PERIODIC SENSING AND GREEDY ACCESS POLICY USING CHANNEL MODELS WITH GENERALLY DISTRIBUTED ON AND OFF PERIODS IN COGNITIVE NETWORKS[†]

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ABSTRACT. One of the fundamental issues in the design of dynamic spectrum access policy is the modeling of the dynamic behavior of channel occupancy by primary users. Under a Markovian modeling of channel occupancy, a periodic sensing and greedy access policy is known as one of the simple and practical dynamic spectrum access policies in cognitive radio networks. In this paper, the primary occupancy of each channel is modeled as a discrete-time alternating renewal process with generally distributed on- and off-periods. A periodic sensing and greedy access policy is constructed based on the general channel occupancy model. Simulation results show that the proposed policy has better throughput than the policies using channel models with exponentially distributed on- or off-periods.

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1. Introduction

Reports of spectrum efficiency reveal that a considerable region of the spectrum remains unused across both space and time [3]. As a solution for such inefficient spectrum usage, dynamic spectrum access (DSA) has been intensively studied [1, 3, 4, 6, 7, 8]. Under this system, a secondary user that does not have a license to use the spectrum is allowed to opportunistically occupy an idle spectrum band owned by a licensee that is termed the primary user [7, 8].

The fundamental issue in the design of DSA is the modeling of the dynamic behavior of channel occupancy. Most of the existing cognitive radio channel models assume target channels to be a simple two-state on-off Markovian model with exponentially distributed on- and off-periods [7]. Such Markovian models,

though analytically tractable, may not be sufficiently accurate, and more general channel occupancy models are preferred [7]. Experimental studies on the IEEE 802.11 Wireless LAN support a semi-Markovian model for various traffic patterns [3]. For channel occupancy model, Cai and Alfa [2] introduced a phase-type distribution as the sojourn time at each channel state. Lee [4, 5] assumed an alternating renewal process (or two-state semi-Markov process) with generally distributed on-periods and exponentially distributed off-periods. Tehrani et al. [6] modeled the primary occupancy of each channel as an alternating renewal process with generally distributed on- and off-periods.

Sensing and access policies are two other fundamental issues in DSA networks where there is no central coordinator or dedicated control channel [8]. Zhao et al. [7] proposed a periodic channel sensing policy, under which each secondary user senses channels in an increasing order of channel index. Imposing a periodic sensing policy simplifies the design of DSA protocol considerably [7]. Zhao et al. [7] considered an optimal access policy under the periodic channel sensing and a Markovian channel occupancy model, and also considered a suboptimal greedy access policy, under which each secondary user chooses an action to maximize the expected immediate reward such as per-slot throughput in slotted transmission structure. Although the greedy access policy is simple, it is proven to be optimal under certain conditions on channel parameters [1]. Ahmad et al. [1], Lee [4], and Zhao et al. [8] also considered a greedy access policy.

Zhao et al. [7] proposed a periodic channel sensing policy under a continuous-time Markov chain modeling of the channel occupancy, where both the on- and off-periods are assumed to be exponentially distributed. In this paper, we propose a periodic sensing and greedy access policy using a discrete-time alternating renewal process for channel occupancy model, where both the on- and off-periods are generally distributed. We also compare the proposed policy and the policies using channel tracking models with exponentially distributed on- or off-periods.

2. System model

It is assumed that there are N parallel channels available for primary and secondary users. The occupancy of each channel by primary users evolves independently according to a discrete-time alternating renewal process with generally distributed on- and off-periods. Each channel can be in any one of two states, 0 (off, idle) or 1 (on, busy). Each time channel i, $i = 0, 1, ..., N - 1$, enters state j, $j = 0, 1$, it remains there for a random sojourn time $H_j^{(i)}$ and then goes to the other state. The sojourn time $H_j^{(i)}$ is generally distributed with probability mass function $h_j^{(i)}(k) \equiv \mathrm{P}\{H_j^{(i)} = kT\}$ for $k = 1, 2, ..., M$, where T denotes time unit and M is the maximum sojourn time measured in time units. Note that the discrete model itself is not only available in many discrete systems, but the model with small T can also be a good approximation to the corresponding continuous systems. The secondary users have a slot structure with slot length T_s, which can be assumed to be the integral multiple of T, say $T_s = N_s T$ for a

positive integer N_s. We assume that the sojourn time distributions are known by secondary users, which is implicitly or explicitly assumed in much literature including [2, 4, 7, 8]. In every slot, each secondary user senses a channel at the beginning of the slot, uses the sensing result to decide if and in which channel to transmit, and possibly receives an acknowledgement by a receiver [8]. Note that an ongoing secondary transmission can be interrupted by primary users.

