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일반화된 허프변환의 임계값 선택을 위한 확률적 접근방식

(A Selection of Threshold for the Generalized Hough Transform: A Probabilistic Approach)

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요 약

허프변환은 이미지 영역에서 패러미터 영역으로의 변환을 통해 주어진 이미지에서 모델 인스턴스를 추출해내는 방식으로 허프변환된 결과는 패러미터 영역 좌표에 해당하는 Cell 카운터들의 히스토그램 형태가 된다. 다음 단계로 임계값을 정한 후 이를 상회하는 카운터 값에 해당하는 패러미터 값을 통해 모델 인스턴스를 추출하게 되는데 일반적으로 그 임계값은 최고 Cell 카운터 값의 일정 부분에 해당하는 값을 주로 선택하게 된다. 임계점이 너무 낮을 경우 잘못된 모델 인스턴스를 추출할 가능성이 있으며(*false positives*) 반대로 너무 높은 임계점을 선택할 경우 존재하는 모델 인스턴스를 추출해내지 못하는 오류 (*false negatives*)를 초래하게 된다. 본 논문에서는 일반화된 허프변환(Generalized Hough Transform) 적용 시 패러미터 영역 에서의 Cell 카운터 값의 임계점 선택을 위한 방법으로 확률적인 접근방식을 제시하며 이를 위해 Cell 카운터 분포에 해당하는 조건부 확률을 도출하여 과학적인 임계점 선택이 가능함을 입증한다.

Abstract

When the Hough transform is applied to identify an instance of a given model, the output is typically a histogram of votes cast by a set of image features into a parameter space. The next step is to threshold the histogram of counts to hypothesize a given match. The question is "What is a reasonable choice of the threshold?" In a standard implementation of the Hough transform, the threshold is selected heuristically, e.g., some fraction of the highest cell count. Setting the threshold too low can give rise to a false alarm of a given shape(Type I error). On the other hand, setting the threshold too high can result in mis-detection of a given shape(Type II error). In this paper, we derive two conditional probability functions of cell counts in the accumulator array of the generalized Hough transform(GHough), that can be used to select a scientific threshold at the peak detection stage of the Ghough.

Keywords : Machine Vision, Object Recognition, Generalized Hough Transform, Hypothesis Tesing

I. Introduction

The Hough transform (HT) is a useful technique for the detection of parametric curves, such as straight lines, circles, and ellipses[1]. Duda and Hart introduced the polar parameterization which makes the method more efficient for the line detection[2]. Kimme, Ballard and Sklansky exploited the gradient information of edge pixels and thus reduced the search space significantly[3]. Merlin and Farber generalized the HT for the detection of an arbitrary shape at a given orientation and a given scale[4]. Ballard extended the work of Mellin and Farber and proposed the generalized Hough transform (GHough)

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that could detect arbitrary shapes with any scale and orientation[5]. To date, hundreds of papers related to the HT have been published. A comprehensive survey by Illingworth and Kittler[6] includes extensive discussion of variants and extensions of HT. It is well-known that the main drawbacks of GHough are heavy computation due to both the enumeration of all possible rotation and scale parameters requiring 4-dimensional parameter space and a 1-to-n voting strategy. Thus, there have been many attempts to improve the time and space complexity of the GHough. A randomized generalized Hough transform was proposed to improve the GHough, but it still employs 1-to-n voting strategy[7]. Tsai proposed an improved generalized Hough transform[8] that employs a circle fitting method to eliminate false matches of points and uses the center of a circle and a edge point to form a vector for estimating rotation angle and translations. Some n-to-1 mapping approaches based on geometric constraints have been proposed to successfully reduce the parameter space from 4-D to 2-D[9~11]. A randomized approach using a line segment has been suggested[12] to detect ellipses using segment merging and randomized approach for pairing the segments. A color information has been used to detect a shape not only from a boundary, but also from an internal ques such as chromatic information[13].

The above approaches have been successful in both improving the time and space requirement of the GHough and making the algorithm rich enough to handle internal information inside the boundary of a shape. However, they do not provide any guidelines for selecting a threshold at the peak detection stage of their methods. The time and space-efficient GHough or its variants are of no use, if they either detect a wrong instance of a model(*false positives*) or fail to detect a correct instance of a model(*false negatives*).

In GHough, the *false positives* and *false negatives* are determined by the threshold used at the peak detection stage. In most of the research so far, the threshold tends to be selected heuristically by the human being(i.e., some fraction of the highest cell count or simply the maximum of the cell counts). Therefore, this paper is an attempt to provide a scientific guidelines for selecting a threshold when GHough searches for the local maxima in the parameter space.

