# Damage Detection in High-Rise Buildings Using Damage-Induced Rotations

Seung Hun Sung\*, Ho Youn Jung\*, Jung Hoon Lee\* and Hyung Jo Jung\*<sup>⊤</sup>

Abstract In this paper, a new damage-detection method based on structural vibration is proposed. The essence of the proposed method is the detection of abrupt changes in rotation. Damage-induced rotation (DIR), which is determined from the modal flexibility of the structure, initially occurs only at a specific damaged location. Therefore, damage can be localized by evaluating abrupt changes in rotation. We conducted numerical simulations of two damage scenarios using a 10-story cantilever-type building model. Measurement noise was also considered in the simulation. We compared the sensitivity of the proposed method to localize damage to that of two conventional modal-flexibility-based damage-detection methods, i.e., uniform load surface (ULS) and ULS curvature. The proposed method was able to localize damage in both damage scenarios for cantilever structures, but the conventional methods could not.

Keywords: Damage Localization, Damage-Induced Rotation, Cantilever-Type Structures, Modal Flexibility

#### 1. Introduction

Civil infrastructure is exposed to severe external loads that cause cumulative damage over time. In order to avoid catastrophic failure, and to inform maintenance to prolong the life of such structures, it is necessary to obtain explicit data about their condition. To evaluate the status and the remaining lifetime of structures, structural health monitoring (SHM) has received considerable attention in civil-engineering research communities. Due to improved SHM techniques, novel methodologies involving changes in dynamic parameters have been presented [1-3]. Vibration-based damage evaluation includes three processes: (1) examining the damage, (2) localizing the damage, and (3) evaluating the severity of the damage in specific locations.

Due to its high sensitivity, modal flexibility has recently become a promising descriptor of damage [4]. In an earlier study, change of the modal flexibility was used to detect damage [5]. Notable variants have used the uniform load surface (ULS) estimated from modal flexibility [6,7], ULS curvature [8], and the damage locating vector (DLV) based on changes in modal flexibility using an intact finite-element model [9-12]. The ULS method has much less truncation effect, and fewer experimental errors, than do changes in modal flexibility [13]. However, its sensitivity to damage of cantilevertype structures is not adequate to localize damage. For the localization of adjoining damage sites, the ULS curvature method is preferable because of its high sensitivity to multiple damage locations, even if close to each other [13]. Moreover, the DLV method localizes damage for elements having negligible internal forces. On the other hand, the ULS curvature method is vulnerable to noise in the process of curvature estimation and the DLV method needs a finite-element model to estimate damage [13].

<sup>[</sup>Received: November 13, 2014, Revised: December 15, 2014, Accepted: December 18, 2014] \*Department of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea †Corresponding Author: hjung@kaist.ac.kr

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In this study, the new modal-flexibility-based damage-detection method, using damage-induced rotation (DIR) is proposed. It can be applied to general cantilever-type structures. The newly proposed method (npDIR) has several specific advantages. First, npDIR is more sensitive to damage than deflection-based damage-detection approaches, and more robust to noise than curvature-based damage-detection approaches. Second, npDIR is widely applicable to general cantilever-type structures (e.g., shear structures, uniform bending structures, and non-uniform bending structures) that lack an intact finiteelement model.

In this paper, we fist present the theory for the estimation of modal flexibility from vibration data. Next, we present the damage-localization criteria for using DIR. Finally, we explain the numerical simulation result for a 10-story building model, by comparing the results from npDIR with those generated using a conventional modal-flexibility-based damage-detection method.

