http://dx.doi.org/10.7468/jksmed.2014.18.4.237
J. Korean Soc. Math. Educ., Ser. D, Res. Math. Educ.
Vol. 18, No. 4, December 2014, 237–256

Primary School Students' Understanding of Equation Structure and the Meaning of Equal Sign: A Chinese Sample Study[1]

YANG, Xinrong*
School of Mathematics and Statistics, Southwest University, Beibei,
Chongqing 400715, China; Email: xinrongy@gmail.com

HUO, Yujia
Faculty of Psychology, Southwest University, Beibei,
Chongqing 400715, China; E-mail: hyj106@163.com

YAN, Yanxiong
School of Mathematics and Statistics, Southwest University, Beibei,
Chongqing 400715, China; E-mail: 2003yyx@163.com

(Received August 18, 2014; Revised November 21, 2014; Accepted November 27, 2014)

This paper reports findings from a written assessment which was designed to investigate Chinese primary school students' understanding of the equal sign and equation structure. The investigation included a sample of 110 Grade 3, 112 Grade 4, and 110 Grade 5 students from four schools in China. Significant differences were identified among the three grades and no gender differences were found. The majority of Grades 3 and 4 students were found to view the equal sign as a place indicator meaning "write the answer here" or "do something like computation", that is, holding an operational view of the equal sign. A part of Grade 5 students were found to be able to interpret the equal sign as meaning "the same as", that is, holding a relational view of the equal sign. In addition, even though it was difficult for Grade 3 students to recognize the underlying structure in arithmetic equation, quite a number of Grades 4 and 5 students were able to recognize the underlying structure on some tasks. Findings in this study suggest that Chinese primary school students demonstrate a relational understanding of the equal sign and a strong structural sense of equations in an earlier grade. Moreover, what found in the study support the argument that students' understanding of the equal sign is influenced by the context in which the equal sign is presented.

---

1 The research reported in this paper was supported by Natural Science Foundation Project of CQ CSTC 2011BB0010; the Fundamental Research Funds for the Central Universities (XDJK 2011B013); and Chongqing Postdoctoral Science Foundation (XM2014029).
* Corresponding author

Key words: equal sign, equations, Chinese primary school students, early algebra
MESC Classification: C33
MSC2010 Classification: 97C30

1. INTRODUCTION

The need for developing students' algebraic thinking in early grades at primary school level has been widely accepted in mathematics education community due to the importance of algebra in students' future learning and life (Cai, Ng & Moyer, 2011; Jones & Pratt, 2012; Knuth et al., 2006; McNeil et al., 2011). In recent years, there has been an ever increasing research interest in the investigation of primary school students' understanding of core algebraic concepts and the development of their algebraic reasoning in Western countries, like in the US and UK (e.g., Asquith et al., 2007; Jones & Pratt, 2012; Stephens et al., 2013). The most widely investigated algebraic concept at primary school level should be the equal sign or mathematical equivalence. The concept of equality and, in particular, the understanding of the equal sign, is commonly believed to be the fundamental to students' mathematics development, especially to the understanding of algebra (Kieran, 1981; Knuth et al., 2006; McNeil & Alibali, 2005). However, previous studies have repeatedly found that most primary school students, especially students from Western countries (e.g., the US and UK), tend to hold misconceptions about the equal sign and have difficulties with mathematical equivalence, like viewing the equal sign as a place indicator meaning "write the answer here" or "do something" (i.e., compute) other than a relational symbol of mathematical equivalence (Behr et al., 1980; Jones & Pratt, 2012; Knuth et al., 2006; McNeil et al., 2011; Stephens et al., 2013).

A main factor which leads to students' difficulties with mathematical equivalence and influences the development of their understanding of the equal sign is the structure of equations they have experienced during primary school mathematics learning (Knuth et al., 2006; McNeil & Alibali, 2005; McNeil et al., 2011). For example, previous studies have found that the way widely used to present the equal sign in standard format (e.g., $a \pm b = c$) in American textbooks and in teachers' mathematics lessons makes primary school students hold an operational view of the equal sign (Li et al., 2008; McNeil et al., 2006; Powell, 2012; Rittle-Johnson et al., 2011). This further reflects a fact that the learning environment or learning experiences affects the development of primary school students' understanding of the equal sign and the ability to solve equations (McNeil & Alibali, 2005). From this point of view, it is reasonable to say that like the learning of other mathematical concepts, the learning of the equal sign and mathematical equivalence is also a cultural activity since "all mathematics learning is cultural" (Stigler & Baranes, 1988, p. 300).

Therefore, to obtain a more complete picture of primary school students' understanding of the equal sign and its development, studies from social and cultural contexts outside of Western countries are needed. Yet, to date, even though there have been studies compared the differences of performance of solving equivalent problems between Chinese students (mainly sixth Grade students) and American or British students (e.g., Li et al., 2008; Jones et al., 2012), very few studies have ever tried to explore how Chinese primary school students, especially students at earlier grades, interpret the equal sign and their understanding of the equation structure. Studies focusing on the sixth Grade Chinese students provide insufficient information for the understanding of Chinese primary school students' interpretation of the equal sign and its development. In view of this, the present study, which mainly replicates Stephens et al. (2013)'s study, explores what understandings Grade 3–5 students hold about the meaning of the equal sign and equation structure in China.