When the Hough transform is applied to identify an instance of a given model, the output is typically a histogram of votes cast by a set of image features into a parameter space. The next step is to threshold the histogram counts to hypothesize a given match. These counts can be considered as evidence that an instance of the model exists in a given image.

The question is "What is a reasonable choice of the threshold?" In a standard implementation of the Hough transform, the threshold is selected heuristically, e.g., some fraction of the highest cell count. However, the distribution of cell counts in the parameter space varies from one method to the other depending on the usage of image features. It also depends on the complexity of an image. Setting the threshold too low can give rise to a false alarm of an instance of a given shape. On the other hand, setting the threshold too high can give rise to a false detection of an instance of a given shape. Therefore, whenever we generate a match hypothesis (e.g., by setting a threshold), we must take the risk of making two types of error: one is not identifying an instance when one exists (*false negatives*, Type II error), and the other is identifying an instance when one does not exist (*false positives*, Type I error). Both of these probabilities are determined by the threshold selected. In order to obtain the probabilities of the two types of error when setting a threshold, two conditional probability functions are required: one is the probability function of a cell count given no instance of an object exists in an image(p_0), and the other is the probability function of a cell count given an instance of an object exists in an image(p_1).

In this paper, we analyze the generalized Hough Transform(GHough) in order to derive the two conditional probability functions(p_0 and p_1) of the peaks in the Hough parameter space. These probability functions can be used quantitatively when setting a threshold at the peak detection stage of the GHough.

II. Analysis of the GHough Method

The generalized Hough Transform(GHough) associates point features in an image to point features in a given model to accumulate statistical evidence for a particular set of Euclidean transformation parameters. Gradient direction information at each point provides additional cues related to model orientation. For the encoding of a given shape, GHough selects an arbitrary reference point and stores the displacement vectors into a hash table (R-table) indexed by a gradient angle at each model point. These displacement vectors pointing to the reference point are later used to deduce the possible locations of the reference point as seen from each edge pixel in a given image. If we assume that the origin of the model coordinate system is at the reference point, then it is easy to see that the location of the reference point relative to the image coordinate system corresponds to the translation required to align a model point with an image point. Therefore, the location of the reference point deduced from an image point implicitly specifies a possible translation of the model.

1. The Region of Spread in Parameter Space

In this section, we consider what effects the uncertainties in measuring image features have on the accuracy of the GHough method. We use the following notation:

- p_i is the ith model point, $i = 1 ... n$.
- ϕ_i is the gradient angle at p_i.

- d_i is the displacement vector as seen from the ith model point.
- q is the quantization size of the R-table.

[Note] : We use capital letters corresponding to small letters to mean they are related to the image plane, e.g., P_I and Φ_I .

We assume that the GHough method has a priori knowledge about the rotation and scale parameters. Suppose an image contains an instance of the model which is rotated by θ and scaled by s. If the image were ideal, then it would contain a point P_I with gradient angle Φ_I, which is a correct instance of the ith model point p_i with a gradient angle ϕ_i (assuming no occlusion). Since Φ_I is a correct instance of the model point scaled by s and rotated by θ, $\phi_i + \theta$ must be equal to Φ_I, and, therefore, GHough would vote into a point in 2-dimensional translation space $P_{\theta,s}$ that corresponds to a rotation of θ and a scale of s. This point t in $P_{\theta,s}$ can be computed as

$$t = P_I + s R_\theta d_i \ ,$$

by indexing into the R-table using the angle $\Phi_I - \theta$ and retrieving the displacement vector d_i, which needs to be rotated by θ and scaled by s.
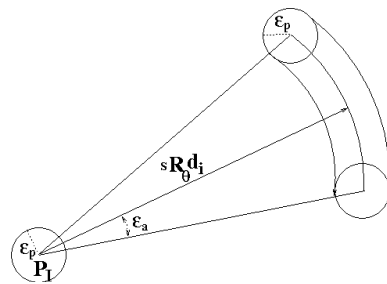

그림 1. 패러미터 영역으로의 Voting은 한 점이 아닌 일정영역으로 분산되며 이는 엣지 위치와 Gradient 각도 계산상의 오차에 기인함(ϵ_p와 ϵ_a에 해당)

Fig. 1. Pairing the ith model point with the correct I^{th} image point results in a set of translations because of measurement errors.

