

MEAN SQUARE EXPONENTIAL DISSIPATIVITY OF SINGULARLY PERTURBED STOCHASTIC DELAY DIFFERENTIAL EQUATIONS

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ABSTRACT. This paper investigates mean square exponential dissipativity of singularly perturbed stochastic delay differential equations. The L -operator delay differential inequality and stochastic analysis technique are used to establish sufficient conditions ensuring the mean square exponential dissipativity of singularly perturbed stochastic delay differential equations for sufficiently small $\varepsilon > 0$. An example is presented to illustrate the efficiency of the obtained results.

1. Introduction

Singularly perturbed delay differential equations is ordinary differential equations in which the highest derivative are multiplied by a small parameter and involving at least one delay term. These equations arise naturally in a wide variety of engineering applications, representative examples include catalytic continuous stirred-tank reactors [1], biochemical reactors [3], fluidized catalytic crackers [15], flexible mechanical systems [5], electromechanical networks [2], etc. In recent years, in a number of papers [4, 8, 10, 18, 19, 20], the stability, dissipativity and other behaviors of singularly perturbed delay differential equations are considered.

However, in addition to delay effects and singular perturbation, stochastic effects likewise exist in real systems. Many dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc. Much effort has been devoted to extend many fundamental results for deterministic systems to stochastic systems [9, 11, 12, 13, 14, 22]. In the past decades, increasing attention has been devoted to the problems of stability and other

Received January 18, 2013; Revised October 29, 2013.

2010 *Mathematics Subject Classification.* 34K50, 34K20, 34K26.

Key words and phrases. delay, stochastic, singularly perturbed, mean square exponential dissipativity, L -operator delay differential inequality.

The work is supported by the National Natural Science Foundation of China (Grant No. 11101367) and the China Scholarship Council (Grant No. 201208330001).

behaviors of singularly perturbed stochastic differential systems by many researchers [6, 7, 16, 17].

Unfortunately, up to now, very little is known on the exponential dissipativity of singularly perturbed stochastic delay differential equations. It is, therefore, our main aim in this paper is to investigate the exponential dissipativity analysis problem of singularly perturbed stochastic delay differential equations. By establishing an L -operator delay differential inequality and stochastic analysis technique, some sufficient conditions are given such that singularly perturbed stochastic delay differential equations are mean square exponentially dissipative for sufficiently small $\varepsilon > 0$. An example is presented to illustrate the efficiency of the obtained results.

2. Model and preliminaries

Throughout the paper, unless otherwise specified let $\tau \geq 0$, $\mathbb{R}_+ = [0, \infty)$, and $C[[-\tau, 0], \mathbb{R}^n]$ denote the family of all continuous functions ϕ from $[-\tau, 0]$ into \mathbb{R}^n with the norm $\|\phi\| = \sup_{-\tau \leq \theta \leq 0} |\phi(\theta)|$, where $|\cdot|$ is Euclidean norm in \mathbb{R}^n . Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is right continuous and \mathcal{F}_0 contains all P -null sets). Let $C_{\mathcal{F}_0}^b[[-\tau, 0], \mathbb{R}^n]$ denote the family of all bounded \mathcal{F}_0 -measurable, $C[[-\tau, 0], \mathbb{R}^n]$ -valued random variables ϕ , satisfying $\|\phi\|_{L^2}^2 = \sup_{-\tau \leq \theta \leq 0} \mathbf{E}|\phi(\theta)|^2 < \infty$, where \mathbf{E} denotes the expectation of stochastic process. Let L denote the well-known L -operator given by the Itô formula.

In this paper, we consider the following singularly perturbed Itô stochastic delay differential equations:

$$(1) \quad \begin{cases} \varepsilon dx(t, \varepsilon) = f(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon))dt \\ \quad + \sqrt{\varepsilon}g(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon))d\omega(t), \quad t \geq t_0, \\ x(t_0 + s, \varepsilon) = \phi(s), \quad s \in [-\tau, 0], \end{cases}$$

where $0 \leq \tau(t) \leq \tau$, $\varepsilon \in (0, \varepsilon_0]$ is a small parameter, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$, $g : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$, $\omega(t) = (\omega_1(t), \dots, \omega_m(t))^T$ is an m -dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$. And the initial function $\phi(s) = (\phi_1(s), \dots, \phi_n(s))^T \in C_{\mathcal{F}_0}^b[[-\tau, 0], \mathbb{R}^n]$.

Throughout this paper, we assume that for any $\phi \in C_{\mathcal{F}_0}^b[[-\tau, 0], \mathbb{R}^n]$, there exists at least one solution of (1), which is denoted by $x(t, t_0, \phi, \varepsilon)$, or, $x(t)$, if no confusion occur.