2. LITERATURE REVIEW

Invented by Robert Recorde in 1557, the equal sign, "=", has become a universally recognized symbol for indicating mathematical equality (Cajori, 1928). The concept of equality and the equal sign is ubiquitous in school mathematics at all levels (Alibali et al., 2007) and it serves as a key link between arithmetic and algebra (Baroody & Ginsburg, 1983; Carpenter et al., 2003; Kieran, 1981; Matthews et al., 2012). Thus, the knowledge of the equal sign is foundational to students' mathematical development, especially to algebra understanding (Alibali et al., 2007; Kieran, 1981; Knuth et al., 2006). Due to its importance, primary school or middle school students' understanding of the equal sign has received increasing research attention in recent years in Western countries (e.g., Alibali et al., 2007; Jones et al., 2012; Knuth et al., 2006; Li et al., 2008; McNeil & Alibali, 2005). In these studies, several kinds of views of students' understanding of the equal sign have been proposed by researchers from different point of views.

The most common view regarding to students' understanding of the equal sign in previous studies is a dichotomy between an *operational view* and a *relational view* (Alibali et al., 2007; McNeil & Alibali, 2005). Students who hold the former view were found to tend to view the equal sign as a place indicator meaning "write the answer here" or "do something" (e.g., compute) in previous studies (Alibali et al., 2007; Knuth et al., 2006). This kind of view is consistent with the view of the equal sign as an announcement of the result of an arithmetic operation and this view will heavily influence primary school students' performance in solving mathematical equation (Alibali et al., 2007; Knuth et al., 2005). For example, previous studies conducted in the US repeatedly found that students with such view of the equal sign put 23 (instead of 9) in the blank of the problem "7+4+5=7+__" (e.g., Falkner et al., 1999; Matthews et al., 2012). Meanwhile, students who hold such view were found to reject equations as false in the type of 8=8 or 15=5+10 (Matthews et al., 2012).

Although the operational view of the equal sign may suffice to solve typical arithmetic problems at primary school level, an exclusive operational view could lead to inflexible thinking about arithmetic (Li et al., 2008; McNeil & Alibali, 2005). In addition, the operational view of the equal sign will cause difficulties for students when they encounter algebraic equations in later grades (Knuth et al., 2006). Therefore, students need to develop a more mature understanding of the equal sign, that is, the *relational view*, which refers to interpret the equal sign as meaning "the same as" (Carpenter et al., 2003). However, recently, Rittle-Johnson and colleagues (2011) further argued that there should be a transition phase between operational view and relational view. They named the transition phase as *flexible operational view*, which means that students become less rigid and will successfully solve, evaluate, and encode atypical equation structures, for example, equations that are "backwards" as in the format of __ = 2+5, or contain no operations as in the format of 8=8, but remain compatible with an operational view of the equal sign.

Some researchers, however, argued that only students with relational view of the equal sign can show more flexibility when work with equations in non-standard types such as 8=8 or 8+4=□+5 than those students who do not have this understanding (Falkner et al., 1999; Stephens et al., 2013). From this point of view, the flexible operational view should be classified into relational view, which includes two levels: *relational-computational view* and *relational-structural view* (Stephens et al., 2013). Students who hold the relational-computational view understand that the equal sign represents an equivalent relation between two sides of an equation and will confirm the equivalence by computing (Stephens et al., 2013). For example, students holding such view will accept 2+3=4+1 through justifying that both sides have the same value by calculating that both sides make 5 (Kieran, 1981). In other words, students who hold this view will understand the equal sign to "symbolize a relation between answers to two calculations" (Stephens et al., 2013, p. 174).

Essentially, the relational-computational view is similar to basic relational view as named by other researchers, which means that students consider the equal sign to indicate a numerical sameness (e.g., Baroody & Ginsburg, 1983; Rittle-Johnson et al., 2011). However, Baroody and Ginsburg (1983, p. 208) further argued that treat the equal sign as "the same as" in this way "is not a full relational understanding from a mathematician's point view". When students learn algebra, like learn how to solve algebraic equations (operating on or with the unknown or variable), the way to calculate and compare the value on both sides of the equal sign is not workable since the situation is quite different from solving arithmetic equations (operating on or with numbers). Therefore, the development of a more sophisticated view or a deeper level of understanding of the equal sign is necessary which requires a movement from the understanding of arithmetic equivalence to the understanding of algebraic equivalence (Jones & Pratt 2012; Kieran, 1981; Knuth et al., 2006). Previous studies generally labeled this level of understanding of the equal sign as relational-structural view (Stephens et al., 2013), or full relational view (Carpenter et al., 2003), or comparative relational view (Rittle-Johnson et al., 2011).

No matter what kind of labels were given to this view of the equal sign, fundamentally, students who hold this view will understand the equal sign "as a symbol expressing an equivalence relation between two expressions rather than two calculations" (Stephens et al., 2013, p. 174). In other words, students with such view can solve equations and evaluate equation structure by comparing the expressions on the two sides of the equal sign rather than carrying out specific computations (Rittle-Johnson et al., 2011). Thus, students start to realize the meanings as being carried by the equal sign within algebra, such as the properties of symmetry, reflexivity and transitivity (Baroody & Ginsburg, 1983; Kieran, 2007; Jones & Pratt, 2012). For example, Alibali et al. (2007) found that when students were asked to answer whether the number in the in the equations $2\times\square+15=31$ and $2\times\square+15-9=31-9$ is the same, students who demonstrated a relational-structural view of the equal sign were more likely to give the right answer by recognizing that the transformation performed on the second equation preserves the equivalence relation expressed in the first equation. In other words, students with relational-structural view can recognize that performing the same operation on both sides of an equation will maintain its equivalence without any computation (Rittle-Johnson et al., 2011). In addition, Carpenter et al. (2003) found that when students were presented the equation $28+32=27+\square$, students holding such view of the equal sign can recognize that 27 is 1 less than 28, so the unknown on the right side should be 1 more than 32. That is, students with relational-structural view can use compensatory strategies to ease calculations with larger numbers since they can recognize the algebraic feature of equation (Rittle-Johnson et al., 2011).