However, in practice, an image cannot be ideal. The observed gradient angle Φ_I^{ob} will deviate from the ideal by ϵ_a and the observed location of P_I can be anywhere within a disk of radius ϵ_p , i.e.,

$$|\Phi_I^{ob} - \Phi_I| \leq \epsilon_a , \quad \text{and}$$

$$\|P_I^{ob} - P_I\| \leq \epsilon_p .$$

If we account for both positional and angular error, we can see that GHough should vote into a region instead of a point in $P_{\theta, s}$. This region of spread is shown in Fig. 1 , and is given by

$$V_{(d_i, s)} = \left\{ P_I^{ob} + s R_\theta d_i \mid \left(|P_I^{ob} - P_I| \leq \epsilon_p, |\theta_{true} - \theta| \leq \epsilon_a \right) \right\}$$

As we can see, the area of this region depends on the length of the ith displacement vector d_i and the amount of positional and angular uncertainty. This is the region into which a single image point P_I can vote. Since the shape of a model is arbitrary, and displacement vectors can have different lengths, the region of spread for a given model can be overlapping and quite complicated as shown in Fig. 2. However, we can obtain the worst case area of
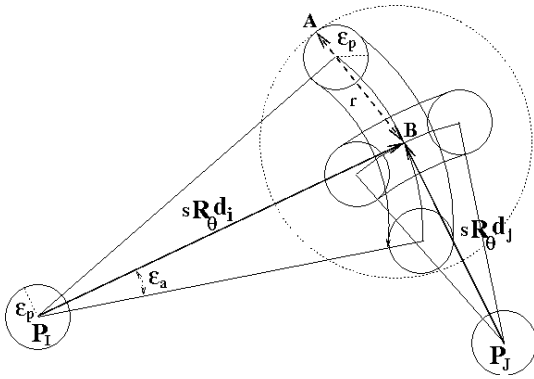

그림 2.　각각의 모델 포인트가 Voting가능한 영역은 상호 겹치게 됨. 모델 포인트에서 Reference Point 까지의 거리가 가장 긴 Displacement 벡터를 기준으로 최대 Voting 가능한 영역을 산출함

Fig. 2.　The region of spread for an arbitrary model can be overlapping and quite complicated. But we can obtain the worst case area using the circle of radius r that corresponds to the longest displacement vector.

spread by computing the area of a circle with radius r, where r is the distance between A and B in Fig. 2.

Since r can be obtained as

$$r = 2 \parallel sd_{\max} \parallel \sin\left(\frac{\epsilon_a}{2}\right) + \epsilon_p ,$$

where $d_{\max}$ is the longest displacement vector in a given model, the worst case area of spread is

$$G_{spread} = \pi \left[2 \parallel sd_{\max} \parallel \sin\left(\frac{\epsilon_a}{2}\right) + \epsilon_p \right]^2 .$$

2. Peak Detection Strategy

The phenomenon of spread into a region in the parameter space suggests that we must use a moving circle with radius r (given above) to receive the votes cast by possible instances of the model points. Therefore, we will assume in this analysis that, in the peak detection stage of the GHough method, we find a maximum sum of cells inside the moving circle of radius r, where r is a function of both measurement errors and the model.

3. Hashing Probability

The moving circle will take votes not only from image points that correspond to an instance of a model (signal), but also from image points that does not belong to a particular instance of a model (noise). Even though we adopt the above peak detection strategy, P_I will not always vote into the moving circle; voting into the moving circle can occur only if gradient angles match. That is, G_{spread} is the region into which the Ith image point can vote only if the angle $\Phi_I^{ob} - \theta$ hashes into the same bin in the R-table as ϕ_i does. Since

$$\left| \Phi_I^{ob} - \Phi_I \right| \leq \epsilon_a \quad ,$$

the indexing into the R-table is not guaranteed to occur. Thus, for a given quantization size q of the
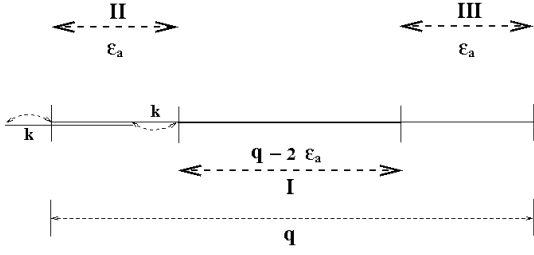
그림 3. R-table의 quantization 크기가 q이고 Gradient 각도 계산상의 에러(ϵ_a) 존재 시 세가지 Event ($E1, E2, E3$) 를 고려할 수 있으며 각각 Voting 이 일어날(Hashing) 확률 계산이 가능함.

Fig. 3. The whole quantization interval q of the R-Table can be divided into three non-intersecting sub-intervals (I, II, and III).

R-table, we want to compute the probability that the Ith point casts a vote into the region G_{spread}.

Let us assume that $2\epsilon_a \leq q$, so, the quantization size q of a gradient angle in the R-table is large enough to compensate for possible error due to an angular uncertainty ϵ_a. Then, depending on the true value of ϕ_i, we can consider three cases as shown in Fig. 3, where the whole quantization interval q is divided into three non-intersecting sub-intervals (I, II, and III).

