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Weak and Strong Form of D-Irresolute Functions

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Abstract. In this paper, we introduce two new types of irresolute functions namely completely D-irresolute functions and weakly D-irresolute functions.We obtain their characterizations and their basic properties.

1. Introduction

Functions and of course irresolute functions give new path towards research. In 1972, S. G. Crossely and S. K. Hildebrand [2] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various of mathematics and related sciences. Recently, as generalization of closed sets, the notion of D-closed sets were introduced and this notion was further studied by Dass et al [1]. In this paper, we will continue the study of related irresolute functions with D-open sets. We introduce and characterize the concepts of completely D-irresolute functions and weakly D-irresolute functions.

2. Preliminaries

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise metioned and $f : (X, \tau) \to (Y, \sigma)$ (or simply $f : X \to Y$) denotes a function f of a space (X, τ) into a space (Y, σ). Let A be a subset of a space X. The closure, the interior and

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the complement of A are denoted by $cl(A)$, $Int(A)$ and A^c, respectively.

Definition 2.1. Let (X, τ) be a topological space. A subset A of the space X is said to be

1. pre-open [12] if $A \subseteq Int(cl(A))$ and pre-closed if $cl(Int(A)) \subseteq A$.
2. semi-open [9] if $A \subseteq cl(Int(A))$ and semi-closed if $Int(cl(A)) \subseteq A$.
3. regular open [16] if $A = Int(cl(A))$ and regular closed if $A = cl(Int(A))$.

Definition 2.2.([5]) Let (X, τ) be a topological space and $A \subseteq X$ is said to be

1. The pre-interior of A, denoted by $pInt(A)$, is the union of all preopen subsets of A.
2. The pre-closure of A, denoted by $Pcl(A)$, is the intersection of all preclosed sets containing A.

Definition 2.3. Let (X, τ) be a topological space. A subset $A \subseteq X$ is said to be

1. $\omega-$closed [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
2. $D-$closed [1] if $Pcl(A) \subseteq Int(U)$ whenever $A \subseteq U$ and U is $\omega-$-open in X.

The complements of the above mentioned sets are called their respective open sets.

Definition 2.4.([1]) Let (X, τ) be a topological space and A be a subset of (X, τ). $D - cl$ of A is the intersection of all $D-$closed sets containing A.

Lemma 2.5.([1]) If A is a D-closed set then $D - cl(A) = A$. Converse need not be true.

Theorem 2.6.([1]) *A subset A of X is regular open if and only if A is open and D-closed.*

Theorem 2.7.([14]) *A set A is $\omega-$open iff $F \subseteq Int(A)$ whenever F is semi-closed and $F \subseteq A$.*

Theorem 2.8. *Every closed set of (X, τ) is D-closed.*

Proof. Let A be a closed set. Then A is semi-closed. Let $A \subseteq U$ and U is $\omega-$open in X. By theorem 2.7, $A \subseteq Int(U)$. Hence $cl(A) \subseteq Int(U)$. Thus $Pcl(A) \subseteq cl(A) \subseteq Int(U)$. Hence A is D-closed □

Remark 2.9. The converse of the above theorem need not be true as seen from the following example

Example 2.10. Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then the set $A = \{b\}$ is D-closed but not closed.

Definition 2.11. A function $f : (X, \tau) \to (Y, \sigma)$ is called strongly continuous [8] if $f^{-1}(V)$ is clopen in X for every subset V of Y.

3. Completely D-irresolute Functions

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called perfectly D-continuous if $f^{-1}(V)$ is clopen in X for every D-closed (resp.D-open) subset V of Y.

Example 3.2. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is perfectly D-continuous.

Definition 3.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called D-irresolute if $f^{-1}(V)$ is D-closed (resp.D-open) in X for every D-closed (resp.D-open) subset V of Y.

Example 3.4. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, X\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$; $f(b) = a$; $f(c) = b$. Then the function f is D-irresolute.

Definition 3.5. A function $f : (X, \tau) \to (Y, \sigma)$ is called contra-D-irresolute if $f^{-1}(V)$ is D-closed in X for every D-open subset V of Y.

Example 3.6. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$; $f(b) = a$; $f(c) = c$. Then the function f is contra-D-irresolute.