## 2. Theory

2.1 Deflections and Rotations Estimated by Modal Flexibility

The modal flexibility matrix Gm using m lower modes can be obtained as

$$\mathbf{G}_{\mathrm{m}} = \boldsymbol{\Phi}_{\mathrm{m}} \boldsymbol{\Lambda}_{\mathrm{m}}^{-1} \boldsymbol{\Phi}_{\mathrm{m}}^{\mathrm{T}} \tag{1}$$

where is  $\Lambda_m = [\langle \omega_i^2 \rangle]$ ,  $\omega_i$  is the *i*-th natural frequency,  $I = 1, 2, \dots, m$ ;  $\Phi_m = \{\phi_1, \phi_2, \dots, \phi_m\}$ ; and  $\phi_i$  is the *i*-th mass-normalized mode shape, obtained by the existing mass-normalization method [14-18]. The deflections under an arbitrary load f using modal flexibility matrix can be estimated as

$$\mathbf{u} = \mathbf{G}_{\mathrm{m}}\mathbf{f} \tag{2}$$

where u is the deflection vector corresponding



Fig. 1 Uniform load surface (ULS) [19]

to the force vector f.

The ULS is used as the force vector f to produce modal flexibility-based deflections, as shown in Fig 1. More detailed information on ULS can be found in previous studies [6,7].

Using modal-flexibility-based deflections, the rotation between sensor locations can be estimated as

$$\tan \theta_i \cong \theta_i = \frac{\mathbf{u}_{i+1} - \mathbf{u}_i}{l_i} \tag{3}$$

where  $\theta_i$  and  $u_i$  are the rotation and the deflection obtained by modal flexibility, respectively.

#### 2.2 Damage Localization Using DIR

From Hooke's law, the relationship between the deflection  $u_0$  and the applied force F can be expressed as

$$K_0 u_0 = F \tag{4}$$

where  $K_0$  is the structural stiffness matrix in the intact state.

A similar relationship under the same externalforce F, with a reduction in the stiffness-matrix  $\Delta K$  due to damage, can be expressed as

$$(K_0 - \Delta K)(u_0 + \Delta u) = F$$
(5)

where  $\Delta u = u^{D} \cdot u^{I}$  is the damage-induced deflection (DID), and  $u^{I}$  and  $u^{D}$  are deflections for intact and damaged, respectively.

By subtracting Eq. (4) from (5), the general equation of the DID can be obtained as

$$\Delta u = G_D (\Delta K u_0) = G_D \Delta F$$
(6)

where  $G_D = (K_0 - \Delta K)^{-1}$  is the structural flexibility matrix in the damaged states, and  $\Delta F = \Delta K u_0$ is the force induced by the stiffness loss of the intact system producing the DID of the damaged system. Thus,  $\Delta F$  may be considered the resisting force lost to damage [19].

The DIR can be evaluated from (3) and (6) as

$$\Delta \boldsymbol{\theta} = \operatorname{diff}(\Delta \mathbf{u}) / \mathbf{L} = \boldsymbol{\theta}^{\mathrm{D}} - \boldsymbol{\theta}^{\mathrm{I}}$$
(7)

where  $\Delta \theta$  is the DIR, and  $\theta^{I}$  and  $\theta^{D}$  are rotations for intact and damaged, respectively.

For a general cantilever-type structure with column damage  $\Delta K$ , the forces  $\Delta F$  due to damage, proportional to the reduction in stiffness, can be obtained as

$$\Delta \mathbf{F} = \Delta \mathbf{K} \mathbf{u}_{0} = \begin{cases} \mathbf{0} \\ \alpha_{e} \mathbf{f}_{e} \\ \mathbf{0} \end{cases}$$
(8)

where  $\Delta K = \text{diag} (0, \alpha_e \text{ke}, 0), \alpha_e$  is the damage ratio,  $0 < \alpha_e < 1$ , ke is the elementary stiffness matrix representing the intact columns at the damaged system, and fe = keue = {V, M,-V, M} is the stress resultant of the element in the intact state, as shown in Fig. 2(a).

Eqs. (6)-(8) indicate that DID and DIR can be evaluated by applying  $\alpha_e f_e$  to the damaged structure. It is notable that  $f_e$  only acts on the damaged location, not on the rest of the structure; and that  $f_e$  is a self-equilibrium force. Thus, DID and DIR occurred initially at the damaged location, as depicted in Fig. 2(b).