Definition 2.1. System (1) is said to be mean square exponentially dissipative for sufficiently small $\varepsilon > 0$ if there exist positive constants M and K , which are independent of $\varepsilon \in (0, \varepsilon_0]$ for some $\varepsilon_0 > 0$, and a constant $\lambda > 0$ such that for any solution $x(t, t_0, \phi, \varepsilon)$ with the initial condition $\phi \in C_{\mathcal{F}_0}^b[[-\tau, 0], \mathbb{R}^n]$,

$$\mathbf{E}|x(t, t_0, \phi, \varepsilon)|^2 \leq K\|\phi\|_{L^2}^2 e^{-\lambda(t-t_0)} + M, \quad t \geq t_0.$$

Definition 2.2. System (1) is said to be mean square exponentially stable if there is a pair of positive constants λ and K , which are independent of $\varepsilon \in (0, \varepsilon_0]$ for some $\varepsilon_0 > 0$, such that for any solution $x(t, t_0, \phi, \varepsilon)$ with the initial condition $\phi \in C_{\mathcal{F}_0}^b[[-\tau, 0], \mathbb{R}^n]$,

$$\mathbf{E}|x(t, t_0, \phi, \varepsilon)|^2 \leq K\|\phi\|_{L^2}^2 e^{-\lambda(t-t_0)}, \quad t \geq t_0.$$

Lemma 2.1 ([20, A generalized Halanay inequality]). *Suppose that*

$$u'(t) \leq \gamma(t) + \alpha(t)u(t) + \beta(t) \sup_{t-\tau \leq \theta \leq t} u(\theta)$$

for $t \geq t_0$. Here $\gamma(t)$, $\alpha(t)$, $\beta(t)$ are continuous and there exist constants $\gamma^* \geq 0$, $\alpha^* > 0$, and $q \in [0, 1)$ such that

$$0 \leq \gamma(t) \leq \gamma^*, \alpha(t) \leq -\alpha^*, 0 \leq \beta(t) \leq -q\alpha(t), \quad \forall t \geq t_0.$$

Then

$$u(t) \leq \frac{\gamma^*}{(1-q)\alpha^*} + Ge^{-\mu^*(t-t_0)}, \quad t \geq t_0.$$

Here $G = \sup_{t_0-\tau \leq \theta \leq t_0} |u(\theta)|$, and $\mu^* > 0$ is defined as

$$\mu^* = \inf_{t \geq t_0} \{\mu : \mu + \alpha(t) + \beta(t)e^{\mu\tau} = 0\}.$$

3. Main results

Definition 3.1. We define the class of Lyapunov-like functions \mathbb{V} by $\mathbb{V} = \{V(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}_+$ are twice continuously differentiable in x and once in $t\}$.

Lemma 3.1. *Assume that there exist functions $V(t, x) \in \mathbb{V}$ such that*

$$(2) \quad LV(t, x) \leq \gamma(t) + \alpha(t)V(t, x) + \beta(t) \sup_{t-\tau \leq \theta \leq t} V(t, x(\theta)), \quad t \geq t_0.$$

Here $\gamma(t)$, $\alpha(t)$, $\beta(t)$ are continuous and there exist constants $\gamma^* \geq 0$, $\alpha^* > 0$, and $q \in [0, 1)$ such that

$$0 \leq \gamma(t) \leq \gamma^*, \alpha(t) \leq -\alpha^*, 0 \leq \beta(t) \leq -q\alpha(t), \quad \forall t \geq t_0.$$

Then

$$\mathbf{E}V(t, x) \leq \frac{\gamma^*}{(1-q)\alpha^*} + Ge^{-\mu^*(t-t_0)}, \quad t \geq t_0.$$

Here $G = \sup_{t_0-\tau \leq \theta \leq t_0} \mathbf{E}V(t_0, x(\theta))$, and $\mu^* > 0$ is defined as

$$(3) \quad \mu^* = \inf_{t \geq t_0} \{\mu : \mu + \alpha(t) + \beta(t)e^{\mu\tau} = 0\}.$$

Proof. Since $x(t)$ is the solution process of (1) and $V(t, x) \in \mathbb{V}$, by the Itô formula, we can get (for convenience, throughout this proof, we assume $t \geq t_0$)

$$V(t, x(t)) = V(t_0, x(t_0)) + \int_{t_0}^t LV(s, x(s))ds$$

$$+ \int_{t_0}^t \frac{\partial V(s, x(s))}{\partial x} \frac{1}{\sqrt{\varepsilon}} g(s, x(s), x(s - \tau(s))) d\omega(s).$$

