

## SOMEWHAT FUZZY $\gamma$ -IRRESOLUTE CONTINUOUS MAPPINGS

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ABSTRACT. We define and characterize a somewhat fuzzy  $\gamma$ -irresolute continuous mapping and a somewhat fuzzy irresolute  $\gamma$ -open mapping on a fuzzy topological space.

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### 1. Introduction

The concept of fuzzy  $\gamma$ -continuous mappings on a fuzzy topological space was introduced and studied by I. M. Hanafy in [2]. Also, the concept of fuzzy  $\gamma$ -irresolute continuous mappings on a fuzzy topological space were introduced and studied by Y. B. Im et al. in [8] and fuzzy irresolute  $\gamma$ -open mappings on a fuzzy topological space was introduced and studied by Y. B. Im in [3].

Recently, somewhat fuzzy  $\gamma$ -continuous mappings on a fuzzy topological space were introduced and studied by G. Thangaraj and V. Seenivasan in [9].

In this paper, we define and characterize a somewhat fuzzy  $\gamma$ -irresolute continuous mapping and a somewhat fuzzy irresolute  $\gamma$ -open mapping which are stronger than a somewhat fuzzy  $\gamma$ -continuous mapping and a somewhat fuzzy  $\gamma$ -open mapping respectively. Besides, some interesting properties of those mappings are also given.

### 2. Preliminaries

A fuzzy set  $\mu$  on a fuzzy topological space  $X$  is called *fuzzy  $\gamma$ -open* if  $\mu \leq \text{ClInt}\mu \vee \text{IntCl}\mu$  and  $\mu$  is called *fuzzy  $\gamma$ -closed* if  $\mu^c$  is a fuzzy  $\gamma$ -open set on  $X$ .

A mapping  $f : X \rightarrow Y$  is called *fuzzy  $\gamma$ -continuous* if  $f^{-1}(\nu)$  is a fuzzy  $\gamma$ -open set on  $X$  for any fuzzy open set  $\nu$  on  $Y$  and a mapping  $f : X \rightarrow Y$  is

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called *fuzzy  $\gamma$ -open* if  $f(\mu)$  is a fuzzy  $\gamma$ -open set on  $Y$  for any fuzzy open set  $\mu$  on  $X$ . It is clear that every fuzzy continuous mapping is a fuzzy  $\gamma$ -continuous mapping. And every fuzzy open mapping is a fuzzy  $\gamma$ -open mapping from the above definitions. But the converses are not true in general [2].

A mapping  $f : X \rightarrow Y$  is called *fuzzy  $\gamma$ -irresolute continuous* if  $f^{-1}(\nu)$  is a fuzzy  $\gamma$ -open set on  $X$  for any fuzzy  $\gamma$ -open set  $\nu$  on  $Y$  and a mapping  $f : X \rightarrow Y$  is called *fuzzy irresolute  $\gamma$ -open* if  $f(\mu)$  is a fuzzy  $\gamma$ -open set on  $Y$  for any fuzzy  $\gamma$ -open set  $\mu$  on  $X$ . It is clear that every fuzzy  $\gamma$ -irresolute continuous mapping is a fuzzy  $\gamma$ -continuous mapping. And every fuzzy irresolute  $\gamma$ -open mapping is a fuzzy open mapping from the above definitions. But the converses are not true in general [8] and [3].

A mapping  $f : X \rightarrow Y$  is called *somewhat fuzzy  $\gamma$ -continuous* if there exists a fuzzy  $\gamma$ -open set  $\mu \neq 0_X$  on  $X$  such that  $\mu \leq f^{-1}(\nu) \neq 0_X$  for any fuzzy open set  $\nu$  on  $Y$ . It is clear that every fuzzy  $\gamma$ -continuous mapping is a somewhat fuzzy  $\gamma$ -continuous mapping. But the converse is not true in general.