Fig. 2 Damage-induced rotation: (a) Intact structure, (b) Damage-induced rotation



Fig. 3 Types of cantilevered structure: (a) Shear structure, (b) Uniform bending structure, (c) Non-uniform bending structure

Cantilevered structures have different behaviors (according to their type) under the forces  $\Delta F$ , as shown in Fig. 3. However, as mentioned above, each structure has in common that additional deflection and rotation induced by damage initially occurs at the damaged location. The trend of subsequent DID and DIR, after its initial appearance, depends on the type of cantilevered structure involved. In other words, the abrupt appearance of  $\Delta u$  or  $\Delta \theta$  in a damaged cantilevered structure at the i-th sensor location indicates that the damage is located at the *i*-th sensor location. In this study,  $\Delta \theta$  was used as the damage metric for damage detection of cantilever-type structures due to its high sensitivity, compared to  $\Delta u$ . Thus, this simple damage-rotation relationship is the keystone of the proposed method.

2.3 Damage Localization Using DIR with Noisy Measurement

From the relationship between damage and DIR in the previous section, damage localization can be performed using DIR  $\Delta\theta(i)$  as follows: Damage occurs at the *i*-th sensor location  $\Leftrightarrow \Delta$  $\theta(i)$  initially occurred at *i*-th sensor location (9).

However, unexpected DIR at the intact location may occur initially due to inevitable measurement noise. Therefore, statistical approaches are preferred to reduce false alarms. In this study, an outlier index  $Z_i$  of the DIR was utilized to signal the existence of damage as follows: Damage-existence alarm issues if  $Z_i > Z_i$  <sup>Threshold</sup> for any *i* (*i* is counted from '1'), from (9)

$$Z_{i} = \frac{\theta(i) - \theta_{0}(i)}{\sigma(\theta_{0}(i))}$$
(10)

$$\Delta \theta(i) = \theta(i+1) - \theta(i) \tag{11}$$

where  $\theta(i)$  is the concurrent rotation,  $\overline{\theta}_0(i)$  is the mean value of the rotation for the intact structure, and  $\sigma(\theta_0(i))$  is the standard deviation of rotation for the intact structures, respectively.

When a damage alert has been issued, the damage location can be identified using the following conditions:

- Once a damage-existence alarm is initially issued at the location of the *i*-th sensor, indices above the *i*-th sensor location are set to zero.
- (2) Damage is located at the *i*-th sensor location about which the existence alarm was issued The proposed damage detection procedure (npDIR) is summarized in Fig. 4.



Fig. 4 Flow chart of the proposed damage detection method

### 3. Numerical Simulation

# 3.1 Numerical Model Properties and Simulation Technique

To evaluate the feasibility of the proposed method, numerical simulations were conducted for a 10-story building model. The material properties of the numerical model are shown in Table 1 and Fig. 5. The numerical model was composed of 1400 elements and 1974 nodes, with beam and shell elements. The model was constructed using ANSYS, and sensors were assumed to be installed at each floor. Under an ambient vibration test, 1% noise of measured natural frequency, and 8-10% noise of measured mode-shape would be expected [20,21]. By using this signal-processing technique, it is possible to filter out as much as 5% of the noise [22]. Therefore, the simulation was done considering 2% of the normally distributed mode-shapes and 1% of the random noise, as natural frequencies.

Thus, the measured natural frequency is contaminated as follows:

$$\widetilde{\omega}_i = \omega_i \times (1 + \alpha_{noise} \times randn(1))$$
(12)

where  $\omega_i$  is the exact natural frequency obtained from ANSYS,  $\alpha_{noise}$  is the noise level of 1% and *randn*(1) is a function in MATLAB that gives random numbers following normal distribution. The measured mode shapes were also simulated by adding noise into the exact mode shapes as follows:

$$\widetilde{\mathbf{\Phi}}_{i} = \mathbf{\Phi}_{i} + \alpha_{noise} \times randn(n, 1) \times \operatorname{var}(\mathbf{\Phi}_{i})$$
(13)

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where  $\Phi_i$  and  $\Phi_i$  are the mass-normalized mode-shape polluted by noise and exact mass-normalized mode-shape, respectively; and  $\alpha_{noise}$  is the noise level. Here, randn(n,1) is a function that generates a vector of normally distributed random numbers of zero mean and

