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RANKING EXPONENTIAL TRAPEZOIDAL FUZZY NUMBERS WITH CARDINALITY

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Abstract. In this paper, we want to represent a method for ranking of two exponential trapezoidal fuzzy numbers. In this study a new Cardinality between exponential trapezoidal fuzzy numbers is proposed. Cardinality in this method is relatively simple and easier in computation and ranks various types of exponential fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In 1965, Zadeh [26] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Ranking fuzzy numbers were first proposed by Jain [10] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Bortolan and Degani [3] reviewed some of these ranking methods [11-15] for ranking fuzzy subsets. Chen [4] presented ranking fuzzy numbers with maximizing set and minimizing set. Wang and Lee [25] also used the centroid concept in developing their ranking index. Chen and Chen [5] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Bodjanova [2] proposed median value and median interval of a fuzzy number and Rezvani ([13]-[22]) evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [19] proposed a new method for ranking in areas of two generalized trapezoidal fuzzy numbers.

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In this paper, we want to represent a method for ranking of two exponential trapezoidal fuzzy numbers. In this study a new Cardinality between exponential trapezoidal fuzzy numbers is proposed. Cardinality in this method is relatively simple and easier in computation and ranks various types of exponential fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2. Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],
(ii) $\mu_A(x) = 0, -\infty < u \leq a$,
(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,
(iv) $\mu_A(x) = w, b \leq x \leq c$,
(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,
(vi) $\mu_A(x) = 0, d \leq x < \infty$.

Where $0 < w \leq 1$ and $a, b, c,$ and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by $A = (a, b, c, d; w)_{LR}$.

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (a, b, c, d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c, d]$. Here, we define theirs general forms as follows:

$$f_A(x) = \begin{cases} we^{-[(b-x)/(b-a)]} & a \leq x \leq b, \\ w & b \leq x \leq c, \\ we^{-[(x-c)/(d-c)]} & c \leq x \leq d, \end{cases} \tag{1}$$

where $0 < w \leq 1$, a, b are real numbers, and c, d are positive real numbers. We denote this type of generalized exponential fuzzy number as $A = (a, b, c, d;\ w)_E$. Especially, when $w = 1$, we denote it as $A = (a, b, c, d)_E$.

We define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a, b, c, d)_E$, where $0 < w \leq 1$, and c, d are positive real numbers, a, b are real numbers as in formula (1). Now, let two monotonic functions be

$$L(x) = we^{-[(b-x)/(b-a)]},\ R(x) = we^{-[(x-c)/(d-c)]}. \tag{2}$$

3. Proposed approach

In this section some important results, that are useful for the proposed approach, are proved.

Definition 1 ([2])**.** Cardinality of a fuzzy number A is the value of the integral

$$card\ A = \int_a^b A(x)\ dx = \int_0^1 (b_\alpha - a_\alpha)\ d\alpha. \tag{3}$$

Now, we use of above definition in exponential trapezoidal fuzzy numbers.

Theorem 1. *Cardinality of a exponential trapezoidal fuzzy number A characterized by* (1) *is the value of the integral*

$$card\ A = w(c-b) + \frac{w}{e}((b-a)(e-1) + (c-d)(1-e)). \tag{4}$$

Proof.

$$\begin{aligned} card\ A &= \int_a^b A(x)\ dx \\ &= \int_a^b we^{-[(b-x)/(b-a)]}\ dx + \int_b^c w\ dx + \int_c^d we^{-[(x-c)/(d-c)]}\ dx \\ &= w(b-a)(1-\frac{1}{e}) + w(c-b) + w(c-d)(\frac{1}{e}-1) \\ &= w(c-b) + \frac{w}{e}((b-a)(e-1) + (c-d)(1-e)). \end{aligned}$$

The article will study location of the cardinality of A. So we can define ranking of cardinality in exponential trapezoidal fuzzy number. □