A mapping  $f : X \rightarrow Y$  is called *somewhat fuzzy  $\gamma$ -open* if there exists a fuzzy  $\gamma$ -open set  $\nu \neq 0_Y$  on  $Y$  such that  $\nu \leq f(\mu) \neq 0_Y$  for any fuzzy open set  $\mu$  on  $X$ . Every fuzzy open mapping is a somewhat fuzzy  $\gamma$ -open mapping but the converse is not true in general [9].

### 3. Somewhat fuzzy $\gamma$ -irresolute continuous mappings

In this section, we introduce a somewhat fuzzy  $\gamma$ -irresolute continuous mapping and a somewhat fuzzy irresolute  $\gamma$ -open mapping which are stronger than a somewhat fuzzy  $\gamma$ -continuous mapping and a somewhat fuzzy  $\gamma$ -open mapping respectively. And we characterize a somewhat fuzzy  $\gamma$ -irresolute continuous mapping and a somewhat fuzzy irresolute  $\gamma$ -open mapping.

**Definition 3.1.** A mapping  $f : X \rightarrow Y$  is called somewhat fuzzy  $\gamma$ -irresolute continuous if there exists a fuzzy  $\gamma$ -open set  $\mu \neq 0_X$  on  $X$  such that  $\mu \leq f^{-1}(\nu)$  for any fuzzy  $\gamma$ -open set  $\nu \neq 0_Y$  on  $Y$ .

It is clear that every fuzzy  $\gamma$ -irresolute continuous mapping is a somewhat fuzzy  $\gamma$ -irresolute continuous mapping. And every somewhat fuzzy  $\gamma$ -irresolute continuous mapping is a fuzzy  $\gamma$ -continuous mapping from the above definitions. But the converses are not true in general as the following examples show.

**Example 3.2.** Let  $\mu_1, \mu_2$  and  $\mu_3$  be fuzzy sets on  $X = \{a, b, c\}$  and let  $\nu_1, \nu_2$  and  $\nu_3$  be fuzzy sets on  $Y = \{x, y, z\}$  with

$$\begin{aligned} \mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\ \mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2, \\ \mu_3(a) &= 0.5, \mu_3(b) = 0.5, \mu_3(c) = 0.5 \text{ and} \\ \nu_1(x) &= 0.3, \nu_1(y) = 0.2, \nu_1(z) = 0.3, \\ \nu_2(x) &= 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5, \\ \nu_3(x) &= 0.5, \nu_3(y) = 0.2, \nu_3(z) = 0.5. \end{aligned}$$

Let  $\tau = \{0_X, \mu_1, \mu_1^c, \mu_2^c, 1_X\}$  be fuzzy topologies on  $X$  and let  $\tau^* = \{0_Y, \nu_1, \nu_2, 1_Y\}$  be fuzzy topologies on  $Y$ . Consider the mapping  $f : (X, \tau) \rightarrow (Y, \tau^*)$  defined by  $f(a) = y$ ,  $f(b) = y$  and  $f(c) = y$ . Then we have  $\mu_1 \leq f^{-1}(\nu_1) = \mu_2$ ,  $f^{-1}(\nu_2) = \mu_3$  and  $\mu_1 \leq f^{-1}(\nu_3) = \mu_2$ . Since  $\mu_1$  is a fuzzy  $\gamma$ -open set on  $(X, \tau)$ ,  $f$  is somewhat fuzzy  $\gamma$ -irresolute continuous. But  $f^{-1}(\nu_1) = \mu_2$  and  $f^{-1}(\nu_3) = \mu_2$  are not fuzzy  $\gamma$ -open sets on  $(X, \tau)$ . Hence  $f$  is not a fuzzy  $\gamma$ -irresolute continuous mapping.  $\square$