As reviewed above, several levels or stages of the development of primary school students' understanding of the equal sign have been discussed in literature. However, even though the development process has been argued as a continuous one and the levels reviewed above should not be interpreted as discrete stages (Rittle-Johnson et al., 2011), in reality, the development process of students' understanding of the equal sign is rather complicated and it is never found to be linear (Knuth et al., 2006). McNeil (2007) identified a U-shaped association between students' age and their performance on equivalence problems. However, Knuth et al. (2006) found that students did not exhibit a growth of a relational understanding of the equal sign as grade level increased. Particularly, in a longitudinal study of students' understanding the equal sign, Alibali et al. (2007) even identified that some students who made a relational definition to the equal sign at an early time point later made an operational definition. One possible reason as suggested by Alibali et al. (2007) is that students' early relational understanding of the equal sign is fragile and students may hold both operational view and relational view of the equal sign at the same time. The coexisted view was also echoed by Rittle-Johnson et al. (2011). They argued that it might be possible that less sophisticated understanding of the equal sign sometimes coexists with more mature understanding of it.

Furthermore, previous studies identified that students' interpretation of the equal sign is clearly related to the equation structure they encounter (Stephens et al., 2013). On the one hand, McNeil and Alibali (2005) found out that for students who just started to have relational understanding of the equal sign, how they interpret the equal sign really depends on the contexts in which the equal sign is elicited. They found that seventh Grade students interpret the equal sign as an operational symbol in the context of 4+8+5+4=__ but interpret it as a relational symbol in the context of 4+8+5=4+__. In other words, students' understanding of the equal sign is related to the format of mathematical equivalence, or the equation structure (Stephens et al., 2013). In return, students' understanding and interpretation of the equal sign also influences the ways or strategies they employ to work with equations as reviewed above.

On the other hand, students' understanding of the equal sign is also influenced or even reinforced by the equation structure they experienced during primary school learning (Jones et al., 2013; McNeil et al., 2011). For example, equations with operations on both sides of the equal sign were found to be especially useful for the development of a relational understanding of the equal sign (McNeil et al., 2006). However, in primary school textbooks and classrooms in Western countries, the majority of arithmetic equations are presented in canonical format, such as a±b=c, which has been argued to be the main reason to make most Western students hold an operational view of the equal sign (Li et al., 2008; McNeil et al., 2006; Rittle-Johnson et al, 2011; Seo & Ginsburg, 2003).

As for how the equal sign is presented and introduced in textbooks, Li et al. (2008) found that the Chinese primary school mathematics textbooks generally introduce the equal sign in a context of relationships and interpret it as "balance", "sameness", and "equivalence", which is quite different from textbooks in the Western countries. These differences have been argued as a main influence for the development of students' understanding of the equal sign (Knuth et al., 2006; Li et al., 2008; McNeil et al., 2006). Indeed, previous comparative studies found that around 70% of American Grade 6 students misunderstood the equal sign as an operator, however, less than 3% of their Chinese counterparts did so (Li et al., 2008). In addition, due to the difference of mathematics textbooks, students from different countries were found to hold different conceptions of the equal sign. Jones et al. (2012) found that Chinese students endorsed the substitutive-relational and sameness–relational conceptions of the equal sign more strongly than English students.

Due to the factor that students' understanding and conceptions of the equal sign are differentially developed across different countries (Jones et al., 2012), it is reasonable to conjecture that Chinese students' understanding of the equal sign and equation structure and its development process should be different from what has been identified in the Western countries. In view of this, under the guide of a development stage of primary school students' understanding of the equal sign as theoretical perspective, the main goal of the present study is to explore the development process of Chinese primary school students' understanding of the equal sign. In addition, to obtain a relatively complete picture of Chinese primary school students' understanding of the equal sign and more importantly, to deeply understand Chinese educational contextual influences which contribute to Chinese students' understanding, findings identified in the study will be further compared with findings found in Stephens et al. (2013)'s study. Specifically, this study focuses on the following two questions:

1) What understanding do Grade 3–5 Chinese primary school students hold about the meaning of the equal sign and equation structure?
2) How does the difference of equation structures influence Chinese primary school students' understanding of the equal sign?

3. METHODOLOGY

3.1. Participants

This study involved a total number of 332 primary school students aged 9 to 12 years (110 Grade 3, 112 Grade 4, and 110 Grade 5; 169 boys and 163 girls) from 4 schools with different academic backgrounds selected from different areas, one rural, two urban, and one suburban in Chongqing, locating in the west part of China. The study was conducted in October, 2013, that is, in the first semester of the school year. Textbooks used in these four schools were published by Southwest Normal University Press (SNUP) which was developed under the guide of National Mathematics Curriculum Standards for Compulsory Education in China and was widely adopted in Chongqing and many other places in China. Since all Chinese primary school mathematics textbooks were developed under the same national mathematics curriculum standard, how the concept of the equal sign was introduced and developed in SNUP textbooks is quite similar to the ways in other Chinese mathematics textbooks (Li et al., 2008).

