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Prospective Primary School Teachers Views on the Nature of Mathematics[1,2]

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This paper examines and presents descriptions of 12 prospective primary teachers' views on the nature of mathematics in USA. All the participants were elementary teacher candidates enrolled in the same mathematics method courses. Interview data show that the prospective primary teachers possess two kinds of views on the nature of mathematics: *primarily traditional* and *even mix of traditional and nontraditional* beliefs in terms of Raymond's (1997) belief criteria. Implications for teacher education were discussed at the end of the paper.

Keywords: teacher education, prospective primary teachers, teachers' beliefs about mathematics
MESC Classification: B55
MSC2010 Classification: 97B50

INTRODUCTION

In the past, there have been several efforts to improve mathematics education, for example, the 'new math movement' and the 'back to basics' effort. However, most movements have been deemed to be unsuccessful to change classroom mathematics teaching and learning. Studies of school practice during the new math movement indicated that the reforms were never authentically implemented in most classrooms (Stanic & Kilpatrick, 1992) because teachers were not well prepared to teach the content that the new math movement required.

Improving mathematics education is an ongoing process and teachers need to be taken into account for successful mathematics education because teachers are the ones who

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actually interact with learners. Especially, to have prospective teachers to be well prepared to teach mathematics in their future classrooms, their initial status needs to be considered for designing teacher education programs. Among many aspects to be considered, one of the most crucial aspects might be prospective teachers' mathematical beliefs (Cross, 2009; Ernest, 1989).

Many studies and reviews of research on teachers' beliefs found that the importance of prospective teachers' initial beliefs (Kagan, 1992; Pajares, 1992). The beliefs that teacher candidates bring to their teacher education program acts as filters when they interpret and internalize their experiences in teacher education programs (Gess-Newsome, 1999, Hollingsworth, 1989; Spangler, Sawyer, Kang, Kim, & Kim, 2012; Weinstein, 1990). Thus it is important to recognize what prospective teachers believe about mathematics when they enter teacher education program for teacher educators to plan to educate prospective teachers. This study attempted to answer the research question, "How prospective primary teachers view the nature of mathematics?"

REVIEW OF LITERATURE

Teachers' Mathematical Beliefs and Teaching Practice

As many scholars agree, teachers' mathematical knowledge is one of the most important aspects and has a lot of impacts on teachers' teaching practice. However, it does not always directly transfer to teachers' teaching practice (Hoy, Davis, & Pape, 2006; König, 2012; Wilkins, 2008). It is understandable when we consider teachers' varying mathematical beliefs. Many scholars view that teachers' beliefs have a critical effect on teaching practice and are one of the most important aspects need to be considered in mathematics teacher education field (Ernest, 1989; Pajares, 1992; Philipp, 2007; Thompson, 1984). For instance, Ernest (1989) claimed, "Teacher knowledge is important but it alone is not enough to account for the differences among mathematics teachers" (Ernest, 1989, p. 249–250).

Evidently, many studies found that teachers' teaching practices are related to their beliefs in some ways even though the relation seems varied. Three junior high school teachers' cases were analyzed by Thompson (1984) to investigate the relationship between teachers' conceptions of mathematics and their teaching practice. Thompson concluded that the teachers' conceptions of mathematics played a critical role in the teachers' teaching practices even though the relationships between the beliefs and teaching practices seemed different from each other.

Stipek, Givvin, Salmon & MacGyvers' (2001) investigated the relations between beliefs and mathematics teaching practice. By comparing the beginning and the end of the school year of fourth through sixth grade teachers, they concluded that the teachers' beliefs and teaching practices were consistent.

Beliefs about the Nature of Mathematics

Among several stems of mathematical beliefs, epistemological beliefs about mathematics, i.e., beliefs about the nature of mathematics play an important role for teachers' mathematical beliefs. For example, Hersh (1986) claimed the importance of epistemological beliefs about mathematics saying, "One's conceptions of what mathematics is affects one's conception of how it should be presented. ... The issue, then, is not, "What is the best way to teach?" but, "What is mathematics really all about?" (Hersh, 1986, p. 13).