**Example 3.3.** Let  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  be fuzzy sets on  $X = \{a, b, c\}$  and let  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  be fuzzy sets on  $Y = \{x, y, z\}$  with

$$\begin{aligned}\mu_1(a) &= 0.2, \mu_1(b) = 0.2, \mu_1(c) = 0.2, \\ \mu_2(a) &= 0.5, \mu_2(b) = 0.5, \mu_2(c) = 0.5, \text{ and} \\ \nu_1(x) &= 0.3, \nu_1(y) = 0.2, \nu_1(z) = 0.3, \\ \nu_2(x) &= 0.5, \nu_2(y) = 0.5, \nu_2(z) = 0.5.\end{aligned}$$

Let  $\tau = \{0_X, \mu_1^c, 1_X\}$  be fuzzy topologies on  $X$  and let  $\tau^* = \{0_Y, \nu_2, 1_Y\}$  be fuzzy topologies on  $Y$ . Consider the mapping  $f : (X, \tau) \rightarrow (Y, \tau^*)$  defined by  $f(a) = y$ ,  $f(b) = y$  and  $f(c) = y$ . Since  $f^{-1}(\nu_2) = \mu_2$  is fuzzy  $\gamma$ -open sets on  $(X, \tau)$ ,  $f$  is fuzzy  $\gamma$ -continuous. But the inverse images  $0_X \leq f^{-1}(\nu_1) = \mu_1$  of a fuzzy  $\gamma$ -open set  $\nu_1$  on  $(Y, \tau^*)$  is not fuzzy  $\gamma$ -open on  $(X, \tau)$ . Hence  $f$  is not a fuzzy somewhat  $\gamma$ -irresolute continuous mapping.  $\square$

**Definition 3.4** ([9]). A fuzzy set  $\mu$  on a fuzzy topological space  $X$  is called fuzzy  $\gamma$ -dense if there exists no fuzzy  $\gamma$ -closed set  $\nu$  such that  $\mu < \nu < 1$ .

**Theorem 3.5.** Let  $f : X \rightarrow Y$  be a mapping. Then the following are equivalent:

- (1)  $f$  is somewhat fuzzy  $\gamma$ -irresolute continuous.
- (2) If  $\nu$  is a fuzzy  $\gamma$ -closed set of  $Y$  such that  $f^{-1}(\nu) \neq 1_X$ , then there exists a fuzzy  $\gamma$ -closed set  $\mu \neq 1_X$  of  $X$  such that  $f^{-1}(\nu) \leq \mu$ .
- (3) If  $\mu$  is a fuzzy  $\gamma$ -dense set on  $X$ , then  $f(\mu)$  is a fuzzy  $\gamma$ -dense set on  $Y$ .

*Proof.* (1) implies (2): Let  $\nu$  be a fuzzy  $\gamma$ -closed set on  $Y$  such that  $f^{-1}(\nu) \neq 1_X$ . Then  $\nu^c$  is a fuzzy  $\gamma$ -open set on  $Y$  and  $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$ . Since  $f$  is somewhat fuzzy  $\gamma$ -irresolute continuous, there exists a fuzzy  $\gamma$ -open set  $\lambda \neq 0_X$  on  $X$  such that  $\lambda \leq f^{-1}(\nu^c)$ . Let  $\mu = \lambda^c$ . Then  $\mu \neq 1_X$  is fuzzy  $\gamma$ -closed such that  $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \lambda = \lambda^c = \mu$ .

(2) implies (3): Let  $\mu$  be a fuzzy  $\gamma$ -dense set on  $X$  and suppose  $f(\mu)$  is not fuzzy  $\gamma$ -dense on  $Y$ . Then there exists a fuzzy  $\gamma$ -closed set  $\nu$  on  $Y$  such that  $f(\mu) < \nu < 1$ . Since  $\nu < 1$  and  $f^{-1}(\nu) \neq 1_X$ , there exists a fuzzy  $\gamma$ -closed set  $\delta \neq 1_X$  such that  $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$ . This contradicts to the assumption that  $\mu$  is a fuzzy  $\gamma$ -dense set on  $X$ . Hence  $f(\mu)$  is a fuzzy  $\gamma$ -dense set on  $Y$ .