3.2. Instrument

Although students' understanding and conceptions of the equal sign are differentially developed across different countries (Jones et al., 2012), as reviewed above, the equal sign ("=") has become a universally recognized symbol for indicating mathematical equality (Cajori, 1928). That is, mathematically, the meaning of the symbol, "=", is the same in every country. Therefore, to investigate Chinese primary school students' understanding of the meaning of equal sign and equation structure, the instrument developed by Stephens et al. (2013) was directly adopted in the study (see Fig. 1). All the items in the instrument were translated into Chinese by the first author and were further checked by three experienced Chinese primary school mathematics teachers. According to their suggestions, slight modifications were made to make sure that Chinese primary school students can fully understand each item.

Item 1 in Stephens et al. (2013)'s study was originally designed by Knuth et al. (2006) to investigate students' interpretation of the equal sign. As shown in Figure 1, the first question of this item required students to name the equal sign symbol and the second question required students to use their own word to interpret the symbol. The main reason to design this item in this way was to preempt students from using the name of the symbol to answer the second question (Knuth et al., 2006).

Items 2 and 3 in Stephens et al. (2013)'s study was adapted from Carpenter et al. (2003) to investigate primary school students' understandings of the equal sign "in use" and their recognition of underlying equation structure as well. The original intention of Stephens et al. (2013)'s was to investigate that whether or not that students' with different view of the equal sign will answer these two items differently. For example, students with operational view of the equal sign would add the numbers on the left side of the equal sign on Item 2 and fill 10 or 8 in the blanks on the right side. However, students with relational-computational view or relational-structural view of the equal sign could provide different reasons for their answers to these two items.

Item 4 in Stephens et al. (2013)'s study was designed in arithmetic context to meet the real situation in primary school mathematics education to assess students' understanding of the preservation of an equivalence relation. Students who recognize the preservation can determine that the second equation is true without computing and comparing the total on both sides. In contrast, students who do not recognize the preservation will compute the total on both sides of the second equation without considering the first equation.

3.3. Data analysis

Students' responses to each of the items as shown in Figure 1 were analyzed first qualitatively and then quantitatively. To analyze students' responses qualitatively, all the authors and two experienced primary school mathematics teachers read all students' responses repeatedly and discussed some common responses made by students. After that, the meaning of these common responses were further discussed with referring to the code system developed in other similar previous studies (e.g., Alibali et al., 2007; Knuth et al., 2006; Stephens et al., 2013) and a code system used for this study was developed.

1. In the number sentence 3+4=7,
   a) what is the name of the symbol "="? b) What does the symbol "=" mean?
2. Fill in the blanks with the value that makes the following number sentences true and explain how did you get your answer?
   a) 7+3=___ + 4 b) 5+3=___+ 3
3. Circle True or False and explain your choice?
   a) 57+22=58+21 True False
   b) 39+121=121+39 True False
4. The following number sentence is true: 15+8=23. Is 15+8+12=23+12 true or false? How do you know?

Figure 1. Equal signs and equation structure problems

First of all, for all the items as mentioned above, if students did not give any responses or they stated that "I don't know" were coded as "*no response/don't know*". For the Items 2–4, if students only simply gave an answer without any explanation were coded as "*answer only*". For other responses, they were coded according to the following code system:

1) Equal sign interpretation coding. Students' responses to the part b of Item 1 were coded as:
   1) *relational*, if the meaning "the same as" is embodied in their explanations;
   2) *operational*, if they expressed the idea that the equal sign means "add the number", "the answer", or "the total"; and
   3) *equals*, if they only simply mentioned something like "equal to" or "equals".

2) Equation structure understanding coding. Similar to the code system developed in Stephens et al (2013)'s study, students' responses to Items 2 and 3 were coded as correct if they gave right answers. Further, students' explanations or strategies were coded as:
   1) *relational-structural*, if students' explanations were based on underlying structure, like saying that 6 should be filled in the blank in 7+3=__+ 4 because addend 4 is 1 more than addend 3 so that the number in the blank on the right side of the equal

sign should be 1 less than 7. Or students use the commutative property of addition to explain why "39+121=121+39" is true;
2) *relational-computational*, if students justify their answers by the strategy of computation; and
3) *operational*, it students treated the equal sign as a signal to write the answer, for example, if students filled 10 in the blank in 7+3=__ + 4.

The first two codes developed in Stephens et al (2013)'s code system were adopted to code students' responses to Item 4 since in the present study, no students used operational strategy to justify their answers. Firstly, students' responses to Item 4 were coded as correct if students answered that the second equation is true. Then, students' strategies to justify their answers to this item were also coded as:

1) *relational-structural*, if students explained that adding 12 to both sides of the original equation preserves the equivalence relation or students explained in a more general way, "adding or subtracting the same value on both sides of an equation preserves the equivalence relation";
2) *relational-computational*, if students justify the second equation was true by computing the total of each side of the equation.

To establish reliability of the coding procedure, the whole coding procedure was carried out by the first author and one of the experienced mathematics teachers who joined to develop the coding system. The agreement between the two coders was 100% for coding the correctness for all the items, 98% for coding students' understanding of the equal sign, at least 96% for coding students' strategies for Items 2–4. The discrepancies were discussed until full agreement was reached. After qualitative analysis, percentage of each category of students' responses to each item was calculated and Chi-square tests were further performed to explore gender and grade differences on each item.

4. RESULTS AND DISCUSSION

This section mainly reports Chinese primary school students' performance on each item and what kind of strategies they employed to work with each item. Other results, including findings of the categories of "*no response/don't know*" and "*answer only*", are not reported in the sub-sections below since the focus of the study is to investigate students' understanding of the equal sign and equation structure.