Let $X^{(i)}(t)$ be the occupancy of channel i by primary users and $\tilde{H}^{(i)}(t)$ the remaining sojourn time in state $X^{(i)}(t)$ of channel i at time t. Then $X_k^{(i)} \equiv X^{(i)}(kT_s)$ and $\tilde{H}_k^{(i)} \equiv \tilde{H}^{(i)}(kT_s)$ are the channel occupancy and the remaining sojourn time, respectively, at the beginning of the kth slot. The stochastic process $\{(X_k^{(i)}, \tilde{H}_k^{(i)}), k = 1, 2, ...\}$ becomes a Markov chain with state space $\{(l, j) \,|\, l = 0, 1, j = 1, 2, ..., M \}$ embedded at the beginning of each slot. To obtain the state transition probabilities

$$ p_{(l,j),(m,n)}^{(i)} \equiv \mathrm{P}\left(X_{k+1}^{(i)} = m, \tilde{H}_{k+1}^{(i)} = nT \,\Big|\, X_k^{(i)} = l, \tilde{H}_k^{(i)} = jT \right) $$

from state (l, j) to state (m, n) of the Markov chain $\{(X_k^{(i)}, \tilde{H}_k^{(i)}), k = 1, 2, ...\}$, we first consider the conditional probability

$$ q_{(l,j),(m,n)}^{(i)}(k) \equiv \mathrm{P}\left(X^{(i)}(kT) = m, \tilde{H}^{(i)}(kT) = nT \,\Big|\, X^{(i)}(0) = l, \tilde{H}^{(i)}(0) = jT \right) $$

that the state of channel i is m and the remaining sojourn time in the state is nT at the beginning of the $k + 1$st time unit, under the condition that channel i is in state l and the remaining sojourn time in the state is jT initially. In the case of $j > k$, channel i stays in state l without transition during the first $k + 1$ time units. Thus, $q_{(l,j),(m,n)}^{(i)}(k) = \delta_{l,m}\delta_{j-k,n}$, where $\delta_{a,b} = 0$ for $a \neq b$ and 1 for $a = b$. In the case of $j \leq k$, conditioning on the sojourn time $(h_{1-l}^{(i)}(s))$ in new state after the first state transition at the end of the jth time unit, we have $q_{(l,j),(m,n)}^{(i)}(k) = q_{(1-l,s),(m,n)}^{(i)}(k - j)$. Thus,

$$ q_{(l,j),(m,n)}^{(i)}(k) = \delta_{l,m}\delta_{j-k,n} + \sum_{s=1}^{\min(k-j+n,M)} h_{1-l}^{(i)}(s) q_{(1-l,s),(m,n)}^{(i)}(k - j). \qquad (1) $$

Since each slot consists of N_s time units, we have $p_{(l,j),(m,n)}^{(i)} = q_{(l,j),(m,n)}^{(i)}(N_s)$. The stationary probabilities

$$ \pi_{(l,j)}^{(i)} \equiv \lim_{k \to \infty} \mathrm{P}\left(X_k^{(i)} = l, \tilde{H}_k^{(i)} = jT \right) $$

are obtained by solving the following balance equations

$$ \sum_{l=0}^{1} \sum_{j=1}^{M} \pi_{(l,j)}^{(i)} p_{(l,j),(m,n)}^{(i)} = \pi_{(m,n)}^{(i)} $$

for $m = 0, 1$ and $n = 1, 2, \cdots, M$ with normalization condition

$$\sum_{l=0}^{1} \sum_{j=1}^{M} \pi_{(l,j)}^{(i)} = 1.$$

3. Periodic sensing and greedy access policy

The state of $(X_k^{(i)}, \tilde{H}_k^{(i)})$ is unknown to secondary users. At the beginning of the kth slot, the secondary user's knowledge of $(X_k^{(i)}, \tilde{H}_k^{(i)})$ can be summarized by a belief matrix