(3) implies (1): Let  $\nu \neq 0_Y$  be a fuzzy  $\gamma$ -open set on  $Y$  and  $f^{-1}(\nu) \neq 0_X$ . Suppose there exists no fuzzy  $\gamma$ -open  $\mu \neq 0_X$  on  $X$  such that  $\mu \leq f^{-1}(\nu)$ . Then  $(f^{-1}(\nu))^c$  is a fuzzy set on  $X$  such that there is no fuzzy  $\gamma$ -closed set  $\delta$  on  $X$

with  $(f^{-1}(\nu))^c < \delta < 1$ . In fact, if there exists a fuzzy  $\gamma$ -open set  $\delta^c$  such that  $\delta^c \leq f^{-1}(\nu)$ , then it is a contradiction. So  $(f^{-1}(\nu))^c$  is a fuzzy  $\gamma$ -dense set on  $X$ . Then  $f((f^{-1}(\nu))^c)$  is a fuzzy  $\gamma$ -dense set on  $Y$ . But  $f((f^{-1}(\nu))^c) = f(f^{-1}(\nu^c)) \neq \nu^c < 1$ . This is a contradiction to the fact that  $f((f^{-1}(\nu))^c)$  is fuzzy  $\gamma$ -dense on  $Y$ . Hence there exists a  $\gamma$ -open set  $\mu \neq 0_X$  on  $X$  such that  $\mu \leq f^{-1}(\nu)$ . Consequently,  $f$  is somewhat fuzzy  $\gamma$ -irresolute continuous.  $\square$

A fuzzy topological space  $X$  is *product related* to a fuzzy topological space  $Y$  if for fuzzy sets  $\mu$  on  $X$  and  $\nu$  on  $Y$  whenever  $\gamma^c \not\leq \mu$  and  $\delta^c \not\leq \nu$  (in which case  $(\gamma^c \times 1) \vee (1 \times \delta^c) \geq (\mu \times \nu)$ ) where  $\gamma$  is a fuzzy open set on  $X$  and  $\delta$  is a fuzzy open set on  $Y$ , there exists a fuzzy open set  $\gamma_1$  on  $X$  and a fuzzy open set  $\delta_1$  on  $Y$  such that  $\gamma_1^c \geq \mu$  or  $\delta_1^c \geq \nu$  and  $(\gamma_1^c \times 1) \vee (1 \times \delta_1^c) = (\gamma^c \times 1) \vee (1 \times \delta^c)$  [1].

**Theorem 3.6.** *Let  $X_1$  be product related to  $X_2$  and  $Y_1$  be product related to  $Y_2$ . Then the product  $f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2$  of somewhat fuzzy  $\gamma$ -irresolute continuous mappings  $f_1 : X_1 \rightarrow Y_1$  and  $f_2 : X_2 \rightarrow Y_2$  is also somewhat fuzzy  $\gamma$ -irresolute continuous.*

*Proof.* Let  $\lambda = \bigvee_{i,j}(\mu_i \times \nu_j)$  be a fuzzy  $\gamma$ -open set on  $Y_1 \times Y_2$  where  $\mu_i \neq 0_{Y_1}$  and  $\nu_j \neq 0_{Y_2}$  are fuzzy  $\gamma$ -open sets on  $Y_1$  and  $Y_2$  respectively. Then  $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$ . Since  $f_1$  is somewhat fuzzy  $\gamma$ -irresolute continuous, there exists a fuzzy  $\gamma$ -open set  $\delta_i \neq 0_{X_1}$  such that  $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$ . And, since  $f_2$  is somewhat fuzzy  $\gamma$ -irresolute continuous, there exists a fuzzy  $\gamma$ -open set  $\eta_j \neq 0_{X_2}$  such that  $\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$ . Now  $\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$  and  $\delta_i \times \eta_j \neq 0_{X_1 \times X_2}$  is a fuzzy  $\gamma$ -open set on  $X_1 \times X_2$ . Hence  $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{X_1 \times X_2}$  is a fuzzy  $\gamma$ -open set on  $X_1 \times X_2$  such that  $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$ . Therefore,  $f_1 \times f_2$  is somewhat fuzzy  $\gamma$ -irresolute continuous.  $\square$

