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The Story of a South Korean Elementary Teacher's Knowledge of Mathematics Curriculum

Rina Kim (Seoul Mogun Elementary School)†
Hang Gyun Sihn (Seoul National University of Education)

The aim of the case study presented in this paper was to explore mathematics curriculum knowledge of a South Korean elementary teacher. An in-depth case study is applied to examine mathematics curriculum knowledge that influences teachers' instructional process including analysis of diverse artifacts such as lesson plan, observation and interviews. Findings of this study suggest that mathematics curriculum knowledge has direct relevance to teaching a lesson, designing a lesson and assessing students' work. In addition, this study identified that mathematics curriculum knowledge may be divided into two sub-categories: vertical mathematics curriculum knowledge and horizontal mathematics curriculum knowledge. The results of this case study help our understanding of South Korean elementary teachers' mathematics curriculum knowledge, which has a deep impact on their teaching practice. Moreover, this cross-national research offers implications for researchers, policymakers, and teachers in U.S. as well as those in South Korea.

I. Introduction

Recent research confirms that there is a positive correlation between elementary teachers' mathematical knowledge for teaching and student learning of mathematics (Moris, Hibert & Spitzer 2009; Hill, Blunk, Charalambos, Lewis, Phelps, Sleep & Ball 2008; River & Sanders 2002; Hill, Rowan & Ball 2005). Studies have investigated teachers' mathematical knowledge for teaching to identify the effectiveness of mathematics teachers (Hill, Rowan & Ball 2005). Considering these perspectives, different approaches have been applied to understand and define elementary teachers' mathematical knowledge for teaching (e.g. Ball, Lubienski & Mewborn 2001; Bass 2005; Hill, Ball & Schilling 2004; Hill, Blunk, Charalambos, Lewis, Phelps, Sleep & Ball 2008; Kahan, Cooper, Betha 2003; Kennedy, Ball, Mcdiarmid 1993; Kılıç 2011; Ma 1999). However, the findings from these studies may not be enough for explaining elementary teachers' mathematical knowledge for teaching, because most of these studies only focused on pedagogical alteration of mathematics concepts, which has been identified as Pedagogical Content Knowledge (PCK) by Shulman (1987). Also, there are limitations of these studies in terms of the effect of elementary teachers' PCK for teaching mathematics. While elementary teachers' PCK affected most students' mathematics achievement, there was little effect on minority students and students that received low scores from previous achievement tests (e.g. Hill 2008; Hill & Ball 2005; Tanase 2011).

In order to overcome this restriction, this study focuses on curriculum knowledge for teaching mathematics, which is a different aspect of teachers' knowledge for teaching mathematics from PCK. Earlier work by Borko and Putnam (1996) and confirmed later by Ball et al. (2000) argue that teachers should know how to take advantage of

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† Corresponding Author : rina98@naver.com

curriculum in order to support students develop their mathematical understanding (Kılıç 2011). However, the importance of curriculum knowledge is unnoticed (Castro 2006), and the definition of curriculum knowledge is still unclear (Ball, Thames & Phelps 2008).

For the purpose of uncovering teachers' knowledge of mathematics curriculum and their practical use of it during a mathematics instruction, we focus on a elementary teacher of South Korea. Recent research suggests that international research might provide opportunities to understand various issues about instructions (Cai 2001). In addition, international research is useful to illustrate diverse mechanisms in teaching and learning, although there may not be a difference among mathematics itself as a subject from country to country (Delaney, Ball, Hill, Schilling & Zopf 2008; Kulm, & Li 2009). In particular, investigating elementary teachers of South Korea may offer implications for researchers, policymakers, and teachers, especially in the U.S., which is in the implementation phase of a nation-wide mathematics curriculum, the Common Core State Standards. Although South Korea's National Mathematics Curriculum was developed based on the U. S. curriculum about 60 years ago, South Korean students show higher achievement on international assessments (e.g., PISA and TIMSS) than U.S. students in mathematics (Kim, Ham, & Paine 2011; Peterson, Woessmann, Hanushek, & Lastra-Anadón 2011). If it is possible to find a reasonable explanation for this considerable difference, then we might identify how South Korean teachers' implementation of the national curriculum interacts with student performance. Therefore, in regards to the Common Core State Standards, the investigation presented in this paper might provide directions for doing research about elementary teachers' knowledge of mathematics curriculum in the United States.