Let $E1$, $E2$, $E3$ be the events, "ϕ_i was in the interval I," "ϕ_i was in the interval II," and " ϕ_i was in the interval III," respectively, and $E0$ be the event "Hashing occurs." Then, the probability that the Ith image point casts a vote is given by

$$P_{vote} = P(E0|E1)P(E1) + P(E0|E2)P(E2) \\ + P(E0|E3)P(E3)$$

Since $P(E0|E2)$ depends on exactly where ϕ_i was in the interval II, we can consider the case where ϕ_i was within a distance k from the end point of the interval I. Depending on the value of k, there are two extreme cases; one is when ϕ_i was as close to the end point of the interval I as possible, the other is when ϕ_i was almost at the end point of the quantization size of the R-table (q). The former gives the probability close to 1, and the latter gives the probability close to 1/2. By averaging, we can

approximate $P(E0|E2)$ as 3/4. Therefore, the total probability that an Ith image point casts a vote is the sum of $P(E0|E1)P(E1)$, $P(E0|E2)P(E2)$ and $P(E0|E3)P(E3)$, and is given by

$$P_{vote} \approx \frac{q - 2\epsilon_a}{q} + 2\left(\frac{3}{4}\right)\frac{\epsilon_a}{q}$$
$$= 1 - \frac{\epsilon_a}{2q}, (q \geq 2\epsilon_a).$$

This is the probability that a single model point can cast a vote into the region G_{spread}.

4. Derivation of the Probability Function of Cell Counts

In this section, we derive the probability function of the cell counts inside a moving circle with radius r at a certain location of the parameter space using the previous analysis. Assume that the size of an input image is $M \times M$ in pixels. Let n be the number of model points, N be the number of image points. Let $k1$ and $k2$ be random variables that represent the number of votes cast by $\lfloor f \times n \rfloor$ model points appearing in an image, where $0 \leq f \leq 1$, and $\lfloor N - f \times n \rfloor$ noise points, respectively, into the moving circle of radius r at a certain location of the accumulator array. Then we can model $k1$ and $k2$ using the binomial distribution. The probability that $k1$ points of the signal cast votes into the moving circle is

$$P(k_1) = \binom{\lfloor f \times n \rfloor}{k_1} P_{vote}^{k_1} \left(1 - P_{vote}\right)^{\lfloor f \times n \rfloor - k_1},$$

where P_{vote} is given above. This is the signal contribution to the moving circle.

For the noise contribution, let us consider how many votes can be cast into the moving circle by a single noise point. Since there are n entries (displacement vectors) in the R-table whose size is equal to $\lfloor 2\pi/q \rfloor$, on the average, $\dfrac{n}{\lfloor 2\pi/q \rfloor}$ votes can be cast into the moving circle by a single noise point. Hence, a total number of votes that can

be cast into the moving circle is given by

$$\beta(n, N, f, q) = \frac{n}{\lfloor 2\pi/q \rfloor}(N - \lfloor f \times n \rfloor).$$

Therefore, the probability that $k2$ noise points cast a vote into the moving circle is

$$P(k_2) = \binom{\beta(n, N, f, q)}{k_2} P_{spread}^{k_2}(1 - P_{spread})^{\beta(n, N, f, q) - k_2},$$

where $P_{spread} = \dfrac{G_{spread}}{M^2}$. This is the noise contribution to the moving circle. Therefore, $p_1(k)$, which is the probability function of the sum of all the cell counts inside the moving circle with radius r is given by the convolution of $P(k_1)$ and $P(k_2)$. (This is true because $k1$ and $k2$ are assumed to be independent and the probability function of the sum of independent random variables can be obtained by convolving the two probability functions[14].) Hence,

$$p_1(k) = P(k_1) \otimes P(k_2),$$

where $\otimes$ is the convolution operator. For even moderate values of n and N, the computation of both $P(k1)$ and $P(k2)$ becomes cumbersome, so we approximate them using the Poisson approximation:

$$P(k_1) \approx \frac{\mu^{k_1}}{k_1!}e^{-\mu}, \text{ and}$$

$$P(k_2) \approx \frac{\nu^{k_2}}{k_2!}e^{-\nu},$$

where $\mu = \lfloor f \times n \rfloor P_{vote}$ and $\nu = \beta(n, N, f, q)P_{spread}$. Thus,

$$p_1(k) \approx \frac{\mu^{k_1}}{k_1!}e^{-\mu} \otimes \frac{\nu^{k_2}}{k_2!}e^{-\nu}.$$

The p_0 can also be obtained by setting $f = 0$ in $P(k_2)$ (i.e., no object exists in an image),

and is given as

$$p_0(k) =$$
$$\binom{\beta(n, N, 0, q)}{k} P_{spread}^k(1 - P_{spread})^{\beta(n, N, 0, q) - k},$$

which we approximate as

$$p_0(k) \approx \frac{[\beta(n, N, 0, q)P_{spread}]^k}{k!}e^{-\beta(n, N, 0, q)P_{spread}}.$$

Given p_0 and p_1, we can set the threshold t by considering both the probability of *false positives* and the probability of *false negatives*.