4.1. Item 1: Equal sign interpretations

As shown in Table 1, the majority of Grade 3 and 4 students in the study provided an operational definition of the equal sign (e.g., "it means the sum of two numbers" (Grade 4 student), or "it means the total you need to write" (Grade 3 student)), whereas substantially fewer Grade 3 and 4 students provided a relational definition of the equal sign (e.g., "it means that the value on its both sides is equal" (Grade 4 student), or "it means the same" (Grade 3 student)). For Grade 5 students, around 45% of them made an operational definition to the equal sign (e.g., "it means the total of an operation like addition, subtraction, multiplication, and division") and more than 30% of them provided a relational definition (e.g., "it means that its left side is equal to its right side").

Table 1. Proportion of students at each grade who made each type of the equal sign definition

Definition	Grade 3	Grade 4	Grade 5
Relational	3.6%	6.3%	33.6%
Operational	63.6%	68.8%	45.5%
"Equals"	30.0%	24.1%	19.1%

Chi-square test results show that there exist significant differences among Grade 3, 4 and 5 participants ($\chi^2 = 51.42$, $p < 0.000$) and no differences between boys and girls ($\chi^2 = 0.91$, $p < 0.635$). As shown in Table 1, compared with Grade 3 and 4 students, relatively less Grade 5 students provided operational definition. On the contrary, substantially more Grade 5 students could make a relational definition. The big change between Grade 3 and 4 and Grade 5 students may suggest that there is a huge improvement of students' understanding of the equal sign at Grade 5. The reason for the change may mostly come from the influence of textbook content. In the second semester of Grade 4, students start to learn some principles of four arithmetic operations. Therefore, for Grade 5 students in this study, they already learned some of the properties of the equal sign, like symmetry and reflexivity. Learning experience like this should have already deepened Grade 5 students' relational understanding of the equal sign.

In addition, findings found in this study are quite different from the findings found in previous studies in the US. In Stephens et al. (2013)'s study, only 1% Grades 3 & 4 and 5% Grade 5 American students provided relational definition and around 40% Grade 3 and 4 and 56% Grade 5 students provided operational definitions. In Knuth et al. (2006)'s study, they found out that only 32% Grade 6 and 31% Grade 8 American students could define the equal sign relationally and more than half of them make operational definition to the equal sign. Differences of content arrangement and the ways to introduce the equal sign in textbooks between China and the US may be the main factor which leads to the differences of students' understanding of the equal sign.

Li et al. (2008) found that multiple concrete contexts were used to illustrate the concept “the same as” even in Grade 1 and the formal symbol “=” was introduced together with the symbol “>” and “<” in three different ways: concrete, symbolic, and verbal. The comparison of the equal sign with other relational symbols, however, has been found out to be an effective way to facilitate students’ understanding of the equal sign in previous studies. For example, Hattikudur & Alibali (2010) found that provide students an opportunity to compare the equal sign (“=”) with greater than (“>”) and less than (“<”) symbols in primary school can promote students’ deeper conceptual understanding of the equal sign than teach students the equal sign alone. Thus, this way of introducing the equal sign in Chinese mathematics textbooks can make Chinese primary school students understand the equal sign relatively more deeply.

In addition, a main goal of teaching algebraic concepts in primary school in China is to deepen students’ understanding of quantitative relationships, both numerically and symbolically (Cai, Ng & Moyer, 2011). To achieve this goal, equations and equation solving permeate the curriculum in Grades 1–4 before they are formally introduced in the first semester of Grade 5. The emphasis of the relationships in earlier grades may also facilitate the development of Chinese primary school students’ relational understanding of the equal sign. The differences as identified in the study may further suggest an assumption that due to the influence of curriculum and textbook, Chinese primary school students may develop the relational understanding of the equal sign much earlier than their American counterparts. However, one thing need to be pointed out is that participants in Stephens et al. (2013)’s study did not receive any formal algebra instructional intervention; therefore, more systematical and comparative studies are needed on this topic before a clear conclusion could be made.

4.2. Item 2: Open number sentences

As for the two questions on Item 2, almost all the students responded correctly, that is, filled 6 or 5 in the blank respectively. For the same two questions, Stephens et al (2013) found that less than 2% Grade 3 students, around 25% Grade 4 and around 55% Grade 5 students in the US could make right responses. Differences between the present study and Stephens et al (2013)’s study are consistent with the differences as identified by Li et al. (2008). They also found that Chinese Grade 6 students perform much better than American Grade 6 students on the items designed to investigate students’ understanding of the equal sign, such as the equation in the format of 6+9=__+4.

However, similar to Stephens et al (2013)’s findings, almost the same amount of students made correct responses to the two questions even the structure of Item 2a is different from the structure of Item 2b. Students’ solving strategies were further examined to investigate how students approached these two questions differently (see Table 2). As shown in Table 2, the majority of students used computational strategy to solve both of the two questions. In the meantime, similar to what was found in Stephens et al (2013)'s study, relatively more students used structural strategies when they were working on Item 2b than with Item 2a. This suggests that Chinese primary school students' understanding of the equal sign also depends on the context in which the conception is elicited (McNeil & Alibali, 2005).

