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PI Controller Design of the Refrigeration System Based on Dynamic Characteristic of the Second Order Model

Young-Mi Jung*, Seok-Kwon Jeong*† and Joo-Ho Yang**

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Abstract: This paper deals with deterministic PI controller design based on dynamic characteristics for refrigeration system. The temperature control system of an oil cooler is described as a typical 2nd order model of the refrigeration system which has zeros in a transfer function. PI controller gains satisfying control specifications are represented by the dynamic characteristic functions using relationship between the parameters and the control specifications in the model. Phase margin was considered to increase robustness of the oil cooler control system. Furthermore, the influence of zeros in the model to the control specifications was analyzed in detail for improving control performance. The validity of the suggested PI controller design was investigated using the four types of gains which had been already confirmed their control performances through experiments.

Key Words : Refrigeration System, Dynamic characteristic, PI controller, Oil cooler control system

1. Introduction

Several controller design methods have been suggested for a refrigeration system. They are simply classified into two types, model based PID and artificial intelligent technique such as fuzzy logic. The PID controller is widely used in many industrial fields because of its reliable control performance in spite of simple logic.[1~3] The representative PID gain decision methods for the oil cooler control system are Matlab PID tuner[4] depending on trial and error manner, optimized method using an evaluation function such as IAE(Integral of the Absolute magnitude of the Error),[5,6] and GA(Genetic Algorithm)[7]. The Matlab tuner is actually easy to design appropriate gains and to confirm control performances in time domain. The IAE is suitable for obtaining optimum gain satisfying control specifications. Also, the GA can provide optimum gain through wide area search for solution in the case of complicated control specifications. However, all of them are only focused on optimized gain satisfying control specifications under the assumption that plant model is correct. Practically, it is really difficult to get the best control performance without gain retuning even though the optimized control gain is applied to the real system. Therefore, gain retuning is inevitable in final step of controller design because of difference between a used ideal model and a real system. The conventional PID controller design methods are not

*† Seok-Kwon Jeong(corresponding author) : Department of Refrigeration and Air-conditioning Engineering, Pukyong National University.
E-mail : skjeong@pknu.ac.kr Tel : 051-629-6181
*Young-Mi Jung : Department of Refrigeration and Air-conditioning Engineering, Pukyong National University.
**Joo-Ho Yang : Department of Mechanical System Engineering, Pukyong National University.

enough for retuning the controller gain because all of these methods do not provide any useful information how the gain should be rearranged to obtain desirable control performances. In the control system design process, the control specifications and control gain were not connected with the system dynamic characteristics.

To solve the problem, a novel PI controller design method based on the dynamic characteristic parameters is suggested in this paper. Generally, refrigeration system can be treated as the 1st order system with large time constant[5,8] which effect of the D controller gain in PID can be ignored, then the closed loop transfer function of the refrigeration system can be described as the 2nd order model which has zeros. Dynamic behaviour of the 2nd order system is dominated by its characteristic parameters which consist of damping ratio and natural frequency. The characteristic parameters include two gains of PI controller in company with system inherent parameters which consist of time constant and DC gain. It is well known that the main control specifications such as rising time, settling time and maximum overshoot are represented by the two characteristic parameters. Using the relationship between characteristic parameters and control specifications, PI controller gains can be described only by the two characteristic parameters because the inherent system parameters are already known values. Therefore, PI controller gain can be simply decided in deterministic manner without depending on an algorithm or trial and error method. Sophisticated retuning of the PI gains is possible as the control specifications, characteristic parameters, and control gains are closely connected in this design manner. Furthermore, the retuning will be performed very simple because the variation effect of gains to the dynamic behaviour of the control system can be understood intuitively.

Moreover phase margin to increase control system robustness was discussed. And influence of the zeros in closed loop transfer function was investigated in detail. Four types of PI gains which had been already confirmed their control performances through experiments were applied to the oil cooler system in order to verify the validity of the suggested design idea. The simulation results were analyzed in detail in the view point of control performances and control robustness.

2. PI Controller design based on dynamic characteristic parameters

2.1 Dynamic parameters and PI controller

Refrigeration system can be generally modeled as Eq. (1).

$$G(s) = \frac{K}{\tau s + 1} e^{-Ls} \tag{1}$$

Where, τ, K, and L represent time constant, DC gain, and dead time respectively. If L has small value, Eq. (1) can be simply described as Eq. (2). As the D controller in PID is ignorable in a refrigeration system with big time constant, PI controller can be shown in Eq. (3).

$$G(s) = \frac{K}{\tau s + 1} \tag{2}$$

$$C(s) = \frac{K_p s + K_i}{s} \tag{3}$$

Where K_p is a proportional gain, and K_i gives an integral gain.

Block diagram of feedback control system for the refrigeration cycle is depicted in Fig. 1.

