$. Then, M is slant if and only if*

(i) *the endomorphism $Q|_D$ has only one eigen value at each point of M,*

(ii) *there exists a function $\lambda : M \to [0, 1]$ such that*

$$(\nabla_X Q)Y = \lambda[\alpha(g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X)],$$

for any $X, Y \in TM$. If θ is the slant angle of M, then $\lambda = \cos^2\theta$.

Proof. Let M be slant, then the statements (i) and (ii) follow directly from Theorem 3.2 and equation (3.5) respectively.
Conversely, suppose that $D = \langle\xi\rangle^\perp$ and assume that statements (i) and (ii) hold. Let λ_1 be the eigenvalue of $Q|_D$, then $QY = \lambda_1 Y$ for each $Y \in D$. Then from (ii), we have

$$\begin{aligned}\nabla_X QY &= Q\nabla_X Y + \lambda[\alpha g(X, Y)\xi], \\ \text{i.e. } (X\lambda_1)Y + \lambda_1\nabla_X Y &= Q\nabla_X Y + \lambda\alpha g(X, Y)\xi,\end{aligned}$$

for any $X \in TM$. Since $\nabla_X Y$ and $Q\nabla_X Y$ are perpendicular to Y, we observe that λ_1 is a constant on M.

Now, let $X \in TM$. Then we can write

$$X = \overline{X} + \eta(X)\xi,$$

where $\overline{X} \in D$. Hence $QX = Q\overline{X}$. Since $Q|_D = \lambda_1 I$, we have $Q\overline{X} = \lambda_1\overline{X}$ and so $QX = \lambda_1\overline{X}$ which implies that $QX = \lambda_1(X - \eta(X)\xi)$. By taking $\mu = -\lambda_1$, the above equation can be written as

$$QX = -\mu(X - \eta(X)\xi).$$

As $\lambda_1(= -\mu)$ is constant, by Theorem 3.1, M is slant in $\overline{M}$ and $\mu = \cos^2\theta$. □

4. Semi-slant Submanifolds

A non-degenerate submanifold M of a $(LCS)_n$-manifold $\overline{M}$ is said to be semi-slant submanifold if there exist two orthogonal distributions D_1 and D_2 on M such that
(i) TM admits the orthogonal direct decomposition i.e.

$$TM = D_1 \oplus D_2 \oplus \langle\xi\rangle,$$

(ii) the distribution D_1 is an invariant distribution, i.e. $\phi(D_1) = D_1$,

(iii) the distribution D_2 is slant with slant angle $\theta \neq 0$ and $\langle\xi\rangle$ denotes the distribution spanned by the structure vector field ξ.

It is clear that if $\theta = \frac{\pi}{2}$, then a semi-slant submanifold is a semi-invariant submanifold. Moreover, if the dimension of $D_2 = 0$, then M is an invariant submanifold. If the dimension of $D_1 = 0$ and $\theta = \frac{\pi}{2}$, then M is an anti-invariant submanifold and M is a proper slant submanifold with slant angle θ, if dimension of $D_1 = 0$ and $\theta \neq \frac{\pi}{2}$.

A semi-slant submanifold is called a proper semi-slant submanifold if dimension of D_1 and D_2, both are not equal to zero and $\theta \neq \frac{\pi}{2}$.

Let M be a non-degenerate semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then in view of Theorem 3.1 [9], the slant angle $\theta \neq \frac{\pi}{2}$, i.e., D_2 is not anti-invariant. For $X \in TM$, we can write

(4.1) $\quad X = P_1X + P_2X + \eta(X)\xi,$

where $P_1X \in D_1$ and $P_2X \in D_2$. Now, applying ϕ on (4.1), we obtain

(4.2) $\quad \phi X = \phi P_1X + PP_2X + FP_2X.$

Then, it is easy to observe that

(4.3) $\quad \phi P_1X = PP_1X$, $FP_1X = 0$ and $PP_2X \in D_2$.

Thus, we have

(4.4) $\quad PX = \phi P_1X + PP_2X$ and $FX = FP_2X$.

Let μ denotes the orthogonal complement of ϕD_2 in $T^\perp M$, i.e., $T^\perp M = \phi D_2 \oplus \mu$. Then μ is an invariant subbundle of $T^\perp M$.

Now, we are in position to workout the integrability conditions of the distributions D_1 and D_2 involved in definition of a non-degenerate semi-slant submanifold of a $(LCS)_n$-manifold.