III. Usage of p_0 and p_1

In this section, we show how p_0 and p_1 can be used to set the threshold at the peak detection stage of the GHough by considering a representative example.

For simplicity, we assume the GHough has a priori knowledge about the rotation(θ) and no scale(s) of the model occurs(i.e., $s = 1$). Also no occlusion occurs in an input image (i.e. , $f = 1$).

Let the size of an input image be 256 x 256, the quantization size (q) of the R-table be 10 degrees. Also, let the positional uncertainty ϵ_p and the angular uncertainty ϵ_a be 5 pixels and 3 degrees, respectively. The number of model points used is set to 100 and the length of the longest displacement vector of the model (d_{max}) is set to 50 pixels. These parameter values are chosen arbitrarily in order to show how the probability distribution p_0 and p_1 can be computed. For example, we can set the positional uncertainty ε_p to be 3 pixels instead of 5 pixels if the imaging condition is good enough and the edge operator can generate more accurate edge points. For demonstration purpose, let us consider the case where the number of edge pixels extracted from an input image is 5,000, 10,000, 15,000, and 20,000 points. The plot in Fig. 4 (a) shows the conditional probability function of the cell counts within a moving circle of radius r when no instance of the model exists(p_0) and when an instance of the model exists(p_1) in a given image. Fig. 4 (b) through (d) shows how the p_0 and p_1 change as we increase the number of edge

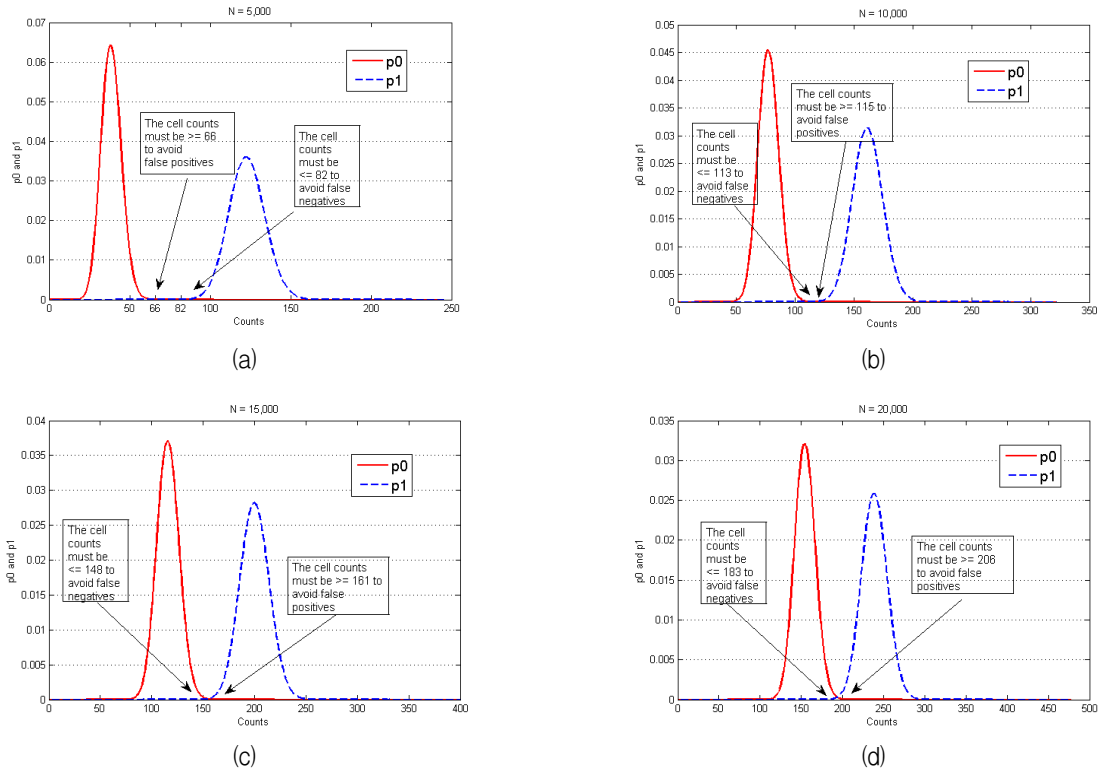(a)　　　　　　　　　　　　　　　　(b)