II. Theoretical Background

There are not many discussions about knowledge of mathematics curriculum, while Shulman (1986) and Ball et al., (2008) have provided groundwork for defining curriculum knowledge (See Fig. 1)

Shulman's Key Categories of Teachers' Knowledge · General pedagogical knowledge · Knowledge of learners · Knowledge of education context · Knowledge of educational, philosophical and historical grounds · Content knowledge · Curriculum knowledge · PCK
Ball, et al.'s Key Domains of Teachers' Knowledge for Teaching Mathematics · Subject matter knowledge (SMK) o Common content knowledge o Horizon content knowledge o Specialized content knowledge · PCK o Knowledge of content and students o Knowledge of content and teaching o Knowledge of content and curriculum

[그림 1] 교사 지식에 있어 교육과정 관련 지식의 위치
[Fig. 1] The Location of Curriculum Knowledge in Teachers' Knowledge

Shulman (1987) explains curricular knowledge as "the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those program, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10), distinguishing it from the other categories of knowledge including PCK. Shulman also suggests that there are two sub-categories of curricular knowledge, lateral curriculum knowledge and vertical curriculum knowledge (p.10). Lateral knowledge represents the relationship among curriculum in diverse subject areas. Vertical curriculum knowledge illustrates familiarity with the topics and concepts in a same subject. In addition, Shulman argues that curricular knowledge may provide base information for PCK.

Whereas Shulman (1987) distinguishes curricular knowledge from PCK, Ball, et al. (2008) point out that there is a certain differences between curriculum knowledge embedded in subject matter knowledge and PCK. The former, identified as horizon content knowledge illustrates an understanding of the relationship among mathematical topics over the span of the mathematics curriculum (Ball 1993). The latter, knowledge of content and curriculum is similar to Shulman's definition of curricular knowledge (Ball, et al. 2008). However, Ball, et al. (2008) demonstrate that there needs more investigation regarding how teachers use curriculum knowledge in their classroom teaching. Shulman (1987) and Ball, et al. (2008) use antithetic vocabularies, however, for this study knowledge of mathematics curriculum implies understanding the relationship among topics in the mathematics curriculum. In this case, mathematics curriculum is a way of organizing mathematics knowledge to support students' learning (Donovan & Bransford 2005).

Since there are diverse definitions about curriculum knowledge, this research refers to teachers' curriculum knowledge for teaching mathematics as knowledge of mathematics curriculum (KMC). KMC indicates teachers' understanding about the sequence among mathematical concepts that exist in both inter-grade and intra-grade. However, this does not indicate that MCK simply implies teachers' memorization of the order of mathematics topics, which are introduced according to the curriculum. Recent studies of teachers' curriculum use show that teachers apply curriculum on their classroom teaching based on their interpretation rather than simply deliver mathematics content according to the curriculum (Lloyd 2012). Therefore, MCK should be defined as in a classroom teaching. The range of teaching process, which is affected by elementary teachers' KMC, consists of developing the instructional process (e.g. Stylianides and Ball 2008; Ball, et al. 2008), classroom teaching (e.g. Turner 2008; Izsa ́k 2006; Polly 2011), and assessing students' work (e.g. Empson, & Junk 2004; Anderson, & Kim 2003; Kleve 2010).

III. Methodology

We conducted an intensive case study with a South Korean elementary teacher in this research. The detailed process of the research is as follows.

1. Participants

This study chose a teacher volunteer based according to the criteria for inclusion as follows. The participant has to have teaching experience, because teaching experience might affect elementary teachers' mathematical knowledge for teaching (e.g. Bell, Wilson, Higgins & McCoach, 2010; Li & Huang 2008). In particular, Ng (2011) reveals that the relationship between years of teaching experience and teachers' mathematical knowledge for teaching geometry is represented as a quadratic curve rather than a linear; the average of mathematical knowledge for teaching of teachers who had taught from five to fifteen years is significantly higher than the other teachers ($p < .05$). Although Ng did not focus on KMC and concentrates on one area of mathematics subject, the case study presented in this paper set a criteria for teaching experience ranging from five to fifteen years. A female participant would be proper for this case study, because the proportion of female teachers in South Korea elementary schools is up to 75.8% (KEDI 2012). Regarding the elementary school that the participant is working, the location of the school may not be significant because teachers' quality and distribution are highly controlled by the South Korea Government. Also, by the law, elementary teachers are required to change schools every five years and change teaching grade each year. Based on these criteria, this study selected a female teacher who has ten years of teaching experience. The teacher works at an elementary school, which is located in Seoul, South Korea.