Then we have

$$(4) \quad \mathbf{E}V(t, x(t)) = \mathbf{E}V(t_0, x(t_0)) + \int_{t_0}^t \mathbf{E}LV(s, x(s)) ds.$$

So, for small enough $\Delta t > 0$, we have

$$(5) \quad \mathbf{E}V(t + \Delta t, x(t + \Delta t)) = \mathbf{E}V(t_0, x(t_0)) + \int_{t_0}^{t+\Delta t} \mathbf{E}LV(s, x(s)) ds.$$

Thus from (2), (4) and (5), we have

$$(6) \quad \begin{aligned} & \mathbf{E}V(t + \Delta t, x(t + \Delta t)) - \mathbf{E}V(t, x(t)) \\ &= \int_t^{t+\Delta t} \mathbf{E}LV(s, x(s)) ds \\ &\leq \int_t^{t+\Delta t} \left\{ \gamma(s) + \alpha(s)\mathbf{E}V(s, x(s)) + \beta(s) \sup_{s-\tau \leq \theta \leq s} \mathbf{E}V(s, x(\theta)) \right\} ds. \end{aligned}$$

From (6), we obtain that

$$(\mathbf{E}V(t, x(t)))' \leq \gamma(t) + \alpha(t)\mathbf{E}V(t, x(t)) + \beta(t) \sup_{t-\tau \leq \theta \leq t} \mathbf{E}V(t, x(\theta)).$$

By a similar argument with the proof of Lemma 2.1 [20], one can know that the μ^* defined by (3) is reasonable. Thus from Lemma 2.1, we know Lemma 3.1 is true. \square

In the following, we will obtain several sufficient conditions ensuring the exponential dissipativity of (1) by employing Lemma 3.1. Here, we shall first introduce the following assumptions.

(A₁) For any $x, y \in \mathbb{R}^n$, there exist continuous functions $\alpha_1(t)$, $\beta_1(t)$, $\gamma(t)$, $t \geq t_0$, such that

$$x^T f(t, x, y) \leq \gamma(t) + \alpha_1(t)|x|^2 + \beta_1(t)|y|^2, \quad t \geq t_0.$$

(A₂) For any $x, y \in \mathbb{R}^n$, there exist continuous functions $\alpha_2(t)$, $\beta_2(t)$, $t \geq t_0$, such that

$$\text{trace}(g(t, x, y))^T(g(t, x, y)) \leq \alpha_2(t)|x|^2 + \beta_2(t)|y|^2.$$

(A₃) There exist constants $\alpha^* > 0$, $\gamma^* \geq 0$ and $q \in [0, 1)$ such that

$$0 \leq \gamma(t) \leq \gamma^*, 2\alpha_1(t) + \alpha_2(t) \leq -\alpha^*, 0 \leq 2\beta_1(t) + \beta_2(t) \leq -q[2\alpha_1(t) + \alpha_2(t)].$$

Theorem 3.1. *Assume that (A₁)-(A₃) hold. Then there exists a small $\varepsilon_0 > 0$ such that system (1) is mean square exponentially dissipative for sufficiently small $\varepsilon \in (0, \varepsilon_0]$.*

Proof. Let $V(t, x) = |x|^2$. Then, by (A_1) and (A_2) , we have

$$\begin{aligned} & LV(t, x(t, \varepsilon)) \\ &= 2x^T \frac{1}{\varepsilon} f(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon)) \\ &\quad + \text{trace}[(\frac{1}{\sqrt{\varepsilon}}g(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon)))^T (\frac{1}{\sqrt{\varepsilon}}g(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon)))] \\ &\leq \frac{2}{\varepsilon} [\gamma(t) + \alpha_1(t)|x(t, \varepsilon)|^2 + \beta_1(t)|x(t - \tau(t), \varepsilon)|^2] \\ &\quad + \frac{1}{\varepsilon} [\alpha_2(t)|x(t, \varepsilon)|^2 + \beta_2(t)|x(t - \tau(t), \varepsilon)|^2] \\ &= \frac{2\gamma(t)}{\varepsilon} + \frac{2\alpha_1(t) + \alpha_2(t)}{\varepsilon} |x(t, \varepsilon)|^2 + \frac{2\beta_1(t) + \beta_2(t)}{\varepsilon} |x(t - \tau(t), \varepsilon)|^2 \\ &\leq \frac{2\gamma(t)}{\varepsilon} + \frac{2\alpha_1(t) + \alpha_2(t)}{\varepsilon} V(t, x(t, \varepsilon)) + \frac{2\beta_1(t) + \beta_2(t)}{\varepsilon} \sup_{t-\tau \leq \theta \leq t} V(t, x(\theta, \varepsilon)). \end{aligned}$$