Definition 3.7. A function $f : (X, \tau) \to (Y, \sigma)$ is called completely D-irresolute if $f^{-1}(V)$ is regular open in X for every D-open subset V of Y.

Example 3.8. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$; $f(b) = f(c) = c$. Then the function f is completely D-irresolute.

Theorem 3.9. *Every strongly continuous function is perfectly D-continuous and so completely D-irresolute.*

Proof. Let V be a D-open subset of Y. Since f is strongly continuous, $f^{-1}(V)$ is clopen in X. Hence f is perfectly D-continuous. Also by Theorem 2.6, f is completely D-irresolute. □

Remark 3.10. The converse of the above theorem need not be true as seen from the following example.

Example 3.11. By example 3.2, f is perfectly D-continuous but not strongly continuous. Observe that for the set $V = \{a, c\}$, $f^{-1}(V)$ is not clopen in X.

Theorem 3.12. *Every completely D-irresolute function is contra-D-irresolute.*

Proof. Let V be a D-open subset of Y. By hypothesis, $f^{-1}(V)$ is regular open in X. Then by theorem 2.6, $f^{-1}(V)$ is both open and D-closed in X. Hence $f^{-1}(V)$ is D-closed in X and so f is contra D-irresolute. □

Remark 3.13. The converse of the above theorem need not be true as seen from the following example.

Example 3.14. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra-D-irresolute but not completely D-irresolute. Observe that for the D-open set $V = \{a\}$, $f^{-1}(V) = \{a\}$ is not regular open in X.

Theorem 3.15. *Every perfectly D-continuous function is D-irresolute.*

Proof. Let G be a D-closed subset of Y. Since f is perfectly D-continuous, $f^{-1}(G)$ is clopen in X. By theorem 2.8, $f^{-1}(G)$ is D-closed in X. Hence f is D-irresolute. □

Remark 3.16. The converse of the above theorem need not be true as seen from the following example.

Example 3.17. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{c\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$; $f(b) = a$; $f(c) = b$. Then f is D-irresolute but not perfectly D-continuous. Observe that for the D-closed set $V = \{c\}$, $f^{-1}(V) = \{a\}$ is not clopen in X.

Theorem 3.18. *Every perfectly D-continuous function is contra-D-irresolute.*

Proof. Let V be a D-open subset of Y. Since f is perfectly D-continuous, $f^{-1}(V)$ is clopen in X. By theorem 2.8, $f^{-1}(V)$ is D-closed in X. Hence f is contra-D-irresolute. □

Remark 3.19. The converse of the above theorem need not be true as seen from the following example.

Example 3.20. By example 3.17, f is contra-D-irresolute but not perfectly D-continuous.

Theorem 3.21. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:*

(i) *f is completely D-irresolute.*

(ii) *$f^{-1}(F)$ is regular closed in X for every D-closed set F in Y.*

Proof. (i) ⇒ (ii) Let F be any D-closed set of Y. Then F^c is D-open in Y. By (i), $f^{-1}(F^c) = (f^{-1}(F))^c$ is regular open in X. Hence $f^{-1}(F)$ is regular closed in X.

Converse is similar. □

Lemma 3.22.([10]) Let S be an open subset of a space (X, τ). Then the following hold:

(i) If U is regular open in X then so is $U \cap S$ in the subspace (S, τ_s).

(ii) If $B \subseteq S$ is regular open in (S, τ_s) then there exists regular open set U in (X, τ) such that $B = U \cap S$.

Theorem 3.23. *If $f : (X, \tau) \to (Y, \sigma)$ is a completely D-irresolute function and A is any open subset of X then the restriction $f|_A : A \to Y$ is completely D-irresolute.*

Proof. Let V be D-open subset of Y. By hypothesis, $f^{-1}(V)$ is regular open in X. Since A is open in X, it follows from the lemma 3.22 that $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ which is regular open in A. Hence $f|_A$ is completely D-irresolute. □

Theorem 3.24. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two functions. Then the following hold:*

(i) *If f is completely D-irresolute and g is perfectly D-continuous then $g \circ f$ is completely D-irresolute.*

(ii) *If f is completely D-irresolute and g is irresolute then $g \circ f$ is completely D-irresolute.*

(iii) *If f is completely D-irresolute and g is strongly continuous then $g \circ f$ is completely D-irresolute.*

Proof. The proof of the theorem is easy and hence omitted. □

Definition 3.25. A space X is said to be

(i) almost connected [4] if there does not exist disjoint regular open sets A and B such that $A \cup B = X$.