2. Settings

This study focuses on the participant's teaching in her tenth year. Her placement was a fifth-grade classroom in an elementary school. The teacher taught every subject including mathematics to her students as a homeroom teacher. The teacher taught a 40-minute mathematics lesson four times per week. All lessons must be prepared based on the National Mathematics Curriculum in regards to the sequence of the content.

The classroom consisted of forty-two 11 years old Korean students. Among them, twenty-one students are males and twenty-one students are females. All of the students participated in the observed lesson. From the information provided by the teacher, most of the students were from a middle class background. For example, thirty-seven students receive extra mathematics lessons from private institutions or tutors, although none of them have any difficulties in learning mathematics. In fact, almost 90% of the students usually receive a grades of A or A- on their mathematics tests.

During the observed lesson, the teacher taught students how to draw a sketch of a rectangular parallelepiped. Two previous lessons focused on basic concepts of a rectangular parallelepiped. From the participant's own survey of her teaching, thirty-four students answered that they had already learned about it before the lesson. However, when the teacher carried out diagnostic assessments, she found that twenty-eight students did not understand the basic concepts of a sketch of rectangular parallelepiped well, although they had learned it already and knew how to draw it.

3. Date Sources

This study applied a case study approach; the purpose of a case study is to understand a large phenomenon through intensive examination of one specific instance (Rossman & Rallis 2003). This case study includes interviews of a practicing teacher, analyses of a lesson plan and subsequent videotaped observations of the participant's teaching.

1) Interviews.

The interviews consist of three stages. The first interview creates space for understanding the participant's knowledge of teaching mathematics in general. The second interview involves a scrutiny investigation of the participants' KMC with students' works. The questionnaire developed in previous studies (An, Klum & Wu 2004; Ball 1988) was adapted for the second interview. The third interview contains subsequent questions about the observations of the participant's teaching for understanding the participant's perspective and clarifying the intention within the context of her teaching.

2) Observation

At an arranged date and time provided by the teacher, classroom observation of the lesson took place. The lesson was videotaped in its entirety whereby field notes were made during the observation. The observation data consists of a 40-minute classroom observation of the teacher by the researcher of the study, whereby the video equipment was placed in the back of the classroom. This study analyzed one videotaped-observation with intensive interview about it in order to present a detailed description of the teacher's use of KMC in one lesson. The collected data from filed notes may enrich and complicate our understanding of the interview data because, as Rossman et al. (2003) observe, the participants' actions can be "purposeful and expressive of [their] deeper values and beliefs (p. 195)."

3) Analyses of a lesson plan

A final data source contains analysis of the teacher's lesson plan, which was developed for the lesson that was observed. It may not always be feasible to observe and videotape teachers' teaching on the topic (Cai 2005). Rather, analyses of lesson plans can provide insight into how teachers conceive and plan their lessons (Leinhardt 1993; Stigler, Fernandez, & Yoshida 1996). Therefore, this study will include the analysis of a lesson plan of the participant that this study observed.

4. Procedures

Before the first interview, the participant was informed about the purpose and process of this study before she decided to participate in this study. The participant was required to participate in three interviews and provide one lesson plan for the observed lesson. Two interviews were conducted before the classroom observation, and each interview took 90-minute. Before the observation, the teacher developed a lesson plan in her own style, following the National Mathematics Curriculum Guidelines. The video camera was located in the back of the classroom and only the teacher and her students were present during the taping of the lesson. During the observations, written notes were taken by the observer to record nonverbal communication as well as writing on the chalkboards and projection screen, which was connected to a desktop computer. After the observation, a 90-minute interview was conducted with the teacher about her lesson plan and teaching. One researcher transcribed all interviews and the observation.