Table 2. Proportion of students at each grade who used each type of problem-solving strategy on Items 2a and 2b

Strategy code	Grade 3	Grade 4	Grade 5
Item 2a: 7+3=__+4			
Structural	0.9%	8.0%	4.5%
Computational	85.5%	85.7%	93.6%
Operational	10.9%	4.5%	1.8%
Item 2b: 5+3=__+3			
Structural	5.5%	16.1%	14.5%
Computational	80.9%	76.8%	80.4%
Operational	10.0%	4.5%	1.8%

In addition, the results of Chi-square tests show that there exist significant differences among Grade 3, 4 and 5 participants on Item 2a (χ^2 = 15.21, p=0.004) and on Item 2b (χ^2 = 13.44, p=0.009) and no differences between boys and girls on both items. As shown in Table 2, relatively more Grade 4 and 5 students used structural strategy and relatively more Grade 3 students used operational strategy (see Figure 2 for the representative cases). As the cases shown in Figure 2, for strategies used by students to work on Item 2a, the Grade 5 example student could recognize that addend 4 is 1 more than addend 3, so the unknown must be 1 less than 7 without any computation. This suggests that this example student already had a comparative relational view of the equal sign as reviewed above (Rittle-Johnson et al., 2011). However, as shown in Table 2, compared with Grade 5 students, relatively more Grade 4 students used structural strategy and less of them used computational strategy. This might due to the factor as argued by Alibali et al. (2007) that students' relational understanding of the equal sign is fragile and various kinds of the understandings coexist during a period of time. The coexistence of mature understanding and less mature understanding will make students hold on to their original conception of the equal sign in some contexts (McNeil & Alibali, 2005).

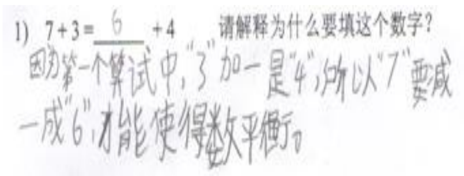
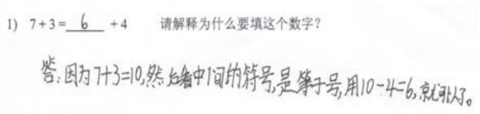
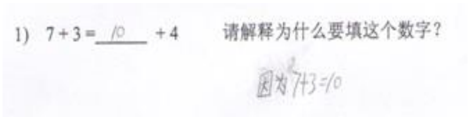
Grade 5 	Grade 5 1) 7+3=____ + 4, please explain your answer. Explanation: because “3” plus “1” is “4”, so “7” should be subtracted “1” to make “6”, therefore, the results on both sides can balance.
Grade 4 	Grade 4 1) 7+3=____ + 4, please explain your answer. Explanation: because 7+3=10, and the equal sign is in the between, so 10-4=6.
Grade 3 	Grade 3 1) 7+3=____ + 4, please explain your answer. Explanation: because 7+3=10.

Figure 2. Examples of different strategies used on Item 2a by Grade 3, 4, and 5 students

4.3. Item 3: True/false number sentences

More than 85% Grade 3 students, around 98% Grade 4 students, and all the Grade 5 students made correct responses to the two questions on Item3. Again, more Chinese primary school students made correct responses to the two questions than their American counterparts did as found in Stephens et al. (2013)’s study. They found that around 10% of Grade 3 students, less than 40% Grade 4 students, and less than 70% of Grade 5 students made correct responses to the same two questions.

Table 3. Proportion of students at each grade who used each of type problem-solving strategy on Items 3a and 3b

Strategy code	Grade 3	Grade 4	Grade 5
Item 3a: 57+22=58+21			
Structural	6.5%	27.3%	36.7%
Computational	86.1%	72.7%	63.3%
Operational	4.6%	0	0
Item 3b: 39+121=121+39			
Structural	26.2%	59.1%	76.9%
Computational	67.3%	40.9%	23.1%
Operational	3.7%	0	0

Strategies employed by students to work on each questions on Item 3 were further examined and the findings were summarized in Table 3. As indicated in Table 3, comparatively speaking, Grade 3 students were more likely to use computational strategy on both of the two questions and relatively more Grade 4 and 5 students tended to use structural strategy on them. The results of Chi-square tests show that there exist significant differences among Grade 3, 4 and 5 participants on Item 2a ($\chi^2 = 42.08$, $p < 0.000$) and on Item 2b ($\chi^2 = 64.77$, $p < 0.000$) and no differences between boys and girls on both items.

Compared with the findings found in Stephens et al. (2013)'s study which was conducted in the US, more Chinese students were found to tend to use computational or structural strategy on these two questions and less Chinese students were found to use operational strategy. Stephens et al. found that around 88% Grade 3 students, 65% Grade 4 students, and 35% Grade 5 students used operational strategy on the same two questions, which was only used by very few Grade 3 Chinese students in this study. In addition, they found that none Grade 3 students, less than 6% Grade 4, and less than 18% Grade 5 students used structural strategy on these two questions, which was used by many Grade 4 and 5 students, especially on Item 3b. As explained above, one main factor which leads to the differences is that the Chinese primary school students have already learned some of the properties of the equal sign and addition, such as symmetry and reflexivity. Actually, some students did use the commutative property of addition; the sum stays the same when the order of the addends is changed, to explain why 39+121=121+39.

In addition, consistent with what identified in Stephens et al. (2013)'s study, Chinese students at each grade were more likely to use structural strategy with Item 3b than they were with Item 3a. As shown in Table 3, more students used computational strategy while they were working with Item 3a; however, more students used structural strategy while they were working with Item 3b. Findings like this further support the argument as proposed by McNeil & Alibali (2005) that students' interpretations of the equal sign depend on the context in which the equal sign is elicited.