$$\mathbf{\Lambda}^{(i)}(k) = \left[\begin{array}{cccc} \lambda_{0,1}^{(i)}(k) & \lambda_{0,2}^{(i)}(k) & \cdots & \lambda_{0,M}^{(i)}(k) \\ \lambda_{1,1}^{(i)}(k) & \lambda_{1,2}^{(i)}(k) & \cdots & \lambda_{1,M}^{(i)}(k) \end{array} \right],$$

where $\lambda_{l,j}^{(i)}(k)$ is the conditional probability of $(X_k^{(i)}, \tilde{H}_k^{(i)}) = (l, j)$ given the decision and observation history [8]. If no information about the initial channel state is unavailable, each entry $\lambda_{l,j}^{(i)}(1)$ can be set to the stationary distribution $\pi_{(l,j)}^{(i)}$ of the underlying Markov chain $\{(X_k^{(i)}, \tilde{H}_k^{(i)}), k = 1, 2, ...\}$.

We assume the secondary user senses only one channel periodically and in increasing order in each slot, i.e., at the beginning of the kth slot, channel $q(k)$ is sensed, where $q(k) = k \bmod N$ and $q(k) < N$. If channel $q(k)$ is sensed to be idle, then $\mathbf{\Lambda}^{(q(k))}(k)$ is updated as

$$\mathbf{\Lambda}^{(q(k))}(k) = \left[\begin{array}{cccc} \frac{\lambda_{0,1}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{0,j}^{(q(k))}(k)} & \frac{\lambda_{0,2}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{0,j}^{(q(k))}(k)} & \cdots & \frac{\lambda_{0,M}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{0,j}^{(q(k))}(k)} \\ 0 & 0 & \cdots & 0 \end{array} \right].$$

Otherwise, $\mathbf{\Lambda}^{(q(k))}(k)$ is updated as

$$\mathbf{\Lambda}^{(q(k))}(k) = \left[\begin{array}{cccc} 0 & 0 & \cdots & 0 \\ \frac{\lambda_{1,1}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{1,j}^{(q(k))}(k)} & \frac{\lambda_{1,2}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{1,j}^{(q(k))}(k)} & \cdots & \frac{\lambda_{1,M}^{(q(k))}(k)}{\sum_{j=1}^{M} \lambda_{1,j}^{(q(k))}(k)} \end{array} \right].$$

After updating $\mathbf{\Lambda}^{(q(k))}(k)$ based on the sensing result, the secondary user chooses a channel to transmit on. Given

$$\mathbf{\Lambda}(k) \equiv (\mathbf{\Lambda}^{(0)}(k), \mathbf{\Lambda}^{(1)}(k), ..., \mathbf{\Lambda}^{(N-1)}(k)),$$

let us define immediate reward $r(\mathbf{\Lambda}(k), a)$, accrued by accessing channel a, as the probability that channel a stays idle during the kth slot, that is,

$$r(\mathbf{\Lambda}(k), a) \equiv \mathrm{P}\left(X_k^{(a)} = 0, \tilde{H}_k^{(a)} \geq N_s \right)$$

for $a = 0, 1, ..., N - 1$. The immediate reward can be evaluated by

$$r(\mathbf{\Lambda}(k), a) = \sum_{j=N_s}^{M} \lambda_{0,j}^{(a)}(k).$$

A simple greedy access policy is introduced [7]. Given $\mathbf{\Lambda}(k)$, the secondary user first computes the immediate reward $r(\mathbf{\Lambda}(k), i)$ for each channel i and chooses the channel

$$i^* = \operatorname{argmax}_i r(\mathbf{\Lambda}(k), i).$$

The secondary user will transmit in channel i^* with probability $\beta(\mathbf{\Lambda}(k), i^*)$. The transmission probability $\beta(\mathbf{\Lambda}(k), i^*)$ is decided such that the constraint for the collision probability, which is defined as the probability that the transmission from a primary user in a slot sees that from a secondary user, is satisfied while maximizing the throughput for the secondary user, which is defined as the average number of successful secondary transmissions per slot [7]. For a given level of allowed collision γ_{i^*} on channel i^* for primary users, this requires that

$$\frac{\beta(\mathbf{\Lambda}(k), i^*)\left[1 - r(\mathbf{\Lambda}(k), i^*)\right]}{N\left[1 - \sum_{j=N_s}^{M} \pi_{(0,j)}^{(i^*)}\right]} \leq \gamma_{i^*}.$$