Specifically, Ernest (1989) claimed that a teacher's perspectives on the nature of mathematics are basis for the teacher's mathematics teaching practices. He suggested three key belief components of the mathematics teacher. Those are *view or conception of the nature of mathematics*, *model or view of the nature of mathematics teaching*, and *model or view of the process of learning mathematics* (Ernest, 1989). He proposed three philosophies about the nature of mathematics: *instrumentalist*, *Platonist*, and *problem-solving view of mathematics*, and each of them is associated with different views on teaching; *instructor*, *explainer*, and *facilitator* correspondingly.

When a teacher has an *instrumentalist view of mathematics*, the teacher might view his/her role as that of an *instructor* and is likely to view learning as *mastery of skills*. Next, if a teacher possess a *Platonist view of mathematics* in which the teacher views mathematics as a unified body of knowledge, then the teacher is likely to believe that the mathematics teacher is an *explainer* and to consider learning as *passive reception of knowledge*. Finally, if a teacher believes that mathematics is about *problem solving*, then the teacher likely views the teacher's role as a *facilitator* and views *learning as the active construction of understanding* and possibly even as autonomous problem-posing and problem-solving. Hence, Ernest argued that beliefs about the nature of mathematics play a role as a foundation for beliefs about learning and teaching of mathematics.

Green (1971) argued that one's belief system has a quasi-logical structure which means that some beliefs are primary whereas others are derivative. In Green's (1971) terms, Ernest's (1998) beliefs about the nature of mathematics can be seen as primary beliefs and beliefs about mathematics teaching and learning can be understood as derivative beliefs and this also supports the importance of teachers' beliefs about the nature of mathematics.

The importance of epistemological beliefs about mathematics is supported by many studies. Raymond (1997) found differences between beliefs about mathematics teaching and mathematics learning and teaching practice. Raymond categorized teachers' mathematical beliefs into two main types; *traditional* and *nontraditional,* with their subcategories by expanding Ernest's (1989) categories. Raymond argued that teachers who have *traditional mathematical beliefs* view mathematics as an unrelated collection of facts, rules, and skills. Moreover, they believe students receive knowledge passively from the teachers, textbooks, worksheets, and individual work rather than discussion or conversation with their peers. Also the teachers' view that students must engage in repeated practice to mater and memorize skills and algorithms. And teachers with that belief view their main role as carrying out lecture and see that a main goal of education is to evaluate students through standardized tests.

Teachers who possess *nontraditional mathematical beliefs* view that mathematics as dynamic, problem driven, continually expanding, surprising, relative, doubtful, and aesthetic. Teachers with this view believe that students are autonomous and active learners who learn mathematics through problem-solving activities and cooperative group interactions rather than only through paper-and-pencil activities. In this perspective, teachers are supposed to guide students to learn, to ask challenging questions, and to promote students' autonomy. And such teachers help students enjoy math, value it, and they evaluate the students' solving process rather than only final products.

Raymond (1997) studied beginning elementary teachers' mathematical beliefs and teaching practices and concluded that teaching practices are not always consistent with teachers' beliefs about teaching and learning mathematics; rather what Raymond found was that teaching practice is closely related to the beliefs about the nature of mathematics. The finding of her study not only implies the importance of beliefs about the nature of mathematics but also support the claims that epistemological beliefs are an important factor for mathematics teaching practice.

Prospective Teachers' Views on the Nature of Mathematics

Regarding prospective teachers' views on the nature of mathematics, many studies found that their views are limited. As Alfred North Whitehead mentioned (as cited in Ball, Lubienski & Mewborn, 2001), students have few chances to be introduced to the power and beauty of mathematics as a creation of human idea as much as students are supported to obtain skills and procedures of mathematics. Consequently, most adults, including teacher candidates, graduate from school with an 'instrumental understanding', i.e., 'rules without reasons', rather than with a 'relational understanding' of mathematics, of which Skemp (1976) speaks.

Thus it is not surprising that a growing body of research found that most pre-service teachers enter teacher education programs with rather traditional mathematical beliefs.

One of possible explanations for this would be future teachers enter teacher education programs with their own beliefs and views about teaching and learning, subjects, and students, which have been developed through what Lortie (1975) called ‘apprenticeship of observation’.