5. Data Analysis

With data from the interviews, lesson plans and observation, this study established a collection frame of data sources by the range of teaching process: developing the instructional process, classroom teaching, and assessing students' work. According to the collection frame, we organized raw data generated from the lesson plan, observation and interview for data analysis.

The use of multiple sources triangulation was applied to avoid reliance exclusively on a single data source and to defuse any bias inherent in a particular data source, such as the observation (Anfara, Brown, & Mangione 2002). Therefore, we analyzed all types of data sources for each stage of the teaching process. In particular, with the second interview and observation as the center, we examined the other two interviews and the analysis the lesson plan in order to understands the participant's use of KMC. Results of these analyses are presented in the following section of the paper.

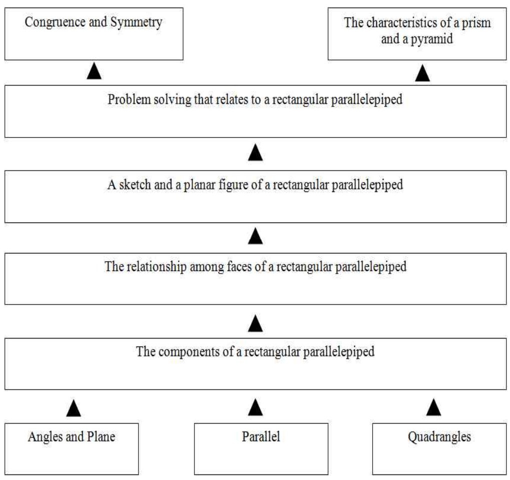
IV. Interpretive Findings and Discussion

This section presents results of the analysis, which includes three stages of instruction: the use of KMC in developing an instructional process, the use of KMC when teaching the lesson, and the use of KMC in assessing students' works.

1. Use of KMC in Developing an Instructional

Process

Analysis of the lesson plan revealed that the teacher used KMC when she prepared lessons. In her lesson plan, the teacher clarified the relationship among mathematical concepts and extracted meaningful implications from it. Specifically, in her lesson plan the teacher used KMC, when presenting the relationship among mathematics concepts between grades referring to it as the flow of learning [Fig. 2].



[그림 2] 학습의 흐름

[Fig. 2] The Flow of Learning

Not only did the teacher indicate the relationship among mathematics concepts, but also she used this information for analyzing her students' mathematical background. The teacher developed survey questions based on the mathematics concepts that her students should learn in their previous grade based on KMC. The teacher seemed to believe that students' should understand the basic concepts of parallel and the way of drawing it before they learn how to draw the sketch of a rectangular parallelepiped. As shown in [Table 1], the teacher developed survey questions and made note of the results in her lesson plan.

[표 1] 설문 1: 평행 개념 이해에 대한 학습자의 성취도

[Table 1] Survey 1: Students' previous achievement about the concept of parallel

Results	No.	%
Students do not understand about the basic concept of parallel or cannot draw well.	8	19.1
Students do understand about the basic concept of parallel or can draw.	27	64.3
Students do understand about the basic concept of parallel and can draw well.	7	16.6

The teacher applied the survey results in planning her lesson and documented what she was going to do during instruction. The teacher reported,

> Student should have learned the basic concept of parallel lines in order to participate in today's lesson. Most students in the class understand the basic concept of parallel lines. However, some students still struggled with drawing parallel lines. Therefore, I need to provide some activities, which relate to drawing parallel lines during the lesson, although they already had learned it. This activity should include detailed instruction about drawing parallel lines in order for students who cannot draw parallel lines can follow my instruction. At the same time, this activity should not bore students who already know the concepts of parallel lines and the way of drawing them.

In addition, the teacher used KMC when she developed the procedures of instruction. In the lesson plan, the teacher organized the sequence of lessons in the chapter and explicated the relationship among them, which is referred to as deployment plan for the chapter as shown in [Table 2].