Column (Beam element)			
Parameters Value			
Mass density $(\rho)$	7850 kg m <sup>-3</sup>		
Poisson's ratio $(\nu)$	0.28		
Elasticity modulus (E)	200 GPa		
Length (L)	0.2 m		
Moment of inertia for One column (I)	490.87 m <sup>4</sup>		
Floor (Shell ele	ment)		
Floor (Shell ele Parameters	ment) Value		
Floor (Shell ele Parameters Mass density (ρ)	ment) Value 7850 kg m <sup>-3</sup>		
Floor (Shell ele Parameters Mass density $(\rho)$ Poisson's ratio $(\nu)$	ment) Value 7850 kg m <sup>-3</sup> 0.28		
Floor (Shell ele Parameters Mass density ( $\rho$ ) Poisson's ratio ( $\nu$ ) Elasticity modulus ( $E$ )	ment) Value 7850 kg m <sup>-3</sup> 0.28 200 GPa		
Floor (Shell ele Parameters Mass density $(\rho)$ Poisson's ratio $(\nu)$ Elasticity modulus $(E)$ Area $(m^2)$	Value           7850 kg m <sup>-3</sup> 0.28           200 GPa           0.2 by 0.2		

Table 1 Structural model parameter



Fig. 5 Numerical model: (a) Locations of damage and assumed sensors, (b) ANSYS model

Table 2 Damage scenarios

Case	Damage location	Reduction in bending stiffness (EI) of the floor
IC	None	None
DC1	first floor	15%
DC2	third floor	35%

unit standard deviation in MATLAB. The var  $(\Phi_i)$  is the variance of a calculated massnormalized mode-shape at *n* measurement points for any mode *i*. Damage was simulated by reducing the bending stiffness (*EI*) of affected columns, as shown in Fig. 5 and Table 2. Two damage scenarios were considered: (a) Damage Case 1 (DC1): 15% *EI* reduction at the fixed support, and (b) Damage Case 2 (DC2): 35% EI reduction at the third floor. As previously mentioned, because measured natural frequencies under ambient conditions may be contaminated by measurement noise of about 1%, compared to the exact natural frequency, the severity of damage that induced a change in the first natural frequency more than about 2% was considered a threshold value.

#### 3.2 Results

For the intact and damage cases proposed in this paper, the numerical simulations were repeated 20 times to evaluate the structural modal parameter under practical conditions. In real structures, the number of measurable modes is limited due to the rigidity of structure and limits on the level of excitation. Therefore, to construct the modal-flexibility matrices of intact and damage cases, the first three natural frequencies and mass-normalized mode-shapes were used. These mass-normalized mode-shapes were constructed using the system mass matrix.

Table 3 and Figs. 6 and 7, describe changes in the modal parameters caused by progressive structural damage. The changes in the natural frequencies were 0.193%-2.658%. However, there were no noticeable changes in the MAC values. The 3-dimensional plot of the modal- lexibility matrix of intact areas is shown in Fig. 8(a), and changes in modal flexibility before and after damage, are shown in Fig. 8(b) and 8(c), respectively. By introducing the baseline parameters of the system, damage locations could not be adequately identified using changes in flexibility.

Rotations under ULS with one deviation were evaluated from modal-flexibility-based deflections

Casa	The first mode		
Case	$f_1$ (Hz)	$\Delta f_l/f_l$ (%)	MAC
IC	1.791	-	1.0000
DC1	1.749	2.345%	0.9999
DC2	1.755	2.010%	0.9999
Case	The second mode		
	$f_2$ (Hz)	$\Delta f_2/f_2$ (%)	MAC
IC	8.791	-	1.0000
DC1	8.682	1.234%	0.9999
DC2	8.774	0.193%	0.9999
Case	Т	he third mode	
	<i>f</i> <sub>3</sub> (Hz)	$\Delta f_3/f_3$ (%)	MAC
IC	23.33	-	1.0000
DC1	22.96	1.586%	0.9998
DC2	22.71	2.658%	0.9992