For example, according to Handal’s (2003) review, future teachers carry traditional beliefs of mathematics and mathematics teaching. Specifically, Benbow (1993) administered three sets of questionnaires to 37 pre-service teachers and interviewed two teacher candidates. Benbow found that pre-service teachers had a limited view of mathematics. That is, the prospective teachers regarded mathematics as mainly facts and procedures to be memorized, and thought there would be only one right way to reach the one and only true answer to a problem. Most of them not only regarded memorization as ‘important’, but also considered mathematics as a dichotomy: absolutely right or absolutely wrong. In addition, Nisbet & Warren (2000) found that most pre-service teachers had either a static view or a mechanistic view of the nature of mathematics, but not a dynamic problem-solving view of mathematics.

In general, in terms of Ernest (1989), teacher candidates show Instrumentalist or Platonist beliefs about mathematics, accordingly, prospective teachers possess Instructor or Explainer views about teachers’ roles, and Passive Reception of Knowledge beliefs about student learning rather than having a Problem Solving view of mathematics, a Facilitator’s view of teachers’ roles, and Active Construction of Knowledge beliefs about learning.

THEORETICAL PERSPECTIVES

Zeichner & Gore (1990) identified three traditions in teacher socialization research; functionalist, interpretive, and critical. This study is based on the interpretive paradigm for teacher socialization. The core idea of interpretivism is that

> “All human activity is fundamentally a social and meaning-making experience, that significant research about human life is an attempt to reconstruct that experience, and that methods to investigate the experience must be modeled after or approximate it.” (Eisenhart, 1988, p.102)

The purpose of this study is to describe “what is *going on*” among prospective primary teachers and then “what intersubjective meanings underlie these *goings on* and render them reasonable” as Eisenhart (1988, p.104) noted.

To analyze the interview data in terms of previous research, criteria for the categorization of teachers’ beliefs about the nature of mathematics that were developed by Raymond (1997) were employed. Raymond’s work was heavily based on Ernest (1989):

(a) *Instrumentalist* view of mathematics,
(b) *Platonist* view of mathematics, and
(c) *Problem solving* view of mathematics.

In detail, *Instrumentalist* view of mathematics refers to the beliefs that consider mathematics as a set of unrelated but utilitarian rules and as an accumulation of facts, rules and skills to be used. Next, *Platonist* view of mathematics means a person who believes mathematics is a static but unified body of certain knowledge so mathematics is discovered, not created. Finally, *Problem solving view of mathematics* is a perspective that considers mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Thus with the problem solving view of mathematics, mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision. Based on Ernest's work, Raymond developed criteria as shown in the following table.

Table 1. Summary of Criteria for the Categorization of Teachers' Beliefs about the Nature of Mathematics (Raymond, 1997)

Traditional ●Mathematics is an unrelated collection of facts, rules, and skills. ●Mathematics is fixed, predictable, absolute, certain and applicable.
Primarily traditional ●Mathematics is primarily an unrelated collection of facts, rules, and skills. ●Mathematics is primarily fixed, predictable, absolute, certain, and applicable.
Even mix of traditional and nontraditional ●Mathematics is a static but unified body of knowledge with interconnecting structures. ●Mathematics is equally both fixed and dynamic, both predictable and surprising, both absolute and relative, both doubtful and certain, and both applicable and aesthetic.
Primarily nontraditional ●Mathematics is primarily a static but unified body of knowledge. ●Mathematics involves problem solving. ●Mathematics is primarily surprising, relative, doubtful, and aesthetic.
Nontraditional ●Mathematics is dynamic, problem driven, and continually expanding. ●Mathematics can be surprising, relative, doubtful, and aesthetic.

METHODS

This paper is a part of a larger study that includes data across the participants' junior year of a teacher education program into their first two years of teaching. For the purpose of the study,12 participants' first interviews about the views about mathematics were analyzed.

Participants

In the U.S., there is no specialized university for educating prospective primary school teachers where there is a specific type of university only for prospective primary teachers in South Korea. People who want to be primary school teachers in the U.S. enter general universities which provide teacher education program and take courses for teacher candidates.