(c)　　　　　　　　　　　　　　　　(d)

그림 4. (a)(b)(c)(d)는 엣지 픽셀 숫자가 5,000개에서 20,000개로 5,000개 단위로 증가할 경우 p_0와 p_1의 변화 추이를 보여줌

Fig. 4. (a) (b) (c) (d) shows how p_0 and p_1 change as the number of edge pixels(N) increases from 5,000 up to 20,000 in steps of 5,000 points in a given image.
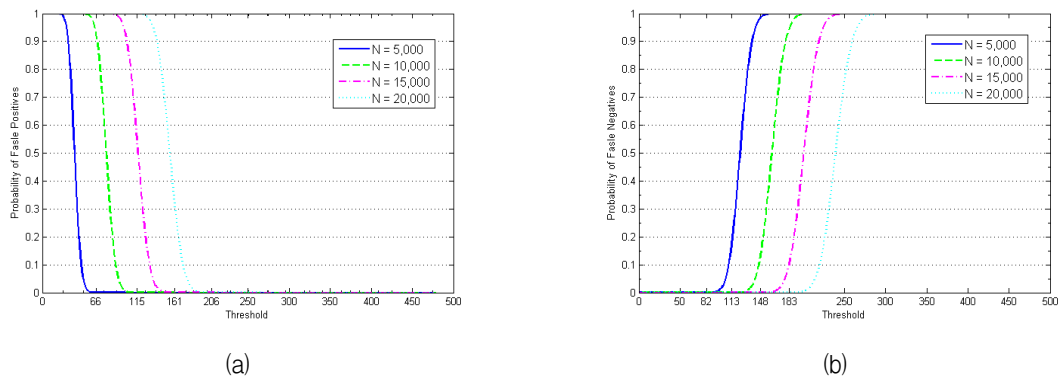

(a)　　　　　　　　　　　　　　　　(b)

그림 5. (a)는 엣지 픽셀이 5,000개에서 20,000개까지 5,000개 단위로 증가할 경우 각각의 선택한 임계값에 해당하는 Type I 에러 확률을 나타냄 (b)는 각각의 Type II 에러 확률을 나타냄

Fig. 5. (a) shows the probability of false positives for each threshold selected when the number of edge pixels is 5,000, 10,000, 15,000 and 20,000, respectively (b) shows the probability of false negatives for each threshold selected when the number of edge pixels is 5,000, 10,000, 15,000 and 20,000, respectively.

pixels from 5, 000 to 20,000 in steps of 5,000.

Fig. 5(a) and (b) shows the probability of both *false positives* and *false negatives* for a given threshold on the x-axis. We can see from Fig. 5(a)

that, to maintain a zero probability of *false positives*, we need to set the threshold higher than 66 when the number of edge pixels is 5,000. As we increase the number of edge pixels up to 20,000 in steps of 5,000,

the GHough requires higher thresholds of 115, 161 and 206 to maintain zero false alarm probability. On the other hand, Fig 5(b) shows the probability of *false negatives* for each selected threshold on the x-axis. We can see that the GHough requires the threshold lower than 82 in order to avoid the *false negatives* and requires thresholds lower than 113, 148 and 183 to maintain zero probability of *false negatives* as the number of edge pixels increases up to 20,000 in steps of 5,000.

IV. Experiments

GHough was tested on a 8-bit grey scale wrench image(256×256) in Fig 6. (b) that has two instances of the model in Fig. 6(a). Total of 83 points were sampled along the boundary of the head of the wrench to construct the R-Table. The quantization interval of the gradient directions was set to 10 degrees. After applying the Sobel operator and thresholding at 70 % of the normalized gradient magnitude to both the original and the Gaussian noise corrupted images(Fig. 6(b) and Fig. 6(c)) with σ=20, two edge maps were obtained as shown in Fig. 6(d) and (e) that has 7,017 and 11,099 edge pixels, respectively. The length of the longest displacement vector was 29 pixels long. Fig 6. (f) and (g) shows that GHough successfully recovers the instances of the model with different rotation and scale parameters((rotation, scale) = (89 degrees, 0.95), (300 degrees, 1.45)). Fig. 7(a) and (b) shows the accumulator arrays for the first instance of the model ((rotation, scale) = (89 degrees, 0.95)) for both the original and noise corrupted images. Fig. 7(c) and (d) are similar for the second instance of the model ((rotation, scale) = (300 degrees, 1.45)).