[표 2] 단원의 차시 계획

[Table 2] A Deployment plan for the chapter

Sequence (Textbook)	Subject	Content and activity
. .	. .	. .
3 (p.56-p.58)	The relationship among faces in a rectangular parallelepiped	Students will find faces, which are parallel to each other in a rectangular parallelepiped. Students will find faces, which are perpendicular to each other in a rectangular parallelepiped.
4 (p.59-p.60) Today's Lesson	The sketch of a rectangular parallelepiped	Students will understand the needs of a sketch of a rectangular parallelepiped. Students will learn the way of drawing a sketch of a rectangular parallelepiped.
. .	. .	. .

Based on her KMC, the teacher specified the range of topics in each lesson. The teacher also developed her teaching strategies and used it in order to predict students' reaction during the lesson. From the third interview, the teacher clarified her intention. The following excerpt from the transcript of the third interview relates to the teacher's use of MCK for teaching a lesson.

Interviewer: How did you motivate your students?

Teacher: At the beginning of the lesson, I intentionally deceived my students. Can you remember a prism, which looked like a cube? I purposely presented a prism, because I wanted students to notice the needs for a sketch of three-dimensional shapes.

Interviewer: Then, you already knew your students' answers when you prepared the lesson? This was a chance for your students to give the right answers. Did you have a Plan B?

Teacher: I did not have a Plan B. I knew that students said wrong answers [because] I showed them a face of a rectangular parallelepiped with similar materials during the previous lesson. My students just believed it was the same face, which they had observed in their previous lesson…I usually plan whole lessons together before I start a new chapter. I check the connection between topics in a chapter or with the other chapters, and I prioritize the mathematics concepts or strategies, which I am going to teach. I decided how much time to spend on each subject. Also, I checked materials that I need for each subject. [Because] A chapter consists of subjects, which are coherent to each other. So, I think that I should make a coherent connection among lessons at least in the chapter. Sometimes, there is a certain relationship among chapters. For example, students learn form one to nine in one chapter first and from ten to ninety-nine in the next chapter in first grade. I believe that teachers help students to find these connections among mathematical concepts.

As stated in the theoretical framework of this paper, Shulman (1987) and Ball, et al. (2008) clarify that curriculum knowledge is presented by the full range of programs designed for teaching particular topics based on understanding of instructional materials. However, as shown above, the teacher focused on the relation among mathematics concepts based on the curriculum and tired to find meaningful instructional implications from it rather than concentrated on the use of instructional materials in order to teach a specific mathematical topic. This may show that there is a need to focus on the curriculum itself regardless of diverse instructional materials in order to reveal its effectiveness of KMC.

In addition, while Ball, et al. (2008) distinguished horizon content knowledge from PCK and categorized it in the range of SMK from the theoretical framework, KMC seems to relate to knowledge of content and students and knowledge of content and teaching, which are in the range of PCK in their knowledge domain; the teacher used her KMC to analyze or understand her students' mathematical background and to use it in her actual teaching. Therefore, there needs to be more discussion about the role of KMC in teachers' instruction and its location in the domain of teachers' mathematical knowledge.

2. Use of KMC When Teaching the Lesson

The teacher started the lesson with reviewing a previous lesson and ends with notification of the next lesson from her lesson plan and teaching. The teacher tried to make connection among the lessons in one chapter based on her KMC. In addition, the teacher followed her lesson plan, and every student could find and draw the edges of a rectangular parallelepiped, which are parallel to each other. The teacher used the information from the survey, which was developed based on her KMC doing her actual teaching. During the third interview, the teacher clarified her intension,

> Based on the survey, I departmentalize the way of drawing a sketch of rectangular parallelepipeds for students who already knew the basic concept and would lead the activity, and at the same time, students who do not know it would follow them. I asked my students how do you find and draw edges, which are parallel to each other, rather than just explain to them how to do it.

By making use of KMC during her lesson, the teacher was able to involve all students during the lesson, considering those who were familiar with the concept and those who were not familiar with it. In addition, the teacher's KMC affected her use of vocabularies in her classroom teaching; the teacher was careful about using manipulatives and mathematical terms in her lesson. The following excerpt from the transcript of the third interview relates to the teacher's use of vocabularies for teaching the lesson.

Interviewer: Is there anything that you are careful about when using mathematical terms?