12 U.S. prospective primary school teachers' data were analyzed. Among the 12 primary school teacher candidates, 10 were female and 2 were male. The participants were reflective of the diversity of students enrolled in the primary school teacher education program to the extent possible. They had taken one mathematics content course for elementary education majors prior to the interview. All of them were juniors in the teacher education program and in the first semester of a mathematics method course when they were interviewed.

Data Collection

The primary prospective teachers were interviewed near the end of the first mathematics methods course in one U. S. university. The interview focused on eliciting the participants' perspectives on mathematics and their prior experiences as mathematics learners.

During the interview participants were asked to choose the most appropriate metaphor for mathematics learning and mathematics teaching. For mathematics learning, participants were asked to choose among working on an assembly line, cooking with a recipe, jigsaw puzzle, building a house, watching a movie, picking fruit from a tree, conducting an experiment, creating clay sculpture or the participants were allowed to choose their own. And then the participants were asked to explain why they chose the metaphor.

For mathematics teaching, participants were asked to choose among news broadcaster, doctor, gardener, missionary, entertainer, orchestra conductor, coach, social worker', or the participants were allowed to choose their own. After the participants made their choices, they were asked to provide the reason why they made their choice.

Analysis

I delved the prospective primary teachers' beliefs about the nature of mathematics from their responses to the metaphors for mathematics learning and teaching. I used both of their response and their explanation of why their metaphor was appropriate to describe mathematics using Raymonds' (1997) criteria (Table1). Even though some prospective primary school teachers chose the same metaphor, their explanations for their choice could be different. I focused on each participants' explanations rather than the metaphor itself because their explanations can show the participants' views on the mathematics.

Note that it is hard to categorize one's beliefs about mathematics because there is no clear cut for one's belief. Many of the participants revealed both of primarily traditional and even mix of traditional and nontraditional in Raymonds' criteria. So I used the term 'Mix of Primarily Traditional and Even Mix of Traditional and Nontraditional'.

FINDINGS

When the primary teacher candidates described mathematics learning and mathematics teaching, most of them referred to their view about the nature of mathematics, which indicates the important role of beliefs about the nature of mathematics for beliefs of learning and teaching of mathematics as Ernest (1989) proposed.

Traditional View on the Nature of Mathematics

Two of the participants revealed Traditional view on the nature of mathematics.

Eleen: I don't really know how much math is changeable, you know, like you pretty much have the formula. ...if I mean you can like coach it and show them how to do it or the teacher would show me how to do it, and…but if you just know that something is not going right you can take that problem and like really do it yourself and guide them through your demonstration

Eleen did not see that mathematics is a still developing scholastic area. She characterized mathematics as a set of formulas and considered mathematics as not changeable. Although she was using the word 'guide' and 'coach' for teaching students, that turned out to mean 'demonstrate'. According to her views on the nature of mathematics, Eleen also thought teaching mathematics is showing how to solve a problem in a predetermined right way.

Rob also demonstrated a Traditional view on the nature of mathematics.

Rob: ... you've got the music there which is like the mathematical concepts and all that. It's already there written for you and you take and try to adapt it ... there are steps you follow and over time you can adapt to your own — like when you're solving an algebra problem there are, like you can learn the short cuts and you know which steps work for you

Rob described mathematics is given set of rules and steps while explaining mathematics teacher as an orchestra conductor. Learning mathematics means accepting and following the given rules and steps to solve a problem for him.

Both of Eleen and Rob viewed mathematics is a fixed set of rules and formulas that need to be followed. For them mathematics is not changeable and according to their view about mathematics, mathematics teachers are supposed to demonstrate how to solve a mathematics problem using predetermined way. Then students are supposed to follow teachers' step by step explanation.

Mix of Primarily Traditional and Even Mix of Traditional and Nontraditional

Ten of the prospective primary teachers mentioned mathematics is a *set of mathematical foundations* and this fits the Primarily Traditional criteria. However, at the same time, they believed that there are *connections* between them and thus fit into the 'Even Mix of Traditional and Nontraditional' criteria also (cf. Table 1).

Meanwhile, the relationship between the 'foundation' and 'basic concepts' in mathematics varied from each participant. Table 2 describes the relationship between the 'foundation' and 'basic concepts'.