Lemma 4.1. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then we have*:

$\nabla_X\xi = \alpha\phi X, \quad h(X,\xi) = 0,$ *for any* $X \in D_1$;

$\nabla_Y\xi = \alpha PP_2Y, \quad h(Y,\xi) = \alpha FP_2Y,$ *for any* $Y \in D_2$;

$\nabla_\xi\xi = 0, \quad h(\xi,\xi) = 0.$

Proof. The lemma follows from (2.7) by using (4.1), (4.2) and (2.15). □

Theorem 4.2. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then the distribution $D_1 \oplus D_2$ is integrable.*

Proof. Let $X, Y \in D_1 \oplus D_2$. Then

$$\begin{aligned} g([X,Y],\xi) &= g(\nabla_X Y - \nabla_Y X, \xi) \\ &= -g(Y, \nabla_X\xi) + g(X, \nabla_Y\xi) \\ &= -g(Y, \alpha\phi X) + g(X, \alpha\phi Y) \\ &= \alpha[-g(X,\phi Y) + g(X,\phi Y)] = 0. \end{aligned}$$

This implies that $[X,Y] \in D_1 \oplus D_2$ and hence $D_1 \oplus D_2$ is integrable. □

Theorem 4.3. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then the invariant distribution D_1 is integrable if and only if $h(X,\phi Y) = h(Y,\phi X)$ for all $X, Y \in D_1$.*

Proof. Let $N \in T^\perp M$. We have

$$g(\overline{\nabla}_X\phi Y - \overline{\nabla}_Y\phi X, N) = g((\overline{\nabla}_X\phi)Y - \phi\overline{\nabla}_X Y - (\overline{\nabla}_Y\phi)X + \phi\overline{\nabla}_Y X, N).$$

By using equations (2.11) and (4.2) in above, we get

$$g(FP_2[X,Y], N) = g(h(X,\phi Y) - h(\phi X, Y), N). \tag{4.5}$$

Thus D_1 is integrable if and only if $h(X,\phi Y) = h(\phi X, Y)$, for $X, Y \in D_1$.

Corollary 4.4. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then, the distribution $D_1 \oplus \xi$ is integrable if and only if $h(X,\phi Y) = h(Y,\phi X)$ for any $X, Y \in D_1 \oplus \xi$.*

Proof. From equation (4.5), we have

$$h(X,\phi Y) - h(\phi X, Y) = FP_2[X,Y], \tag{4.6}$$

for any $X, Y \in D_1 \oplus \{\xi\}$. Hence, if $D_1 \oplus \{\xi\}$ is integrable, then we have $h(X,\phi Y) = h(\phi X, Y)$.

Conversely, let $h(X,\phi Y) = h(Y,\phi X)$ for any $X, Y \in D_1 \oplus \xi$.

Then equation (4.6) gives

$$FP_2[X,Y] = 0.$$

Since D_2 is a slant distribution with slant angle $\theta(\neq 0)$, $P_2[X,Y]$ must vanish. Therefore, $[X,Y] \in D_1 \oplus \{\xi\}$.

This completes the proof. □

Lemma 4.5. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then, for any $X, Y \in TM$, we have*

(4.7) $$P_1(\nabla_X \phi P_1 Y) + P_1(\nabla_X PP_2 Y) = \phi P_1 \nabla_X Y + P_1 A_{FP_2Y} X + \alpha\eta(Y) P_1 X,$$

(4.8) $$P_2(\nabla_X \phi P_1 Y) + P_2(\nabla_X PP_2 Y) = P_2(A_{FP_2Y} X) + PP_2 \nabla_X Y + th(X,Y) + \alpha\eta(Y) P_2 X,$$

(4.9) $$\eta(\nabla_X \phi P_1 Y) + \eta(\nabla_X PP_2 Y) = \eta(A_{FP_2Y} X) + \alpha(g(X,Y) + 3\eta(Y)\eta(X)),$$

(4.10) $$h(X, \phi P_1 Y) + h(X, PP_2 Y) + \nabla^{\perp}_X FP_2 Y = FP_2 \nabla_X Y + fh(X,Y).$$

Proof. By using equations (2.11), (2.15), (2.16), (2.19), (4.1), (4.2), (4.3) and (4.4), we get

(4.11) $$\nabla_X \phi P_1 Y + h(X, \phi P_1 Y) + \nabla_X PP_2 Y + h(X, PP_2 Y) - A_{FP_2Y} X + \nabla^{\perp}_X FP_2 Y = \phi P_1 \nabla_X Y + PP_2 \nabla_X Y + FP_2 \nabla_X Y + th(X,Y) + fh(X,Y) + \alpha\eta(Y) P_1 X + \alpha\eta(Y) P_2 X + \alpha(g(X,Y)\xi + 3\eta(Y)\eta(X)\xi),$$

for any $X, Y \in TM$. Hence (4.7), (4.8), (4.9) and (4.10) follow from (4.11), by identifying the components on D_1, D_2, $\langle\xi\rangle$ and $T^{\perp}M$ respectively.

Theorem 4.6. *Let M be a semi-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Then the distribution D_2 is integrable if and only if*

(4.12) $$P_1(\nabla_X PY - \nabla_Y PX) = P_1(A_{FY} X - A_{FX} Y),$$

for any $X, Y \in D_2$.

Proof. As in Theorem (4.2), we have

(4.13) $$g([X,Y], \xi) = 0,$$

for any $X, Y \in D_2$.
From equation (4.7), we can easily obtain

(4.14) $$\phi P_1[X,Y] = P_1(\nabla_X PY - \nabla_Y PX) - P_1(A_{FY} X - A_{FX} Y),$$

for any $X, Y \in D_2$.
As $\phi P_1[X,Y] = 0$, $\forall\ X, Y \in D_2$ if and only if $P_1[X,Y] = 0$, in view of (4.13) and (4.14), D_2 is integrable if and only if (4.12) holds. □

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