Application of the inequality in Lemma 3.1 to the above inequality yields

$$\mathbf{E}V(t, x(t, \varepsilon)) \leq \frac{2\gamma^*}{(1-q)\alpha^*} + Ge^{-\mu^*(\varepsilon)(t-t_0)}, \quad t \geq t_0.$$

Here $\mu^*(\varepsilon) > 0$ is defined as

$$(7) \quad \mu^*(\varepsilon) = \inf_{t \geq t_0} \{ \mu : \mu + \frac{2\alpha_1(t) + \alpha_2(t)}{\varepsilon} + \frac{2\beta_1(t) + \beta_2(t)}{\varepsilon} e^{\mu\tau} = 0 \},$$

and $G \geq 0$ only depends on the initial condition ϕ . By a similar argument with the proof of Lemma 2.1 [20], one can know that the $\mu^*(\varepsilon)$ defined by (7) is reasonable.

For any $t \geq t_0$, let $\mu(t, \varepsilon)$ be defined as the unique positive zero of

$$(8) \quad \mu + \frac{2\alpha_1(t) + \alpha_2(t)}{\varepsilon} + \frac{2\beta_1(t) + \beta_2(t)}{\varepsilon} e^{\mu\tau} = 0.$$

Differentiate both sides of (8) with respect to the variable ε , we have

$$\frac{\partial \mu(t, \varepsilon)}{\partial \varepsilon} = \frac{-\mu(t, \varepsilon)}{\varepsilon + (2\beta_1(t) + \beta_2(t))\tau e^{\mu\tau}} < 0,$$

so $\mu(t, \varepsilon)$ is monotonically decreasing with respect to the variable ε . Hence we can deduce that there exists a small $\varepsilon_0 > 0$ such that for any solution $x(t, t_0, \phi, \varepsilon)$ of (1) with the initial condition $\phi \in C_{\mathcal{F}_0}^b [[-\tau, 0], \mathbb{R}^n]$,

$$\mathbf{E}|x(t, \varepsilon)|^2 \leq \frac{2\gamma^*}{(1-q)\alpha^*} + Ge^{-\mu^*(\varepsilon_0)(t-t_0)}, \forall \varepsilon \in (0, \varepsilon_0], t \geq t_0.$$

The proof is completed. □

Corollary 3.1. *Assume that (A_1) - (A_3) with $\gamma(t) = 0$ hold. Then there exists a small $\varepsilon_0 > 0$ such that system (1) is mean square exponentially stable for sufficiently small $\varepsilon \in (0, \varepsilon_0]$.*

Remark 3.1. For system (1), when $g(t, x(t), x(t - \tau)) = 0$, then it degenerates to the deterministic singularly perturbed delay differential equations:

$$(9) \quad \begin{cases} \varepsilon dx(t) = f(t, x(t, \varepsilon), x(t - \tau(t), \varepsilon))dt, & t \geq t_0, \\ x(t_0 + s, \varepsilon) = \phi(s), & s \in [-\tau, 0], \end{cases}$$

the same as the models discussed in [19], for (9), if noting that:

When $g(t, x(t), x(t - \tau)) = 0$, (A_2) is satisfied by taking $\alpha_2(t) = \beta_2(t) = 0$.

And (A_3) degenerates to

(A'_3) There exist constants $\alpha_0 > 0$, $\gamma^* \geq 0$ and $0 \leq q < 1$ such that

$$0 \leq \gamma(t) \leq \gamma^*, \quad \alpha_1(t) \leq -\alpha_0, \quad 0 \leq \beta_1(t) \leq -q\alpha_1(t).$$

Then we can easily obtain the following corollaries.

Corollary 3.2 ([19, Theorem 3.2]). *Assume that (A_1) and (A'_3) hold. Then there exists a small $\varepsilon_0 > 0$ such that (9) is exponential dissipative for sufficiently small $\varepsilon \in (0, \varepsilon_0]$.*

Corollary 3.3. *Assume that (A_1) and (A'_3) with $\gamma(t) = 0$ hold. Then there exists a small $\varepsilon_0 > 0$ such that (9) is exponentially stable for sufficiently small $\varepsilon \in (0, \varepsilon_0]$.*

4. Example

The following illustrative example will demonstrate the effectiveness of our results.