Teacher: I'm trying to use them carefully based on students' [current] grade level. There are certain stages in which to introduce mathematical terms to students according to the mathematics curriculum. [For example] during the lesson as you observed, I did not use the term 'prism', although I showed a prism to my students. I did not use the term on purpose, because my students are going to learn about prism in a higher grade. I focused on the grade level mathematical terms introduce based on curriculum. If I use a mathematical word that is going to be introduced in the future, it may lead students' to develop misconceptions, because they did not learn enough relative mathematics concepts to understand those terms.

The findings of this study contradict Shulman's (1987) basic categories of teachers' knowledge. Shulman (1987) assumes that teachers' curriculum knowledge does not directly relate to mathematics instruction, although it may provide some background knowledge for teaching mathematics. However, for this current study, the teacher's KMC affected the way she organized activities and the use of mathematical vocabularies during the lesson; the teacher tried to make connections among lessons based on her KMC. Furthermore, the teacher was very careful about her use of mathematical terms, because it might encourage students' development of mathematical misconceptions. This indicates that KMC may affect teaching directly, which suggest that the basic assumption of categories should be reexamined.

The findings in this study also help us to understand how SMK support PCK. In earlier research, Even (1993) proposes that the relationship between SMK and PCK have been unknown, therefore the effectiveness of SMK in classroom teaching is unclear. However, this study has shown that the curriculum knowledge, which was categorized in the area SMK by Shulman (1987), does influence the process of instruction.

3. Use of KMC in Assessing Students' Work

The teacher used KMC when she analyzes students' mathematical background and assesses their work. According to her lesson plans, the teacher conducted a diagnostic assessment on her students based on what they learned in their previous grade. In addition, the teacher applied this knowledge when she assessed a student's work (See Appendix 1). The following episode form the analysis of the transcript from the second interview relates to knowledge of mathematics and ideas about teaching and learning mathematics. The teacher is asked to examine work completed by a child in second grade, involving subtraction with regrouping.

Interviewer: How would you respond to this second grade student's work?

Teacher: If she really doesn't know the basic concepts, she has to learn place value, subtraction and regrouping. She had to learn the basic concept of zero when she was in the first grade. So, it is almost impossible for her to learn zero in her second grade. I think I have to find extra time for her.

Interviewer: Do you check the curriculum every time?

Teacher: No, I don't. I have taught almost every grade during the last ten years. Therefore, I just remember it. Although I can't tell you about the curriculum exactly, I know the general sequences of the curriculum.

The teacher's use of her KMC in assessing students' work may show that KMC is integrated with knowledge that relates to understanding students' mathematical background. This finding provides some clues about the reason why elementary teachers' mathematical knowledge may have little affect on improving students' mathematics achievement scores, who received low scores from previous achievement tests (e.g. Hill 2008; Hill et al. 2005; Tanase 2011). If a teacher only focuses on the transformation of mathematical ideas for the current grade based on PCK and does no pay attention to students' previous mathematical experiences, it might be difficult to expect students to develop understanding of new mathematics concepts they did not learn in their previous school grade.

A further insight is this teacher acquired KMC overtime because she taught mathematics at various grade levels. Examining how teachers acquire KMC and their subsequent use of it for planning, teaching, and assessing may help us understand the relationship between teachers' knowledge for teaching mathematics and their experiences. Although there are studies that suggest teachers who have experiences with a wider range of grades in elementary schools have better knowledge for teaching mathematics than who have taught only one or two elementary grades (e.g. Ng 2011; Chinnappan, & Lawson 2005), it is still unclear how teaching experience contribute to teachers' knowledge for teaching. The findings from this study show that teachers' teaching experience at diverse grade levels help teachers to understand the mathematics topics covered in the curriculum.

V. Conclusions and Implications

An analysis of the findings based on the theoretical framework offers the basis for the following conclusions and implications. In attempting to analyze KMC in an actual instructional process with diverse types of data, this study found that KMC is one of the essential categories of an elementary teacher's knowledge for teaching mathematics, and KMC is integrated into elementary teachers' whole process of instruction: planning the lesson, teaching the lesson and assessing student learning of content presented during the lesson.