**Theorem 3.7.** *Let  $f : X \rightarrow Y$  be a mapping. If the graph  $g : X \rightarrow X \times Y$  of  $f$  is a somewhat fuzzy  $\gamma$ -irresolute continuous mapping, then  $f$  is also somewhat fuzzy  $\gamma$ -irresolute continuous.*

*Proof.* Let  $\nu$  be a fuzzy  $\gamma$ -open set on  $Y$ . Then  $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$ . Since  $g$  is somewhat fuzzy  $\gamma$ -irresolute continuous and  $1 \times \nu$  is a fuzzy  $\gamma$ -open set on  $X \times Y$ , there exists a fuzzy  $\gamma$ -open set  $\mu \neq 0_X$  on  $X$  such that  $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X$ . Therefore,  $f$  is somewhat fuzzy  $\gamma$ -irresolute continuous.  $\square$

**Definition 3.8.** A mapping  $f : X \rightarrow Y$  is called somewhat fuzzy irresolute  $\gamma$ -open if there exists a fuzzy  $\gamma$ -open set  $\nu \neq 0_Y$  on  $Y$  such that  $\nu \leq f(\mu)$  for any fuzzy  $\gamma$ -open set  $\mu \neq 0_X$  on  $X$ .

It is clear that every fuzzy irresolute  $\gamma$ -open mapping is a somewhat fuzzy irresolute  $\gamma$ -open mapping. And every somewhat fuzzy irresolute  $\gamma$ -open mapping is a fuzzy  $\gamma$ -open mapping. Also, every fuzzy  $\gamma$ -open mapping is a somewhat fuzzy  $\gamma$ -open mapping from the above definitions. But the converses are not true in general as the following examples show.

**Example 3.9.** Let  $\mu_1$  and  $\mu_2$  be fuzzy sets on  $X = \{a, b, c\}$  and let  $\nu_1$  and  $\nu_2$  be fuzzy sets on  $Y = \{x, y, z\}$  with

$$\begin{aligned}\mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\ \mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2 \text{ and} \\ \nu_1(x) &= 0.0, \nu_1(y) = 0.1, \nu_1(z) = 0.0, \\ \nu_2(x) &= 0.0, \nu_2(y) = 0.2, \nu_2(z) = 0.0, \\ \nu_3(x) &= 0.0, \nu_3(y) = 0.8, \nu_3(z) = 0.0, \\ \nu_4(x) &= 0.0, \nu_4(y) = 0.9, \nu_4(z) = 0.0.\end{aligned}$$

Let  $\tau = \{0_X, \mu_2, 1_X\}$  be fuzzy topologies on  $X$  and let  $\tau^* = \{0_Y, \nu_1, \nu_1^c, \nu_2^c, 1_Y\}$  be fuzzy topologies on  $Y$ . Consider the mapping  $f : (X, \tau) \rightarrow (Y, \tau^*)$  defined by  $f(a) = y$ ,  $f(b) = y$  and  $f(c) = y$ . Since  $f(\mu_1) = \nu_1$ ,  $\nu_1 \leq f(\mu_2) = \nu_2$ ,  $f(\mu_1^c) = \nu_3$  and  $f(\mu_2^c) = \nu_4$ ,  $f$  is somewhat fuzzy irresolute  $\gamma$ -open. But  $f(\mu_2) = \nu_2$  is not a fuzzy  $\gamma$ -open set on  $(Y, \tau^*)$ . Hence  $f$  is not a fuzzy irresolute  $\gamma$ -open mapping.  $\square$