Example 4.1. Consider the following singularly perturbed stochastic delay differential equations:

$$(10) \quad \begin{cases} \varepsilon dx_1(t) = \left[-8x_1(t) + \frac{2\gamma(t)}{3x_1(t)} + (4 - 2\sin^2 t)x_1(t - \tau(t)) + (4 - 2\sin^2 t)x_2(t - \tau(t)) \right] dt \\ \quad + \sqrt{\varepsilon}\sqrt{1 + \cos t}x_1(t)dw_1(t) + \sqrt{\varepsilon}\sqrt{2 - \sin t}x_2(t - \tau(t))dw_2(t), \\ \varepsilon dx_2(t) = \left[-6x_2(t) + \frac{\gamma(t)}{3x_2(t)} + (2 - 2\sin^2 t)x_1(t - \tau(t)) + (2 - 2\sin^2 t)x_2(t - \tau(t)) \right] dt \\ \quad + \sqrt{\varepsilon}\sqrt{2 - \sin^2 t}x_1(t - \tau(t))dw_1(t) - \sqrt{\varepsilon}\sqrt{1 + \cos t}x_2(t)dw_2(t), \end{cases}$$

for $t \geq 0$. Where $\tau(t) = e^{-t} \leq 1 \triangleq \tau$ and $\gamma(t)$ is continuous function and satisfies $0 \leq \gamma(t) \leq \gamma^*$.

For system (10), we have

$$\begin{aligned} f_1(t, x(t), x(t - \tau)) &= -8x_1(t) + \frac{2\gamma(t)}{3x_1(t)} + (4 - 2\sin^2 t)x_1(t - \tau(t)) \\ &\quad + (4 - 2\sin^2 t)x_2(t - \tau(t)), \\ f_2(t, x(t), x(t - \tau)) &= -6x_2(t) + \frac{\gamma(t)}{3x_2(t)} + (2 - 2\sin^2 t)x_1(t - \tau(t)) \end{aligned}$$

$$\begin{aligned}
& + (2 - 2 \sin^2 t)x_2(t - \tau(t)), \\
g_1(t, x(t), x(t - \tau(t))) &= (\sqrt{1 + \cos t}x_1(t), \sqrt{2 - \sin t}x_2(t - \tau(t))), \\
g_2(t, x(t), x(t - \tau(t))) &= (\sqrt{2 - \sin^2 t}x_1(t - \tau(t)), -\sqrt{1 + \cos t}x_2(t)).
\end{aligned}$$

So,

$$\begin{aligned}
& x^T f(t, x, x(t - \tau(t))) \\
& \leq \gamma(t) + (-4 - 2 \sin^2 t)|x(t)|^2 + (3 - 2 \sin^2 t) \sup_{t-\tau \leq \theta \leq t} |x(\theta)|^2, \\
& \text{trace}(g(t, x(t), x(t - \tau(t))))^T g(t, x(t), x(t - \tau(t))) \\
& \leq (1 + \cos t)|x(t)|^2 + \sup_{t-\tau \leq \theta \leq t} |x(\theta)|^2.
\end{aligned}$$

Thus, we can choose the parameters of Condition (A_3) as follows:

$$\begin{aligned}
\alpha_1(t) &= -4 - 2 \sin^2 t, \quad \alpha_2(t) = 1 + \cos t, \\
\beta_1(t) &= 3 - 2 \sin^2 t, \quad \beta_2(t) = 3 - \sin t, \quad \alpha^* = 6, \quad q = \frac{1}{2}.
\end{aligned}$$

And for any $\varepsilon > 0$, the positive constant $\mu(t, \varepsilon)$ is determined by the following equation:

$$(11) \quad \mu(t) + \frac{1}{\varepsilon}(-7 - 4 \sin^2 t \cos t + (8 - 4 \sin^2 t - \sin t)e^{\mu(t)}) = 0.$$

So for a given ε , we can obtain the corresponding $\mu^*(\varepsilon)$ by (11). By the proof of Theorem 3.1, we know that $\mu^*(\varepsilon)$ is monotonically decreasing with respect to the variable ε , then there exists an $\varepsilon_0 > 0$ such that for any $\varepsilon \in (0, \varepsilon_0]$, we have $\mu^*(\varepsilon) > \mu^*(\varepsilon_0)$. Therefore, all the conditions of Theorem 3.1 are satisfied, we conclude that (10) is mean square exponentially dissipative for sufficiently small $\varepsilon > 0$.

Remark 4.1. If $\gamma(t) = 0$ in (10), then from the above discussion, (10) is mean square exponentially stable for sufficiently small $\varepsilon > 0$.

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