Thus,

$$\beta(\mathbf{\Lambda}(k), i^*) = \min\left(\frac{\gamma_{i^*} N \left[1 - \sum_{j=N_s}^{M} \pi_{(0,j)}^{(i^*)}\right]}{1 - r(\mathbf{\Lambda}(k), i^*)}, 1\right).$$

During the kth slot, if the secondary user transmits a packet successfully, then $\mathbf{\Lambda}^{(i^*)}(k)$ is updated as

$$\mathbf{\Lambda}^{(i^*)}(k) = \begin{bmatrix} 0 & \cdots & 0 & \frac{\lambda_{0,N_s}^{(i^*)}(k)}{\sum_{j=N_s}^{M} \lambda_{0,j}^{(i^*)}(k)} & \frac{\lambda_{0,N_s+1}^{(i^*)}(k)}{\sum_{j=N_s}^{M} \lambda_{0,j}^{(i^*)}(k)} & \cdots & \frac{\lambda_{0,M}^{(i^*)}(k)}{\sum_{j=N_s}^{M} \lambda_{0,j}^{(i^*)}(k)} \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Otherwise, $\mathbf{\Lambda}^{(i^*)}(k)$ is updated as

$$\mathbf{\Lambda}^{(i^*)}(k) = \begin{bmatrix} \frac{\lambda_{0,1}^{(i^*)}(k)}{\mu} & \cdots & \frac{\lambda_{0,N_s-1}^{(i^*)}(k)}{\mu} & 0 & 0 & \cdots & 0 \\ \frac{\lambda_{1,1}^{(i^*)}(k)}{\mu} & \cdots & \frac{\lambda_{1,N_s-1}^{(i^*)}(k)}{\mu} & \frac{\lambda_{1,N_s}^{(i^*)}(k)}{\mu} & \frac{\lambda_{1,N_s+1}^{(i^*)}(k)}{\mu} & \cdots & \frac{\lambda_{1,M}^{(i^*)}(k)}{\mu} \end{bmatrix},$$

where

$$\mu = \sum_{j=1}^{N_s-1} \lambda_{0,j}^{(i^*)}(k) + \sum_{j=1}^{M} \lambda_{1,j}^{(i^*)}(k).$$

At the end of the kth slot, the belief matrix is updated as

$$\lambda_{m,n}^{(i)}(k+1) = \sum_{l=0}^{1} \sum_{j=1}^{M} \lambda_{l,j}^{(i)}(k) p_{(l,j),(m,n)}^{(i)}.$$

4. Numerical examples

In this section, we focus on the case $N = 6$ and $T_s = 0.25$ms. Motivated from the experiments conducted in [3], we consider the case that the occupancy of each channel by primary users evolves as follows: The length of off-periods follows a 50/50 mixture distribution of a uniform distribution on the interval $[0, 0.7$ms$]$ and a generalized Pareto distribution with parameters $k = -0.255$ and $\sigma = 0.01$ as in [7]. Its cumulative distribution function (CDF) is given by

$$F(x) = \frac{F_u(x) + F_p(x)}{2}, \tag{2}$$

where $F_u(x)$ is the CDF of the uniform distribution on $[0, 0.7$ms$]$ and $F_p(x)$ denotes the CDF of the generalized Pareto distribution depending on the parameters $k = -0.255$ and $\sigma = 0.01$:

$$F_u(x) = \frac{x}{0.7}, \quad 0 < x < 0.7,$$

$$F_p(x) = 1 - \left(1 + \frac{kx}{\sigma}\right)^{-\frac{1}{k}}, \quad 0 < x < -\frac{\sigma}{k}.$$

The length of on-periods is constant and equal to 1ms as in [7].

For our proposed policy, we assume $T = 0.01$ms and $M = 461$. Since we assume that the distributions of the on- and off-periods are known to secondary users [2, 4, 7, 8], we can choose our channel model as follows. The off-periods are chosen randomly from a discrete version of the mixture distribution (2):

$$h_0^{(i)}(k) = F(kT) - F((k-1)T), \quad k = 1, \cdots, M. \tag{3}$$

The on-period is equal to 1ms:

$$h_1^{(i)}\left(\frac{1}{T}\right) = 1. \tag{4}$$

Note that, even though the distributions of the on- and off-periods are known to secondary users, many existing dynamic spectrum access policies approximate the distributions as exponential ones in modeling and tracking the channel occupancy, which is called *exponential approximation* in this section.