The findings of this study may offer useful guidance for the design of the instructional process for elementary teachers. According to the Common Core State Standards for Mathematics (2010), elementary teachers' should help students develop conceptual understanding of mathematics as well as procedural fluency. In order to develop conceptual understating, students should make connections among mathematics concepts (Hiebert, & Carpenter 1992; Mclellan, & Dewey 1895; Polya 1957; Wertheimer 1950). In this case, it is important that elementary teachers know the relationship among mathematical concepts in order to support their students, because there are direct parallels between the ways teachers connect their mathematical knowledge and the instruction they implement in their classrooms as a result (Carpenter, Fennema, Peterson, Chiang, & Loef 1989; Fennema, Carpenter, & Peterson 1989; Peterson, Fennema, & Carpenter 1991). In addition, regarding conceptual understanding, studies propose that teachers should understand students' thinking (Fennema, & Romberg 1999; Brophy 1997; Carpenter, Franke, Jacobs, & Fennema 1998). Teachers may not be able to engage students in productive discussion when they do not understand or can predict students' responses. As shown in this study, KMC provide some criteria for understating students' mathematical background and their responses. Although the information about students' mathematical background from the analysis based on KMC cannot represent the whole, it helps teachers to understand more about students' mathematical background. The more teachers know about their students' mathematical background, the more opportunities teachers have to provide effective instruction to their students.

The findings of this study are also expected to broaden the range of research areas about elementary teachers' knowledge for teaching mathematics. So far, previous studies (e.g. Izsák 2008; Rowland, Huckstep, &Thwaites 2005; Santiban ~ezm 2005) have focused heavily on PCK. However, as shown in this study, teachers' KMC also relates to elementary teachers' teaching. Especially, KMC seems to relate to knowledge of understanding students' mathematical background and use of mathematical terms in the actual teaching of mathematics. This may imply that teachers use diverse categories of knowledge for teaching mathematics by integrating them rather than rely on a single domain. Therefore, further studies are needed in diverse categories of teachers' knowledge for teaching mathematics and the relations among them.

It is important to acknowledge that a possible limitation of this study is that only one teacher participated in this study; however, several artifacts were examined to build a case for KMC (e.g., lesson plan, observation, interviews), which were use to triangulate the data. Emerging from this case are findings that may be used to inform future studies in this area. Therefore, it is suggested that further studies should be conducted with diverse teachers to develop a broader understanding of KMC and with diverse categories of teachers' knowledge for teaching mathematics. Certainly, new understanding and application of KMC that emerge from further studies and from analysis of the relationship among categories of teachers' knowledge will assist in improving the teaching of mathematics in general. As implied in this study, KMC is one of vital categories of teachers' knowledge for teaching mathematics in which to improve teacher's understanding of mathematics concepts, students' mathematical background, and their use of mathematical terms in actual teaching.

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한국 초등학교 교사의 수학 교육과정 지식에 대한 사례 연구

김리나 (Rina Kim)†
서울목운초등학교 (Seoul Mogun Elementary School)
E-mail : rina98@naver.com

신항균 (Hang Gyun Sihn)
서울교육대학교 (Seoul National University of Education)
E-mail : hkshin@snue.ac.kr

본 사례 연구의 목적은 한국초등교사의 수학교육과정 지식을 알아보는데 있다. 본 연구에서는 수업지도안, 수업 관찰, 인터뷰 등의 다양한 데이터 자료의 분석을 통해 한국 초등교사의 수학교육과정 지식이 어떻게 그들의 수학 교수 활동에 영향을 주는지에 대한 심도 깊은 사례연구를 진행하였다. 본 연구의 데이터 분석 결과 한국 초등교사의 수학교육과정 지식은 수업을 설계하고, 진행하고, 학습자의 과업을 평가하는데 직접적인 영향을 주는 것으로 나타났다. 또한, 본 연구에서는 수학교육과정 지식이 수직적 수학교육과정 지식, 수평적 교육과정 지식의 두 개의 하위 영역으로 구분될 수 있음을 밝혀냈다. 이 국제 사례연구의 결과는 초등 수학 수업과 연관된 교사의 교육과정 지식을 이해하는데 더움을 줄 것으로 기대되며, 한국과 미국 양국의 연구자들, 정책입안자들에게 의미 있는 시사점을 제공할 수 있을 것으로 기대된다.

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† 교신저자

<Appendix 1>

An elementary teacher's knowledge relates to assess a student's work. This example of a student's work is adapted from Ball (1998).