4.4. Item 4: Equivalent equations

Item 4 was originally designed by Stephens et al. (2013) to investigate students' deeper understanding of the equal sign since to make correct response; students need to recognize that performing the same operation on both sides of an equation at the same time will preserve the equivalent relationship. In other words, students need to recognize that 15+8=23 implies 15+8=12=23+12 without any computation, but they need to have a structural understanding of equations to justify their solution. Around 80% Grade 3 and 91% Grade 4 students, and more than 95% Grade 5 made right judgment on Item 4. Again, more Chinese primary school students made correct responses to this item than their American counterparts did as found by Stephens et al. (2013). They found that around 10% of Grade 3 students, 30% Grade 4 students, and less than 50% of Grade 5 students made correct responses to this item.

Table 4. Proportion of students at each grade who used each type of problem-solving strategy on Item 4

Strategy code	Grade 3	Grade 4	Grade 5
Item 4: 15+8=23→15+8+12=23+12			
Structural	11.8%	34.3%	61.7%
Computational	68.6%	56.5%	34.6%

Since students could also perform computations to determine whether or not 15+8+12=23+12 is true without the consideration of the first equation 15+8=23. Students' strategies were further analyzed. The results as shown in Table 4 indicate that Grade 3 and 4 students were more likely to carry out computation to justify their answers, however, Grade 5 students were more likely to use structural strategy to determine that 15+8+12=23+12 is true (see Figure 3 for representative examples). The results of Chi-square tests show that there exist significant differences among Grade 3, 4 and 5 participants on this item (χ^2 = 60.44, $p < 0.000$) and again, no differences between boys and girls on this item.

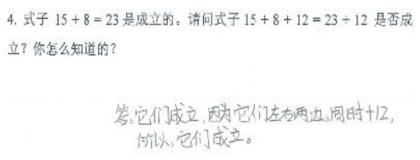
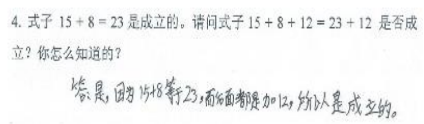
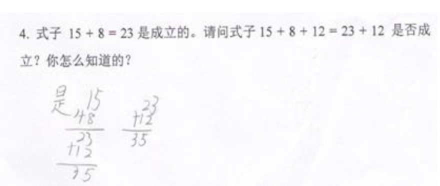
Grade 5  	Grade 5 4. The following number sentence is true: 15+8=23. Is 15+8+12=23+12 true or false? How do you know? Answer: True. Because 12 is added to the left and right side of the first equation at the same time, so it is true.
Grade 4  	Grade 4 4. The following number sentence is true: 15+8=23. Is 15+8+12=23+12 true or false? How do you know? Answer: True. Because 15 plus 8 equals to 23, 12 is added to the both sides, so it is true.
Grade 3  	Grade 3 4. The following number sentence is true: 15+8=23. Is 15+8+12=23+12 true or false? How do you know? Answer: True.

Figure 3. Examples of different strategies used on Item 4 by Grade 3, 4, 5 students

In addition, Chinese primary school students' performance in the present study is quite different from the performance of their American counterparts (Stephens et al., 2013). In their study, the structural strategy was used by only 5% Grade 4 and 9% Grade 5 students and only 2% Grade 3 students, 14% Grade 4 students, and 32% Grade 5 students used computational strategy. However, more than 30% of students in their study used operational strategy, which was not found to be used by Chinese primary school students in the present study. For a similar question, Knuth et al. (2005) also found that only 12% Grade 6, 17% Grade 7, and 34% Grade 8 American students could recognize the equation's equivalence without any computing, which requires students to have a relational understanding of the equal sign. Differences like this may again imply that compared with their American counterparts, Chinese primary school students hold a relatively more mature understanding of the equal sign and this mature understanding might develop at an earlier grade.

5. CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS

With the adoption of the instrument designed in Stephens et al. (2013)'s study, the present study investigated Chinese primary school students' understanding of the equal sign and equation structure. The study is so far, one of the very few studies focusing on systematically exploring how Chinese primary school students understand the equal sign and the equation structure. Generally, like what have been found in most of the Western studies, the operational understanding of the equal sign dominates Chinese primary school students' understanding of the equal sign and the majority of students tend to employ computational strategy when they work with equations. However, findings of the present study indicate that Chinese primary school students were likely to exhibit a relational understanding of the equal sign as grade level increases, which is somehow different from what has been found out in previous studies (e.g., Alibali et al., 2007; McNeil, 2007). Generally, Grade 5 students' understanding of the equal sign was found to be more mature than that of Grade 3 and 4 students. In addition, no gender differences were identified in this study.

In the meantime, as reviewed above, although it has repeatedly found in literature that American primary school students, or even middle school students, tend to hold an operational view of the equal sign, a part of Grade 4 and 5 Chinese students in this study were found to start to hold a relational view of the equal sign. In addition, many Grade 4 and 5 students could recognize the underlying structure of some arithmetic equations and could use more sophisticated strategies to work with equations. Differences like this might suggest that Chinese primary school students demonstrate a relational understanding of the equal sign and a strong structural sense of equations much earlier, even start from Grade 3 or Grade 4. In addition, consistent what have identified in previous studies, the equation structure acts as an important factor which will influence students' interpretation and understanding of the equal sign.