In Figure 1 and 2, the proposed policy 'general on, general off' is evaluated and compared with the policies using three different channel occupancy models, each of which approximates at least one of idle and busy period distributions by an exponential one: 'general on, exponential off', 'exponential on, general off', and 'exponential on, exponential off'. Note that 'general off' and 'general on' imply that the probability mass function of the off-periods and on-periods are given by (3) and (4), respectively; and 'exponential off' and 'exponential on' imply that the off-periods and on-periods are exponentially distributed with average 4.2ms and 1ms, respectively, where 4.2ms is the mean of the distribution (3). We obtain the collision probability in Figure 1 and the throughput in Figure 2. Generally an aggressive spectrum access policy has higher collision probability and higher
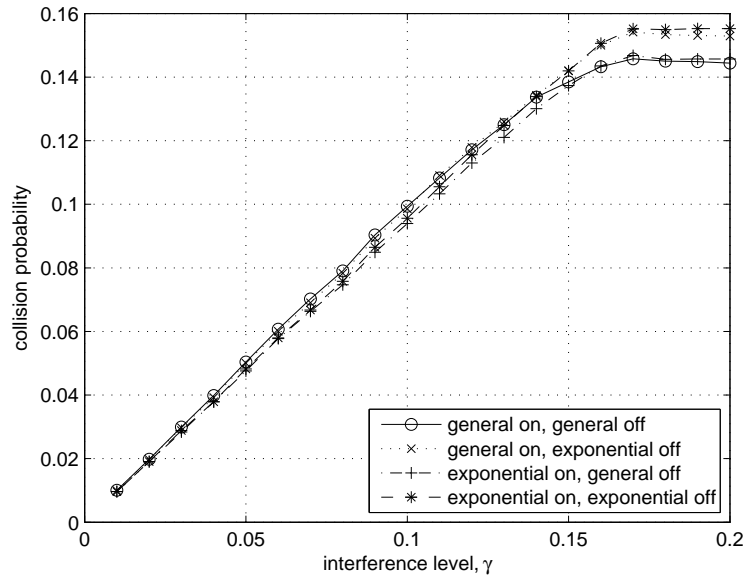
FIGURE 1. Collision Probability

throughput than conservative ones. However, although the proposed policy has better throughput than the policies using channel occupancy models with exponential approximation, it varies the collision probability slightly. Our numerical results show that the exponential approximation for channel occupancy model may degrade significantly the throughput performance of secondary users.

5. Conclusion

In this paper, a discrete-time alternating renewal process with generally distributed on- and off-periods is considered to model channel occupancy by primary users. Using the channel occupancy model, we formulate a periodic sensing and greedy access policy for secondary users. Simulation results demonstrate that the proposed policy has better throughput than the policies using channel occupancy models with exponentially distributed on- and/or off-periods.

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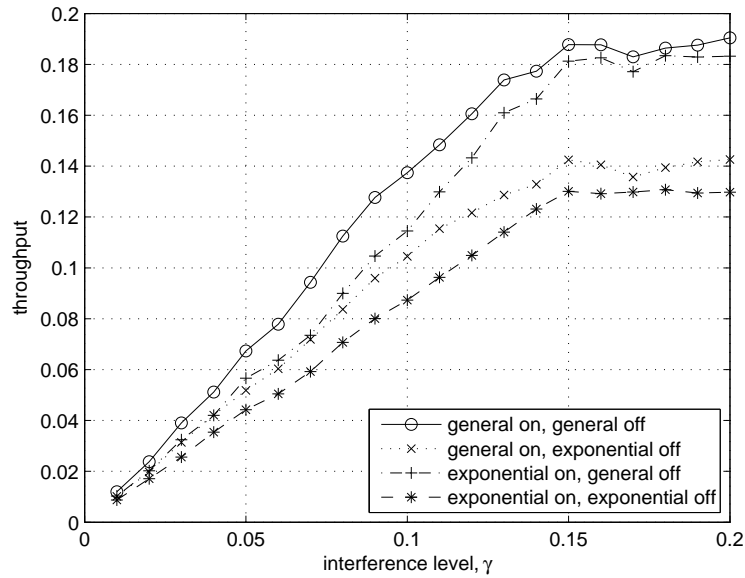
FIGURE 2. Throughput

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