The question is what threshold should be used and how we can select it when GHough searches through the accumulator array for the local maxima in Fig 7 (a) through (d). From Fig. 8(a), we can see that, if GHough select a threshold t lower than 55,
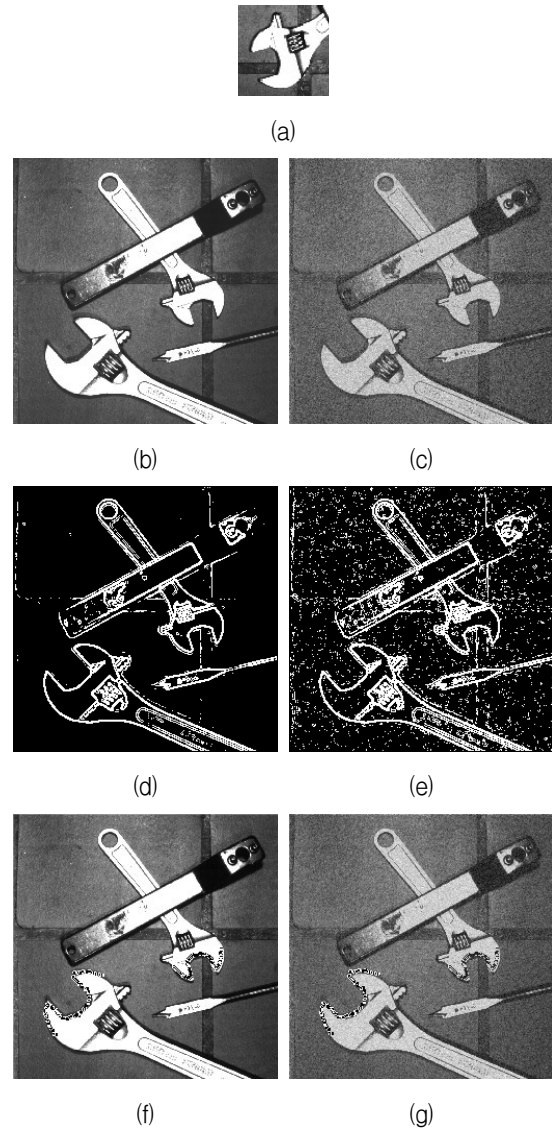

(a)

(b)   (c)

(d)   (e)

(f)   (g)

그림 6. (a) 모델 이미지 (b) 입력 영상 (c) Gaussian Noise 추가된 영상 (d)(e) 엣지 추출된 영상 (f)(g) 각각의 인식결과

Fig. 6. (a) a template image (b) 256×256 input image with two different instances of the model (rotation, scale) = (89 degrees, 0.95), (300 degrees, 1.45) (c) an input image corrupted with Gaussian noise with σ= 20 (d) an edge map consisting of 7,017 edge pixels after applying the Sobel operator to the original image. (e) is an edge map consisting of 11,099 edge pixels obtained from a noise corrupted image (f) and (g) shows the recovered model points superimposed on both the original and noise corrupted images.

there exists a non-zero probability of Type I error.(i.e., the sum of p_0 from t to 55 cannot be
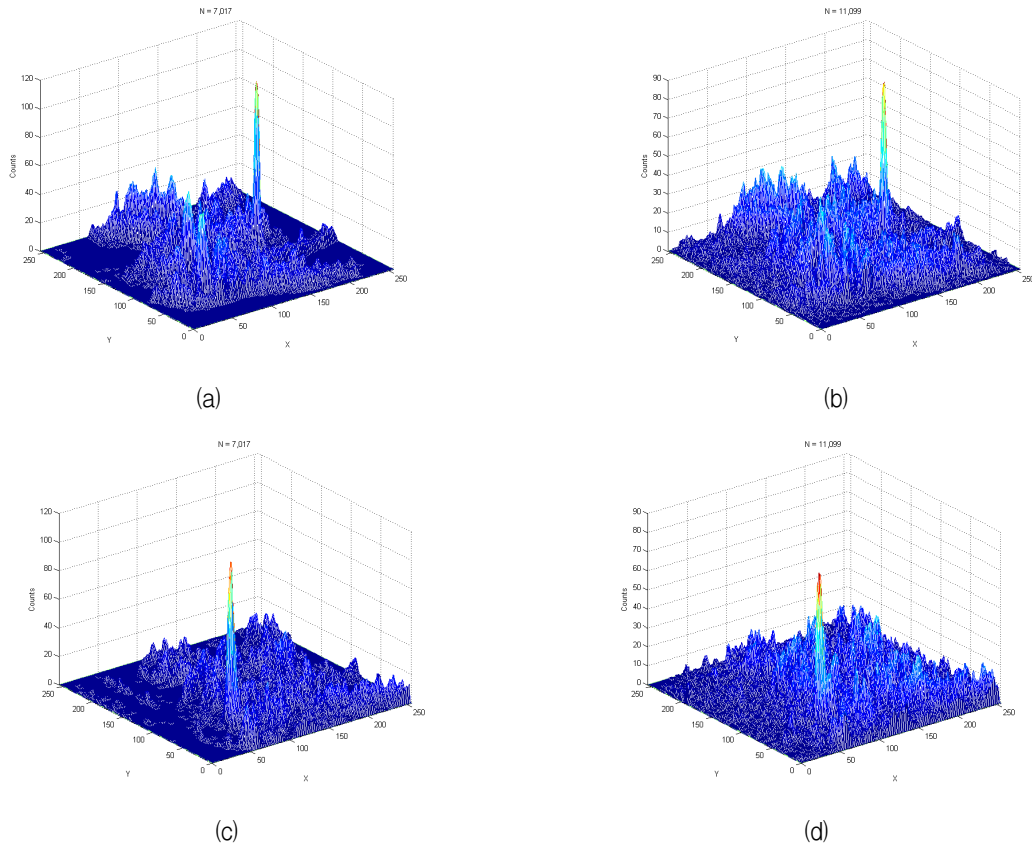
(a)