(ii) D-connected if there does not exist disjoint D-open sets A and B such that $A \cup B = X$.

Theorem 3.26. *Let $f : (X, \tau) \to (Y, \sigma)$ is completely D-irresolute surjective function and X is almost connected then Y is D-connected.*

Proof. Suppose that Y is not D-connected. Then there exist disjoint D-open sets A and B of Y such that $A \cup B = Y$. Since f is completely D-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X. Moreover $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \Phi$ and $f^{-1}(B) \neq \Phi$. This shows that X is not almost connected. Which is a contradiction.

Definition 3.27. A space X is said to be

(i) nearly compact [15] if every regular open cover of X has a finite subcover.

(ii) nearly Lindelof [6] if every regular open cover of X has a countable subcover.

(iii) nearly countably compact [7] if every regular open countable cover of X has a finitesubcover.

(iv) D-compact if every D-open cover of X has a finite subcover.

(v) countably D-compact if every D-open countable cover of X has a finite subcover.

(vi) D-Lindelof if every D-open cover of X has a countable subcover.

Theorem 3.28. *Let $f : (X, \tau) \to (Y, \sigma)$ be a completely D-irresolute surjective function. Then the following statements hold:*

(i) *If X is nearly compact then Y is D-compact.*

(ii) *If X is nearly Lindelof then Y is D-Lindelof.*

(iii) *If X is nearly countably compact, then Y is countably D-compact.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a completely D-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha : \alpha \in \Delta\}$ be any D-open cover of Y. Then $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X. Since X is nearly compact, there exists a finite subcover $\{f^{-1}(U_{\alpha_i})/i = 1, 2, \ldots, n\}$ of X. It follows that $\{U_{\alpha_i}/i = 1, 2, \ldots, n\}$ is a finite subcover of Y. Hence the space Y is D-compact.

The proof of other cases are similar. □

Definition 3.29. A space X is said to be

(i) D-closed compact (resp. S-closed [17]) if every D -closed (resp. regular closed) cover of X has a finite sub cover.

(ii) Countably D-closed compact (resp.countably S-closed compact [3]) if every D-closed (resp. regular closed) countable cover of X has a finite subcover.

(iii) D-closed Lindelof (resp. S-Lindelof [11]) if every D-closed (resp. regular closed) cover of X has a countable subcover.

Theorem 3.30. *Let $f : (X, \tau) \to (Y, \sigma)$ be a completely D-irresolute surjective function. Then the following statements hold:*

(i) *If X is S-closed then Y is D-closed compact.*

(ii) *If X is S-Lindelof then Y is D-closed Lindelof.*

(iii) *If X is countably S-closed compact then Y is countably D-closed compact.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a completely D-irresolute surjective function. Let $\{U_\alpha : \alpha \in \Delta\}$ be any D-closed cover of Y. Then $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular closed cover of X. Since X is S-closed, there exists a finite subcover $\{f^{-1}(U_{\alpha_i})/i = 1, 2, \ldots, n\}$ of X. It follows that $\{U_{\alpha_i}/i = 1, 2, \ldots, n\}$ is a finite subcover of Y. Hence the space Y is D-compact.

The proof of other cases are similar. □

Definition 3.31. A space X is said to be $D - T_1$ (resp. $r - T_1$ [4]) if for each pair of distinct points x and y of X, there exist disjoint D-open (resp.regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 3.32. *Let $f : (X, \tau) \to (Y, \sigma)$ is completely D-irresolute injective function and Y is $D - T_1$ then X is $r - T_1$.*

Proof. Let x, y be any two distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since Y is $D - T_1$, there exist D-open sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$, $f(x) \notin W$ and $f(y) \notin V$. Since f is completely D-irresolute, there exist regular open sets $f^{-1}(V)$ and $f^{-1}(W)$ in X such that $x \in f^{-1}(V)$ and $y \in f^{-1}(W)$, $x \notin f^{-1}(W)$ and $y \notin f^{-1}(V)$. Hence X is $r - T_1$. □

Definition 3.33. A space (X, τ) is said to be $D - T_2$ if for each pair of distinct points x and y in X, there exist disjoint D-open sets U_1 and U_2 in X such that $x \in U_1$ and $y \in U_2$.