Theorem 2. *If $A = (a, b, c, d)_E$ is a exponential trapezoidal fuzzy number, so*

i) *If $card\ A < card\ B$, then $A < B$.*
ii) *If $card\ A > card\ B$, then $A > B$.*
iii) *If $card\ A \sim card\ B$, then $A \sim B$.*

4. Results

Example 1. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= 0.35(0.6-0.4) + \frac{0.35}{2.72}[(0.4-0.2)(2.72-1) + (0.6-0.8)(1-2.72)] \\ &= 0.07 + 0.13[0.344 + 0.344] = 0.16 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= 0.7(0.3-0.2) + \frac{0.7}{2.72}[(0.2-0.1)(2.72-1) + (0.3-0.4)(1-2.72)] \\ &= 0.07 + 0.26[0.172 + 0.172] = 0.16. \end{aligned}$$

So with use of Theorem 2, we have $card\ A \sim card\ B$, then $A \sim B$.

Example 2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= (0.4 - 0.2) + \frac{1}{2.72}[(0.2 - 0.1)(2.72 - 1) + (0.4 - 0.5)(1 - 2.72)] \\ &= 0.2 + 0.37[0.172 + 0.172] = 0.33 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= (0.3 - 0.3) + \frac{1}{2.72}[(0.3 - 0.1)(2.72 - 1) + (0.3 - 0.5)(1 - 2.72)] \\ &= 0 + 0.37[0.344 + 0.344] = 0.25. \end{aligned}$$

So with use of Theorem 2, we have $card\ A > card\ B$, then $A > B$.

Example 3. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (1, 1, 1, 1; 1)$ be two generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= (0.4 - 0.2) + \frac{1}{2.72}[(0.2 - 0.1)(2.72 - 1) + (0.4 - 0.5)(1 - 2.72)] \\ &= 0.2 + 0.37[0.172 + 0.172] = 0.33 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= (1 - 1) + \frac{1}{2.72}[(1 - 1)(2.72 - 1) + (1 - 1)(1 - 2.72)] \\ &= 0 + 0.37[0 + 0] = 0. \end{aligned}$$

So with use of Theorem 2, we have $card\ A > card\ B$, then $A > B$.

Example 4. Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= (-0.3 + 0.3) + \frac{1}{2.72}[(-0.3 + 0.5)(2.72 - 1) + (-0.3 + 0.1)(1 - 2.72)] \\ &= 0 + 0.37[0.344 + 0.344] = 0.25 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= (0.3 - 0.3) + \frac{1}{2.72}[(0.3 - 0.1)(2.72 - 1) + (0.3 - 0.5)(1 - 2.72)] \\ &= 0 + 0.37[0.344 + 0.344] = 0.25. \end{aligned}$$

So with use of Theorem 2, we have $card\ A \sim card\ B$, then $A \sim B$.

Example 5. Let $A = (0.3, 0.5, 0.5, 1; 1)$ and $B = (0.1, 0.6, 0.6, 0.8; 1)$ be two generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= (0.5 - 0.5) + \frac{1}{2.72}[(0.5 - 0.3)(2.72 - 1) + (0.5 - 1)(1 - 2.72)] \\ &= 0 + 0.37[0.344 + 0.86] = 0.44 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= (0.6 - 0.6) + \frac{1}{2.72}[(0.6 - 0.1)(2.72 - 1) + (0.6 - 0.8)(1 - 2.72)] \\ &= 0 + 0.37[0.86 + 0.344] = 0.44. \end{aligned}$$

So with use of Theorem 2, we have $card\ A \sim card\ B$, then $A \sim B$.