**Example 3.10.** Let  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  be fuzzy sets on  $X = \{a, b, c\}$  and let  $\nu_1$  and  $\nu_2$  be fuzzy sets on  $Y = \{x, y, z\}$  with

$$\begin{aligned}\mu_1(a) &= 0.4, \mu_1(b) = 0.1, \mu_1(c) = 0.4, \\ \mu_2(a) &= 0.5, \mu_2(b) = 0.5, \mu_2(c) = 0.5, \\ \mu_3(a) &= 0.1, \mu_3(b) = 0.0, \mu_3(c) = 0.1 \text{ and} \\ \nu_1(x) &= 0.0, \nu_1(y) = 0.1, \nu_1(z) = 0.0, \\ \nu_2(x) &= 0.0, \nu_2(y) = 0.5, \nu_2(z) = 0.0.\end{aligned}$$

Let  $\tau = \{0_X, \mu_1, \mu_2, 1_X\}$  be fuzzy topologies on  $X$  and let  $\tau^* = \{0_Y, \nu_2, 1_Y\}$  be fuzzy topologies on  $Y$ . Consider the mapping  $f : (X, \tau) \rightarrow (Y, \tau^*)$  defined by  $f(a) = y$ ,  $f(b) = y$  and  $f(c) = y$ . Since  $f(\mu_1) = \nu_1$  and  $f(\mu_2) = \nu_2$  are fuzzy  $\gamma$ -open sets on  $(Y, \tau^*)$ ,  $f$  is fuzzy  $\gamma$ -open. But  $\mu_3 \neq 0_X$  is a fuzzy  $\gamma$ -open set on  $(X, \tau)$  and  $f(\mu_3) = 0_Y$ . Hence  $f$  is not a fuzzy somewhat irresolute  $\gamma$ -open mapping.  $\square$

**Example 3.11.** Let  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  be fuzzy sets on  $X = \{a, b, c\}$  with

$$\begin{aligned}\mu_1(a) &= 0.1, \mu_1(b) = 0.1, \mu_1(c) = 0.1, \\ \mu_2(a) &= 0.2, \mu_2(b) = 0.2, \mu_2(c) = 0.2 \text{ and} \\ \mu_3(a) &= 0.3, \mu_3(b) = 0.3, \mu_3(c) = 0.3.\end{aligned}$$

Let  $\tau = \{0_X, \mu_2^c, 1_X\}$  and  $\tau^* = \{0_X, \mu_1, \mu_3, 1_X\}$  be fuzzy topologies on  $X$ . Consider the identity mapping  $i_X : (X, \tau) \rightarrow (X, \tau^*)$ . We have  $\mu_3 \leq i_X(\mu_2^c) = \mu_2^c$ . Since  $\mu_3$  is a fuzzy  $\gamma$ -open set on  $(X, \tau)$ ,  $i_X$  is somewhat fuzzy  $\gamma$ -open. But  $i_X(\mu_2^c) = \mu_2^c$  is not a fuzzy  $\gamma$ -open set on  $(X, \tau^*)$ . Hence  $i_X$  is not a fuzzy  $\gamma$ -open mapping.  $\square$

**Theorem 3.12.** *Let  $f : X \rightarrow Y$  be a bijection. Then the following are equivalent:*

- (1)  *$f$  is somewhat fuzzy irresolute  $\gamma$ -open.*
- (2) *If  $\mu$  is a fuzzy  $\gamma$ -closed set on  $X$  such that  $f(\mu) \neq 1_Y$ , then there exists a fuzzy  $\gamma$ -closed set  $\nu \neq 1_Y$  on  $Y$  such that  $f(\mu) < \nu$ .*

*Proof.* (1) implies (2): Let  $\mu$  be a fuzzy  $\gamma$ -closed set on  $X$  such that  $f(\mu) \neq 1_Y$ . Since  $f$  is bijective and  $\mu^c$  is a fuzzy  $\gamma$ -open set on  $X$ ,  $f(\mu^c) = (f(\mu))^c \neq 0_Y$ . And, since  $f$  is somewhat fuzzy irresolute  $\gamma$ -open, there exists a  $\gamma$ -open set  $\delta \neq 0_Y$  on  $Y$  such that  $\delta < f(\mu^c) = (f(\mu))^c$ . Consequently,  $f(\mu) < \delta^c = \nu \neq 1_Y$  and  $\nu$  is a fuzzy  $\gamma$ -closed set on  $Y$ .