Theorem 3.34. *If $f : (X, \tau) \to (Y, \sigma)$ is completely D-irresolute injective function and Y is $D - T_2$ then X is T_2.*

Proof. Let x, y be any two distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since Y is $D - T_2$, there exist D-open sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$. Since f is completely D-irresolute, there exist regular open sets $f^{-1}(V)$ and $f^{-1}(W)$ in X such that $x \in f^{-1}(V)$ and $y \in f^{-1}(W)$ and $f^{-1}(V) \cap f^{-1}(W) = \Phi$. Hence X is T_2. □

Definition 3.35. A function $f : (X, \tau) \to (Y, \sigma)$ is called D-closed if the image of each D-closed set of X is D-closed in Y.

Theorem 3.36. *A surjective map $f : (X, \tau) \to (Y, \sigma)$ is D -closed iff for each subset B of Y and each D-open set U of X containing $f^{-1}(B)$ there exists a D-open set V in Y containing B such that $f^{-1}(V) \subseteq U$.*

Proof. Suppose that f is D-closed map. Let $B \subseteq Y$ and U be D-open set of X such that $f^{-1}(B) \subseteq U$. Since f is D-closed, $(f(U^c))^c = V$ is a D-open set in Y containing B such that $f^{-1}(V) \subseteq U$. Conversely, let F be any D-closed set of X. Put $B = (f(F))^c$. Then we have $f^{-1}(B) \subseteq F^c$ is D-open in X. By hypothesis there exist V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c = f^{-1}(V^c)$. Hence we obtain $f(F) = V^c$. Since V^c is D-closed, $f(F)$ is D-closed. Hence f is D-closed. □

Definition 3.37. A space X is said to be strongly D-normal (resp. midly D-normal) if for each pair of disjoint D-closed (resp. regular closed) sets A and B of X there exist disjoint D-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 3.38. *If $f : (X, \tau) \to (Y, \sigma)$ is completely D-irresolute, D -closed function from a midly D-normal space X onto a space Y then Y is strongly D-normal.*

Proof. Let A and B be two disjoint D-closed subsets of Y. Since f is completely D-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X. Since X is midly D-normal space, there exist disjoint D-open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is D -closed, $f(U^c)$ and $f(V^c)$ are D-closed sets in Y. Then by theorem 3.36, there exist D-open sets $G = (f(U^c))^c$ and $H = (f(V^c))^c$ containing A and B such that $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$. Clearly G and H are disjoint D-open subsets of Y. Hence Y is strongly D-normal space. □

Definition 3.39. A space X is said to be

(i) strongly D-regular if for each D-closed subset F and each point $x \in F$, there exist disjoint D-open sets U and V in X such that $x \in U$ and $F \subseteq V$.

(ii) almost D-regular if for each regular closed subset F and each point $x \in F$, there exist disjoint D-open sets U and V in X such that $x \in U$ and $F \subseteq V$.

Theorem 3.40. *If f is a completely D-irresolute, D -closed function of an almost D-regular space X onto a space Y, then Y is strongly D-regular space.*

Proof. Let F be D-closed subset of Y and let $y \in F$. Then $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$.Since X is almost D-regular space, there exist disjoint D-open sets U and V in X such that $f^{-1}(y) \in U$ and $f^{-1}(F) \subseteq V$. Since f is D-closed and by theorem 3.36, there exist D-open sets $G = (f(U^c))^c$ such that $f^{-1}(G) \subseteq U$, $y \in G$ and $H = (f(V^c))^c$ such that $f^{-1}(H) \subseteq V$, $F \subseteq H$. Clearly G and H are disjoint subsets of Y. Hence Y is strongly D-regular space. □

4. Weakly D-irresolute Functions

Definition 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly D-irresolute if for each point $x \in X$ and each $V \in DO(Y, f(x))$, there exists a $U \in DO(X, x)$ such that $f(U) \subseteq D - cl(V)$.