However, this study only investigated Grade 3–5 students' understanding of the equal sign and equation structure. Even it found that Grade 4 or 5 students demonstrated a relative strong structural sense of equation structure and could define the equal sign structurally, it is not clear how stable this sense is since previous studies have not found consistent findings so far (e.g., Alibali et al., 2007; McNeil, 2007). Future studies could consider to involve Grade 6 students at least or even secondary school students to explore the stability of Chinese students' understanding of the equal sign. In addition, a longitudinal study design which follows the same group of students for several years in China will provide deeper and more convincing information about for Chinese primary school students, from which grade the relational view of the equal sign starts to emerge and from which grade, it starts to be stable. Moreover, this study only involved students who use the same series textbooks in China, since textbooks have been argued as main influence for the development of students' understanding of the equal sign, future studies could consider to involve students who use several different series of textbooks and compare the differences. Findings like this will make people understand more deeply about how to arrange the content effectively to improve students' understanding of equal sign. Last, this study only employed written assessment to investigate students' understanding, future studies could consider to conduct interviews to collect more information about why students define the equal sign in a certain way and why they employed a certain strategy on a particular item. Interview information will further provide useful resources for the investigation of reasons which influence the development of students' understanding of the equal sign.

REFERENCES

Alibali, M. W.; Knuth, E. J.; Hattikudur, S.; McNeil, N. M. & Stephens, A. C. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Math. Think. Learn.* **9(3)**, 221–246. ME **2007b.**00352

Asquith, P.; Stephens, A. C.; Knuth, E. J. & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Math. Think. Learn.* **9(3)**, 247–270. ME **2007b.**00353

Baroody, A. J. & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *Elementary School Journal* **84**, 199–212.

Behr, M.; Erlwanger, S. & Nichols, E. (1980). How children view the equal sign. *Math, Teaching*, **92**, 13–15. ME **1981x.**00631

Cai, J.; Ng, S. F. & Moyer, J. (2011). Developing students' algebraic thinking in earlier grades: Lessons from China and Singapore. In: J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 25–41). Berlin: Springer. ME **2011d.**00630

Cajori, F. (1928). *A history of mathematical notations: Vol. I, Notations in Elementary Mathematics.* LaSalle, IL, USA: Open Court.

Carpenter, T. P.; Franke, M. L. & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in the elementary school*. Portsmouth, NH, USA: Heinemann.

Falkner, K. P.; Levi, L. & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teach. Child. Math.* **6(4)**, 232–236. ME **2000e.**03407

Hattikudur, S., & Alibali, M. W. (2010). Learning about the equal sign: Does comparing with inequality symbols help?. *Journal of experimental child psychology* **107(1)**, 15–30.

Jones, I.; Inglis, M.; Gilmore, C. & Dowens, M. (2012). Substitution and sameness: Two components of a relational conception of the equals sign. *Journal of experimental child psychology* **113(1)**, 166–176.

Jones, I.; Inglis, M.; Gilmore, C. & Evans, R. (2013). Teaching the substitutive conception of the equals sign. *Res. Math. Educ.* **15(1)**, 34–49. ME **2014c.**00701

Jones, I. & Pratt, D. (2012). A substituting meaning for the equals sign in arithmetic notating tasks. *J. Res. Math. Educ.* **43(1)**, 2-33. ME **2012f.**00762

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics* **12(3)**, 317–326. ME **1982x.**00022

_____ (2007). Learning and teaching algebra at the middle school through college levels. In: F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707–762). Charlotte, NC, USA: Information Age. ME 06332594

Knuth, E. J.; Alibali, M. W.;, McNeil, N. M.; Weinberg, A. & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & variable. *ZDM, Zentralbl. Didakt. Math.* **37(1)**, 68–76. ME **2009b.**00392

Knuth, E. J.; Stephens, A. C.; McNeil, N. M. & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *J. Res. Math. Educ.* **37(4)**, 297–312. ME **2008f.**00383

Li, X.; Ding, M.; Capraro, M. M. & Capraro, R. M. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. *Cogn. Instr.* **26(2)**, 195–217. ME **2012a.**00383

Matthews, P.; Rittle-Johnson, B.; McEldoon, K. & Taylor, R. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *J. Res. Math. Educ.* **43(3)**, 316–334. ME **2012f.**00766

McNeil, N. M. (2007). U-shaped development in math: 7-Year-olds outperform 9-year-olds on equivalence problems. *Developmental Psychology* **43(3)**, 687–695.

McNeil, N. M. & Alibali, M. W. (2005). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition & Development* **6**, 385–406.

McNeil, N. M.; Fyfe, E. R.; Petersen, L. A.; Dunwiddie, A. E. & Brletic-Shipley, H. (2011). Benefits of practicing 4 = 2 + 2: Nontraditional problem formats facilitate children's understanding of mathematical equivalence. *Child development* **82(5)**, 1620–1633.

McNeil, N. M.; Grandau, L.; Knuth, E.; Alibali, M.; Stephens, A.; Hattikudur, S. et al. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction* **24(3)**, 367–385.

Powell, S. R. (2012). Equations and the equal sign in elementary mathematics textbooks. *Elementary School Journal* **112(4)**, 627–648.

Rittle-Johnson, B., Matthews, P., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence. A construct-modeling approach. *Journal of Educational Psychology*, **103**, 85–104.

Seo, K.-H. & Ginsburg, H. P. (2003). "You've got to carefully read the math sentence": Classroom context and children's interpretations of the equals sign. In: A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 161–187). Mahwah, NJ: Lawrence Erlbaum Associates. ME **2003c.**02295

Stephens, A. C.; Knuth, E. J.; Blanton, M. L.; Isler, I.; Murphy Gardiner, A. & Marum, T. (2013). Equation structure and the meaning of the equal sign: The impact of task selection in eliciting elementary students' understandings. *J. Math. Behav.* **32(2)**, 173–182. ME **2013c.**00613

Stigler, J. W., & Baranes, R. (1988). Culture and mathematics learning. In: E. Z. Rothkoft (Ed.), *Review of research in education* (vol. 15, pp. 253–306). Washington, DC: American Educational research Association.