(b)

(c)

(d)

그림 7. (a)(b) 원래 영상과 Noise 추가된 영상에서 첫번째 모델 추출 결과
(c)(d) 원래 영상과 Noise 추가된 영상에서 두번째 모델 추출 결과
Fig. 7. (a) and (b) shows the accumulator arrays for the first target in the original and the noise corrupted image.
(c) and (d) shows the accumulator arrays for the second target in the original and the noise corrupted image.
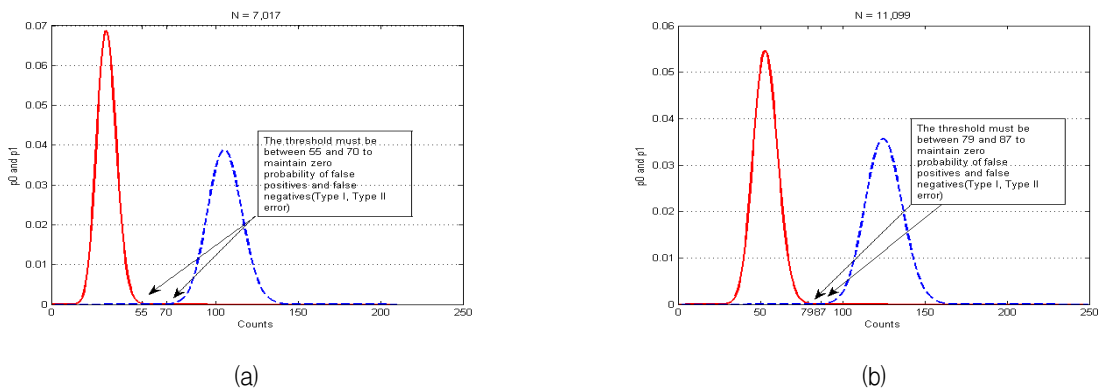

(a)

(b)

그림 8. 원래의 영상과 Noise 추가된 영상에서 엣지 수가 증가함에 따른 p_0와 p_1의 변화 추이를 보여줌. Type I과 Type II를 피하기 위한 임계값 선택의 폭이 좁아짐
Fig. 8. (a) and (b) shows the plot of p_0 and p_1 for both the original and the noise corrupted images. As the number of edge pixels increases due to noise, we need to select a different threshold in order to avoid Type I and Type II error, accordingly.

zero) On the other hand, GHough will not generate *false negatives* because the probability of Type II error which is the sum of p_1 from t lower than 55 to the left direction is zero anyway. That is, GHough

will not miss an instance because the threshold t lower than 55 is sufficiently low. However, if GHough select a threshold t higher than 70, the probability of Type I error is zero, but there exists a non-zero probability of Type II error which is the sum of p_1 from 70 to t. That is, GHough will miss an existing instance because the threshold higher than 70 is too high. Therefore, Fig 8(a) suggests that GHough should select a threshold between 55 and 70 in order to avoid both Type I and Type II error in this case. Similarly, from Fig. 8(b), we can see that GHough should select a threshold between 79 and 87 for the noise corrupted image with 11,099 edge pixels to avoid both Type I and Type II error.

V. Conclusion

In this paper, we derived two conditional probability functions of the peak(cell counts) in the accumulator array of the generalized Hough transform(GHough). The probability functions are related to parameters such as the amount of uncertainty in measuring image features, the number of image features, and the degree of occlusion of the model. Using these probability functions(both p_0 and p_1), we can choose a scientific threshold at the peak detection stage of the GHough considering both the probability of *false positives*(Type I error) and the probability of *false negatives*(Type II error).

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