Example 4.2. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is weakly D-irresolute.

Theorem 4.3. *Every D-irresolute function is weakly D-irresolute.*

Proof. Let $x \in X$ and $V \in DO(Y, f(x))$. Since f is D-irresolute, $f^{-1}(V)$ is D-open in X. Then there exists $U = f^{-1}(V) \in DO(X, x)$ such that $f(U) \subseteq V \subseteq D - cl(V)$. Hence f is weakly D-irresolute. □

Remark 4.4. The converse of the above theorem need not be true as seen from the following example

Example 4.5. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is weakly D-irresolute but not D-irresolute. Observe that for the D-closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{c\}$ is not D-closed in (X, τ).

Remark 4.6. Contra-D-irresolute and weakly D-irresolute are independent. It is shown by the following examples.

Example 4.7. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$; $f(b) = a$; $f(c) = b$. Then the function f is contra-D-irresolute but not weakly D-irresolute. Observe that for the D-open sets $V = \{c\}$ and $U = \{a, b\}$, $f(U) \not\subseteq D - cl(V)$.

Example 4.8. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{b\}, X\}$ and $\sigma = \{\phi, \{c\}, Y\}$. Define the function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$; $f(b) = c$; $f(c) = b$. Then the function f is weakly D-irresolute but not contra D-irresolute. Observe that for the D-open set $V = \{c\}$, $f^{-1}(V) = \{b\}$ is not D-closed.

Remark 4.9. From the above discussion and known results we have the following implication $A \longrightarrow B$ ($A \not\longleftrightarrow B$) represents A implies B but not conversely (A and B are independent of each other).

strongly continuous $\longrightarrow$ perfectly D-continuous $\longrightarrow$ completely D-irresolute
$\downarrow$ $\downarrow$ $\downarrow$
completely D-irresolute $\longrightarrow$ contra-D-irresolute $\not\longleftrightarrow$ weakly-D-irresolute

Definition 4.10. A space (X, τ) is said to be

(i) locally indiscrete space [13] if every open subset of X is closed.

(ii) locally indiscrete D-space if every D-open (resp. D-closed) set is D-closed (resp. D-open).

Theorem 4.11. *A function $f : (X, \tau) \to (Y, \sigma)$ is weakly D-irresolute if the graph function defined by $g(x) = (x, f(x))$ for each $x \in X$ is weakly D-irresolute.*

Proof. Let $x \in X$ and $V \in DO(Y, f(x))$. Then $X \times V$ is a D-open set of $X \times Y$ containing $g(x)$. Since g is weakly D-irresolute, there exists $V \in DO(X, x)$ such that $g(U) \subseteq D - cl(X \times V) \subseteq X \times D - cl(V)$. Hence we have $f(U) \subseteq D - cl(V)$.□

Lemma 4.12. A locally indiscrete D-space is $D - T_2$ if and only if for each pair of distinct points $x, y \in X$, there exist $U \in DO(X, x)$ and $V \in DO(X, y)$ such that $D - cl(U) \cap D - cl(V) = \Phi$.

Proof. This follows immediately from the definition of locally indiscrete D-space and lemma 2.5. □

Theorem 4.13. *If a locally indiscrete D-space Y is $D - T_2$ space and $f : (X, \tau) \to (Y, \sigma)$ is a weakly D-iresolute injection then X is $D - T_2$.*

Proof. Let x, y be any two distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since Y is locally indiscrete D-space and $D - T_2$ space, by lemma 4.12, there exist $V \in DO(Y, f(x))$ and $W \in DO(Y, f(y))$ such that $D - cl(V) \cap D - cl(W) = \Phi$. Since f is weakly D-irresolute $G \in DO(X, x)$ and $H \in DO(X, y)$ such that $f(G) \subseteq D - cl(V)$ and $f(H) \subseteq D - cl(W)$. Hence we obtain $G \cap H = \Phi$. Hence X is $D - T_2$. □

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