Table 2. Description of the Prospective Primary Teachers' Beliefs About the Nature of Mathematics

Types of connection		Description
Mathematical foundations/basic concepts are connected	hierarchically	Prior mathematical foundations are stepping stones for the more advanced level of mathematics learning
	as certain steps	Mathematical foundations represent each step and the steps produce a final answer

Mathematical foundations/basic concepts are connected hierarchically.

Many of the prospective teachers who viewed *mathematics as hierarchical* described mathematics as building a house, or jigsaw puzzle.

Sindy: Because it's like you learn, you see bits and pieces of things, and you know they're all going to fit together, but it takes time for you to learn them first, like after you learn "one plus one is two", then you can build from there ...

Eco: ... everything in math is connected and based on something else, and so I said the foundation of a house, the foundation of math and it just goes up and gets fancier and more extravagant and more detailed and more complex. And I think about like building a house, like everything is built on something else. And if you don't understand or you don't have good fundamentals, the rest isn't going to work, and that's more like a house to me.

Enna: ... in math you have so much information and it's just like figuring out which pieces fit together, you need a foundation before you can move up.

Kara: I would say math is like building a house because you have to have a foundation of basic concepts before you can more on to a higher mathematical learning.

Jayne: ... it's building a house because you start with a foundation in like K through second of number stance, and addition and subtraction. Then you add a little multiplication to that, a little division, a little decimal, a little fraction, and keep building from just learning what a number is all the way up to algebra and trig and calculus. I mean it's just like you start with your foundation and then you add structural elements. And then you can add some more complicated and detailed parts of math. But you have to start simple and then, you know, get bigger as you go and add things as you go along.

Morry: there are things like facts ... you learn your addition facts, you learn your multiplication facts, so that is like your recipes ... because there's so much out there, just as you might learn the basic things in basketball or baseball or something, but there's so much you can do. I mean, you can go to championships or you can just play in your backyard or something.

Seha: I felt like all of the little pieces looked a lot alike and it was hard to find which one should go where and, you know, when to use which one and I always felt like I was having to put one piece in and go oh no, that's not it, that's not the right way to work that problem.

Mathematical foundations/basic concepts are connected as steps.

Participants who described *mathematics as steps* with foundations compared learning mathematics to a jigsaw puzzle, however, the reason why they choose jigsaw puzzle for describing mathematics was different from the prospective teachers who described mathematics is connected hierarchically.

Bena: ... you have this thing and at first it is just this big blob and you don't know what you are going to do with it. (When the interviewer asked that everything sort of comes of pre-packaged and fits together in a predetermined way and the product is always the same to clarify what Bena said then Bena agreed.)

Azit was describing mathematics teacher as orchestra conductor like Rob but his explanation was different from Rob's.

Azit: ... You can say to the flutes, you know, "do this," or "strings, I need to hear this section and this section." "Okay, here's a math problem, this is the answer, you got it right, but show me how you did steps three and four. How did you move that decimal point? Can you show me how you did that? Why did you do that?" "Why does this piece sound the way it does," you know?

Whereas Rob viewed mathematics as a given music note Azit considered the reason why we do to solve mathematics problems as important aspect.

Mathematical foundations/basic concepts are connected hierarchically also as certain steps.

Arin was the one who mentioned all the types of connections in Table 2. She considered mathematics as set of foundations and given steps that need to be accepted. For her, figuring out the final answer is one of the goals of doing mathematics.

Arin: ... it (mathematics) does build on each other but you do have to start like with the foundation and there is certain steps, you know, that you follow to get the final product...it (mathematics) always seem to connect with what we'd done the year before.

Many of the prospective primary teachers recognized the importance of learning mathematics at each certain level and how the previous mathematics learning affects later mathematics learning.

As shown above, many of the prospective teachers believed that there are certain foundations in mathematics which some of them called 'facts', and they can be connected once we learn the basic facts. However, the prospective teachers considered the basics of mathematics are just given and fixed. One of them even explicitly mentioned that the mathematical facts are already there as a given and all we need to do is accept those and use it to solve a given mathematics problem. Also no one mentioned about definitions of mathematics which is one of the most important starting points in mathematics and did not recognized how mathematics definitions are formed.