(2) implies (1): Let  $\mu$  be a fuzzy  $\gamma$ -open set on  $X$  such that  $f(\mu) \neq 0_Y$ . Then  $\mu^c$  is a fuzzy  $\gamma$ -closed set on  $X$  and  $f(\mu^c) \neq 1_Y$ . Hence there exists a fuzzy  $\gamma$ -closed set  $\nu \neq 1_Y$  on  $Y$  such that  $f(\mu^c) < \nu$ . Since  $f$  is bijective,  $f(\mu^c) = (f(\mu))^c < \nu$ . Hence  $\nu^c < f(\mu)$  and  $\nu^c \neq 0_X$  is a fuzzy  $\gamma$ -open set on  $Y$ . Therefore,  $f$  is somewhat fuzzy irresolute  $\gamma$ -open.  $\square$

**Theorem 3.13.** *Let  $f : X \rightarrow Y$  be a surjection. Then the following are equivalent:*

- (1)  *$f$  is somewhat fuzzy irresolute  $\gamma$ -open.*
- (2) *If  $\nu$  is a fuzzy  $\gamma$ -dense set on  $Y$ , then  $f^{-1}(\nu)$  is a fuzzy  $\gamma$ -dense set on  $X$ .*

*Proof.* (1) implies (2): Let  $\nu$  be a fuzzy  $\gamma$ -dense set on  $Y$ . Suppose  $f^{-1}(\nu)$  is not fuzzy  $\gamma$ -dense on  $X$ . Then there exists a fuzzy  $\gamma$ -closed set  $\mu$  on  $X$  such that  $f^{-1}(\nu) < \mu < 1$ . Since  $f$  is somewhat fuzzy irresolute  $\gamma$ -open and  $\mu^c$  is a fuzzy  $\gamma$ -open set on  $X$ , there exists a fuzzy  $\gamma$ -open set  $\delta \neq 0_Y$  on  $Y$  such that  $\delta \leq f(\text{Int}\mu^c) \leq f(\mu^c)$ . Since  $f$  is surjective,  $\delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c$ . Thus there exists a  $\gamma$ -closed set  $\delta^c$  on  $Y$  such that  $\nu < \delta^c < 1$ . This is a contradiction. Hence  $f^{-1}(\nu)$  is fuzzy  $\gamma$ -dense on  $X$ .

(2) implies (1): Let  $\mu$  be a fuzzy open set on  $X$  and  $f(\mu) \neq 0_Y$ . Suppose there exists no fuzzy  $\gamma$ -open  $\nu \neq 0_Y$  on  $Y$  such that  $\nu \leq f(\mu)$ . Then  $(f(\mu))^c$  is a fuzzy set on  $Y$  such that there exists no fuzzy  $\gamma$ -closed set  $\delta$  on  $Y$  with  $(f(\mu))^c < \delta < 1$ . This means that  $(f(\mu))^c$  is fuzzy  $\gamma$ -dense on  $Y$ . Thus  $f^{-1}((f(\mu))^c)$  is fuzzy  $\gamma$ -dense on  $X$ . But  $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$ . This is a contradiction to the fact that  $f^{-1}((f(\mu))^c)$  is fuzzy  $\gamma$ -dense on  $X$ . Hence there exists a  $\gamma$ -open set  $\nu \neq 0_Y$  on  $Y$  such that  $\nu \leq f(\mu)$ . Therefore,  $f$  is somewhat fuzzy irresolute  $\gamma$ -open.  $\square$

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