Example 6. Let $A = (0, 0.4, 0.6, 0.8; 1)$ and $B = (0.2, 0.5, 0.5, 0.9; 1)$ and $C = (0.1, 0.6, 0.7, 0.8; 1)$ be three generalized trapezoidal fuzzy number. Then

$$\begin{aligned} card\ A &= w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e)) \\ &= (0.6 - 0.4) + \frac{1}{2.72}[(0.4 - 0)(2.72 - 1) + (0.6 - 0.8)(1 - 2.72)] \\ &= 0.2 + 0.37[0.688 + 0.344] = 0.58 \end{aligned}$$

and

$$\begin{aligned} card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e-1) + (c_B - d_B)(1-e)) \\ &= (0.5 - 0.5) + \frac{1}{2.72}[(0.5 - 0.2)(2.72 - 1) + (0.5 - 0.9)(1 - 2.72)] \\ &= 0 + 0.37[0.516 + 0.688] = 0.44 \end{aligned}$$

and

$$\begin{aligned} card\ C &= w_C(c_C - b_C) + \frac{w_C}{e}((b_C - a_C)(e-1) + (c_C - d_C)(1-e)) \\ &= (0.7 - 0.6) + \frac{1}{2.72}[(0.6 - 0.1)(2.72 - 1) + (0.7 - 0.8)(1 - 2.72)] \\ &= 0.1 + 0.37[0.86 + 0.172] = 0.48. \end{aligned}$$

So with use of Theorem 2, we have $card\ A > card\ C > card\ B$, then $A > C > B$.

Example 7. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (-2, 0, 0, 2; 1)$ be two generalized trapezoidal fuzzy number. Then

$$card\ A = w_A(c_A - b_A) + \frac{w_A}{e}((b_A - a_A)(e-1) + (c_A - d_A)(1-e))$$

TABLE 1. A comparison of the ranking results for different approaches

Approaches	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
Cheng [7]	$A < B$	$A \sim B$	Error	$A \sim B$	$A > B$	$A < B < C$	Error
Chu [8]	$A < B$	$A \sim B$	Error	$A < B$	$A > B$	$A < B < C$	Error
Chen [4]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$
Abbasbandy [1]	Error	$A \sim B$	$A < B$	$A \sim B$	$A < B$	$A < B < C$	$A > B$
Chen [6]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Kumar [11]	$A > B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Singh [24]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Rezvani [22]	$A > B$	$A > B$	$A < B$	$A < B$	$A < B$	$A < B < C$	$A < B$
Proposed approach	$A \sim B$	$A > B$	$A > B$	$A \sim B$	$A \sim B$	$A > C > B$	$A < B$

$$
\begin{aligned}
&= (0.4 - 0.2) + \frac{1}{2.72}[(0.2 - 0.1)(2.72 - 1) + (0.4 - 0.5)(1 - 2.72)] \\
&= 0.2 + 0.37[0.172 + 0.172] = 0.33
\end{aligned}
$$

and

$$
\begin{aligned}
card\ B &= w_B(c_B - b_B) + \frac{w_B}{e}((b_B - a_B)(e - 1) + (c_B - d_B)(1 - e)) \\
&= (0 - 0) + \frac{1}{2.72}[(0 + 2)(2.72 - 1) + (0 - 2)(1 - 2.72)] \\
&= 0 + 0.37[3.44 + 3.44] = 2.55.
\end{aligned}
$$

So with use of Theorem 2, we have $card\ A < card\ B$, then $A < B$.

5. Conclusion

The main advantage of the ranking fuzzy numbers in Table 1 is that the proposed method provides the correct ordering of generalized and normal trapezoidal fuzzy numbers. With use of cardinality in this method, we can computation ranking exponential fuzzy numbers. Also the proposed method is very simple and easy to apply in the real life problems.

References

[1] S. Abbasbandy and T. Hajjari, *A new approach for ranking of trapezoidal fuzzy numbers*, Comput. Math. Appl. **57** (2009), no. 3, 413–419.

[2] S. Bodjanova, *Median value and median interval of a fuzzy number*, Inform. Sci. **172** (2005), no. 1-2, 73–89.

[3] G. Bortolan and R. Degani, *A review of some methods for ranking fuzzy subsets*, Fuzzy Sets and Systems **15** (1985), no. 1, 1–119.