CONCLUSIONS AND IMPLICATIONS

The prospective primary teachers recognized that the 'basics' of mathematics are *connected*. It is promising that the prospective primary teachers recognized that the basics of mathematics are connected. However, they considered the concepts as given and as connected in predetermined ways. Thus teacher educators might need to consider the prospective teachers' views on the nature of mathematics when designing teacher education programs and create experiences that help prospective teachers to see mathematics in a broader perspective (Bahr, Shaha & Monroe, 2013).

One thing to be considered is what they mean by ‘foundations or basics’ in mathematics. The other thing is none of them mentioned *definitions* in mathematics even though they might mean ‘definition’ by the word ‘foundations/basic facts’ of mathematics. Mathematical definitions play a key role in mathematics (Edwards & Ward, 2008), so the absence of the idea of mathematical ‘definitions’ in the views of teachers might have implications. One possible reason for the absence of definitions in their discussion might be that prospective teachers had few chances to consider mathematical definitions in their previous learning experiences.

Currently, *Common Core State Standards* (CCSS) have been adopted in most of the states in USA in an effort to establish a demanding curriculum so as to improve students’ mathematical learning and thinking. Among the standards, one thing that might need to be spot lighted is that the Common Core State Standards for Mathematics asks students ability to mathematically precise definitions (Common Core State Standards Initiative, 2010). For example, in the *Standards for Mathematical Practice* we find,

> “3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.” (p.6)

> “6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.” (p.7)

Also, at the high school level Common Core State Standards say,

> “During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.” (p.74)

Both of the quotes from Common Core State Standards for Mathematics (CCSSM) require students’ use of mathematical definitions. To be well prepared to teach according to the Common Core State Standards, prospective primary teachers need to understand what mathematics definitions are.

The prospective teachers might mean mathematical definition when they say “foundations or basic concepts”, however, they need to be aware of mathematical definitions even though they are not going to advanced tertiary level mathematics. Primary teachers need to see elementary level mathematics from a higher mathematical view.

Two Types of Defining

Elementary teachers who teach mathematics need to see primary mathematics in a big picture of mathematics and need to consider what mathematics definitions are and where they come from. DeVilliers (1998) proposed two types of defining *descriptive defining* and *constructive defining*.

Descriptive defining

With the descriptive (a posteriori) defining of a concept is meant here that the concept and its properties have already been known for some time and is defined only afterwards. A posteriori defining is usually accomplished by selecting an appropriate subset of the total set of properties of the concept from which all the other properties can be deduced. This subset then serves as the definition and the other remaining properties are then logically derived from it as theorems.

Constructive defining

Constructive (a priori) defining takes places when a given definition of a concept is changed through the exclusion, generalization, specialization, replacement or addition of properties to the definition, so that a new concept is constructed in the process. In other words, a new concept is defined "into being", the further properties of which can then be experimentally or logically explored. Whereas the main purpose or function of a posteriori defining is that of the systematization of existing knowledge, the main function of a priori defining is the production of new knowledge. (p. 250)

Even though teachers are aware that mathematics concepts are connected, if they consider the concepts as given and consider them as something should be accepted, then it is likely that this might affect their way of teaching it. The teacher might tend to present mathematical concepts to younger students as a set of given facts.

Vinner (1991) and many others argued that just knowing the definition of a concept does not necessarily mean understanding the concept behind of the definition. To help prospective primary teachers to grasp and understand what mathematical definitions are, prospective teachers need to have opportunities to develop/construct mathematical definitions by themselves. Vinner (1991) suggested that several examples and non-examples should be presented so that the mathematical concept will be formed.

The result of this study shows that there was no participant who showed primarily nontraditional or nontraditional views about the nature of mathematics at the beginning of teacher education program. This might imply that teacher candidates enter teacher education program with traditional views about mathematics. Teacher educators might need to consider 'inquiry-oriented' paradigm (Zeichner, 1983) for designing teacher education programs to help prospective teachers to view mathematics with different perspectives. Also prospective teachers themselves need to take a moment to consider the question: "In what ways do mathematical definitions need to be taught to students? And how to help learners to grasp the concept of mathematics definitions"

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