Table 3 Changes in modal parameter due to damage



Fig. 6 Changes in natural frequency due to damage with one deviation: (a) First mode, (b) Second mode, (c) Third mode



Fig. 7 Mode shapes: (a) First mode, (b) Second mode, (c) Third mode

before and after damage, as shown in Fig. 9. The solid lines in Fig. 9 and Fig. 10 indicate one deviation of rotation under ULS. Plot shifts were observed, which indicated that DIR occurred at the damaged floor for all the damage cases. The DIRs with one deviation, which was estimated from rotations before and after damage, are also plotted in Fig. 10. For DC1, it was clearly found that change in



Fig. 8 Matrix plots: (a) Flexibility of IC, (b) Change in flexibility between intact IC and DC1, and (c) Change in flexibility between intact IC and DC2

rotation initially occurred at the first floor. It was also apparent that DIR occurred abruptly at the damaged floor (third floor) for DC2. However, modal parameters contaminated by measurement noise, induced variance in rotations and DIRs that could affect the accuracy of damage detection. In addition, the exact locations of damage were not easily distinguishable (Fig. 10).

Therefore, the proposed damage index Zi was used to improve damage-detection performance and to carry out damage detection more conveniently. In Fig. 11, based on the normal distribution, the damage index Zi, with threshold value '2', was proposed to decrease the number of false alarms to less than about 2%. In the end, for all the damage cases, the index Zi accurately identified damaged locations. In other



Fig. 9 Rotation with one deviation under ULS: (a) IC, (b) DC1, (c) DC2



Fig. 10 Damage induced rotation values within one standard deviation: (a) IC, (b) DC1, (c) DC2



Fig. 11 Outlier index for damage detection: (a) DC1 from IC, (b) DC2 from IC

words, outlier indices of undamaged floors remained under the threshold level, indicating no damage. The threshold value of the index Zi is a user choice, in which false-positive errors are traded off against false-negative errors. High threshold values decrease the probability of false-positives, but increase the probability of false-negatives [19].

#### 3.3 Comparative Studies

The damage sensitivity of the proposed method was compared with conventional modal flexibility based damage detection method, like the ULS method [6,7] and the ULS curvature method [8].

Fig. 12 shows the damage indices with two deviations for DC1. The damage at the first floor was successfully identified by the proposed method and the ULS method. However, for the ULS curvature method, damage localization remains impossible, since measurement noises have adverse effect on the damage detection ability.

Fig. 13 shows the damage indices for DC2 with same deviations. For the ULS method, false-positive damage detection was reported at the fourth floor due to its insensitivity to damage. For the ULS curvature method, it also might be difficult to identify correct damage location due to its noise vulnerable characteristic.



Fig. 12 Estimates of damage location for DC1 from three different methods: (a) Proposed method, (b) ULS method, (c) ULS curvature method



Fig. 13 Estimation of damage location for DC2 from three different methods: (a) Proposed method, (b) ULS method, (c) ULS curvature method

In conclusion, the proposed method performed better in detecting damages compared to the other two methods in terms of sensitivity to damage and robust to measurement noises. Since the damage indices might be affected by the optional parameters like a weighting of each mode, the results does not mean the clear advantage of the proposed method.

#### 4. Conclusions

In this study, we proposed a new vibrationbased damage-detection method using DIR of modal flexibility for cantilever-type structures. The proposed method was evaluated using numerical simulation. From the simulation, it was found that DIR initially occurs at the location of a damaged sensor. Therefore, excessive change in rotation indicates the existence of damage in cantilever-type structures. Numerical simulations of a 10-story building model were conducted to validate the feasibility of the proposed method. Two damage scenarios were considered by changing the bending stiffness (EI) at different locations. The sensitivity of the proposed method to damage was compared with two conventional modal-flexibility-based damage-detection methods (i.e., ULS and ULS-curvature) to show the advantages of the proposed method. Finally, it is concluded that the performance of damage localization could be enhanced by using the proposed DIR-based damage-detection approach. It is also concluded that conventional damagedetection methods based on modal flexibility still have limitations for practical damage identification in cantilever-type structures.

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