[4] S.-H. Chen, *Ranking fuzzy numbers with maximizing set and minimizing set*, Fuzzy Sets and Systems **17** (1985), no. 2, 113–129.

[5] S. J. Chen and S. M. Chen, *Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers*, Appl. Intell. **26** (2007), 1–11.

[6] S.-H. Chen and G.-C. Li, *Representation, ranking, and distance of fuzzy number with exponential membership function using graded mean integration method*, Tamsui Oxf. J. Math. Sci. **16** (2000), no. 2, 123–131.

[7] C. H. Cheng, *A new approach for ranking fuzzy numbers by distance method*, Fuzzy Sets and Systems **95** (1998), no. 3, 307–317.

[8] T. C. Chu and C. T. Tsao, *Ranking fuzzy numbers with an area between the centroid point and original point*, Comput. Math. Appl. **43** (2002), no. 1-2, 111–117.

[9] D. Dubois and H. Prade, *The mean value of a fuzzy number*, Fuzzy Sets and Systems **24** (1987), no. 3, 279–300.

[10] R. Jain, *Decision making in the presence of fuzzy variables*, IEEE Transactions on Systems, Man and Cybernetics **6** (1976), no. 10, 698–703.

[11] A. Kumar, P. Singh, A. Kaur, and P. Kaur, *RM approach for ranking of generalized trapezoidal fuzzy numbers*, Fuzzy Information and Engineering **2** (2010), no. 2, 37–47.

[12] C. Liang, J. Wu, and J. Zhang, *Ranking indices and rules for fuzzy numbers based on gravity center point*, Paper presented at the 6th world Congress on Intelligent Control and Automation, Dalian, China, (2006), 21–23.

[13] S. Rezvani, *Graded mean representation method with triangular fuzzy number*, World Applied Sciences Journal **11** (2010), no. 7, 871–876.

[14] ———, *Multiplication operation on trapezoidal fuzzy numbers*, J. Phys. Sci. **15** (2011), 17–26.

[15] ———, *A new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers*, International Journal of Applied Operational Research **2** (2012), no. 3, 83–90.

[16] ———, *A new approach ranking of exponential trapezoidal fuzzy numbers*, J. Phys. Sci. **16** (2012), 45–57.

[17] ———, *A new method for ranking in areas of two generalized trapezoidal fuzzy numbers*, International Journal of Fuzzy Logic Systems **3** (2013), no. 1, 17–24.

[18] ———, *Ranking method of trapezoidal intuitionistic fuzzy numbers*, Annals of Fuzzy Mathematics and Informatics **5** (2013), no. 3, 515–523.

[19] ———, *Ranking generalized trapezoidal fuzzy numbers with Euclidean distance by the incentre of centroids*, Mathematica Aeterna **3** (2013), no. 2, 103–114.

[20] ———, *Ranking exponential trapezoidal fuzzy numbers by median value*, Journal of Fuzzy Set Valued Analysis, Article ID jfsva-00139, (2013), 9 Pages.

[21] ———, *Approach for ranking of exponential fuzzy number with TRD distance*, Journal of Fuzzy Set Valued Analysis, Article ID jfsva-00140, (2013), 11 Pages.

[22] ———, *Arithmetic operations on trapezoidal fuzzy numbers*, Journal of Nonlinear Analysis and Application, Article ID jnaa-00111, (2013), 8 Pages.

[23] R. Saneifard and R. Saneifard, *A modified method for defuzzification by probability density function*, Journal of Applied Sciences Research **7** (2011), no. 2, 102–110.

[24] P. Singh et al, *Ranking of generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread*, Turkish Journal of Fuzzy Systems **1** (2010), no. 2, 141–152.

[25] Y. J. Wang and H. S. Lee, *The revised method of ranking fuzzy numbers with an area between the centroid and original points*, Comput. Math. Appl. **55** (2008), no. 9, 2033–2042.

[26] L. A. Zadeh, *Fuzzy set*, Information and Control **8** (1965), no. 3, 338–353.

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