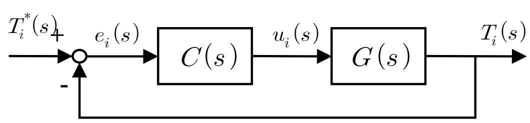

Fig. 1 Block diagram of feedback control system for the refrigeration system

In this figure, $T_i^*(s)$ is temperature set point, $e_i(s)$ means error signal, $u_i(s)$ is manipulated variable and $T_i(s)$ represents output as controlled variable. A closed loop transfer function of feedback system, $T(s)$, is derived as 2nd order system with zeros as shown in Eq. (4) from Eq. (2) and Eq. (3).

$$T(s)=\frac{T_i(s)}{T_i^*(s)}=\frac{as+b}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{4}$$

Where, $\zeta=\dfrac{KK_p+1}{2\sqrt{\tau KK_i}}$, $\omega_n=\sqrt{\dfrac{KK_i}{\tau}}$, $a=\dfrac{KK_p}{\tau}$, $b=\dfrac{KK_i}{\tau}$. Dynamic behaviour of this system is completely governed by characteristic parameters, damping ratio ζ and natural undamped frequency ω_n.

Steady state error for step input is equal to 0 because open loop transfer function $C(s)G(s)$ in Fig. 1 becomes 1 type system from Eq. (2) and Eq. (3). Therefore, design control specification of the oil cooler control system can be given by the transient characteristic index. Rising time(t_r), percent maximum overshoot(M_p), and settling time(t_s) are utilized as the design specifications in general. It is well known that the design specifications are approximately formulated by the characteristic parameters as shown in Eq. (5)~(7) on the assumption that Eq. (4) is prototype.[9)]

$$t_r=\frac{0.8+2.5\zeta}{\omega_n},\quad (0<\zeta<1) \tag{5}$$

$$M_p=100e^{-\pi\zeta/\sqrt{1-\zeta^2}} \tag{6}$$

$$t_s=\frac{4}{\zeta\omega_n},\quad (0<\zeta<0.7) \tag{7}$$

Since left terms in Eq. (5)~(7) are given by the design specification, the parameter ζ is specified from Eq. (6), and ω_n is obtained from Eq. (5). Also $\zeta\omega_n(\sigma)$ refers to the settling time, Eq. (7). Poles

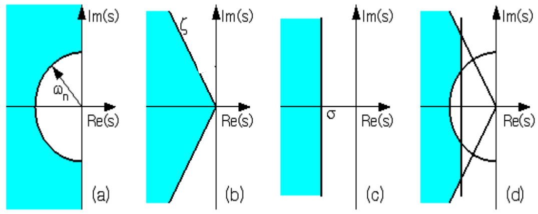

Fig. 2 Pole range satisfying design specification of time domain

which are roots of characteristic equation depend on ω_n and ζ in Eq. (4). If $0<\zeta<1$, the poles can be $s=-\sigma\pm j\omega_d$, real number σ means damping constant while imaginary number $\omega_d(\omega_d=\omega_n/\sqrt{1-\zeta^2})$ represents damped frequency. Thus, each area of poles satisfying each design specification t_r, M_p, and t_s is presented in Fig. 2(a)~(c). Fig. 2(d) indicates common area satisfying these three design specifications. From this figure, desirable location of poles satisfying design specifications can be understood intuitively.

PI controller gains are derived as Eq. (8) and Eq. (9) from Eq. (4) in terms of characteristic parameters and inherent system parameters.

$$K_i=\omega_n^2\frac{\tau}{K} \tag{8}$$

$$K_p=\frac{2\zeta\sqrt{\tau KK_i}-1}{K} \tag{9}$$

Since the parameter τ and K are already known value from system parameter, as a result, PI gains can be decided by the characteristic parameters ζ and ω_n in deterministic way. Design specifications are directly linked by PI gains. ω_n is designed by considering more severe condition of design specification between t_r and t_s because t_r and t_s depend on both ζ and ω_n.

As the control specifications, characteristic parameters, and control gains are closely connected with each other in this design manner, sophisticated retuning of the PI gains is possible. Furthermore, the retuning will be performed very simple because the variation effect of gains to the dynamic behaviour of the control system can be understood intuitively.

2.2 Phase margin considering robustness

The plant model used in design includes modeling error, and control system also has uncertainties such as noise and disturbance from internal or external environment. Therefore, system robustness is considered to assure stability of control system. Phase margin is only considered because the gain margin of the open loop transfer function is infinite. The phase margin is a function of damping ratio ζ, and it can be approximated as Eq. (10) in case of $0<\zeta<0.7$.[10)]

$$PM \cong 100\zeta \tag{10}$$

From Eq. (10), the phase margin is proportional to damping ratio. Actually, increasement of damping ratio reduces percent overshoot. However, it induces the rising time to increase simultaneously as shown in Eq. (5). Moreover, it will cause increase of settling time in case of $\zeta \geq 0.7$. Therefore, an appropriate damping ratio ζ is recommended to decide within satisfying the design specifications.

2.3 Effect of zeros in the 2nd order system

The oil cooler control system model as shown in Eq. (4) has zeros. Hence, it is transformed as Eq. (11). Indicial response of this system can be derived as Eq. (12).[9)]

$$T(s)=\frac{\omega_n^2(s/Z_o+1)}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{11}$$

$$y(t)=1-e^{-\zeta\omega_n t}\cos\omega_d t - e^{-\zeta\omega_n t}\frac{\omega_n}{\omega_d}(\zeta+1/\alpha)\sin\omega_d t \tag{12}$$

Where Z_o means zeros, and α is $\alpha=Z_o/\omega_n$. It is noted here that the 3rd term in Eq. (12) indicates transient response by zeros. In case of $1/\alpha \ll 1$, zeros does not effect to system response. However, in case of $1/\alpha \geq 1$, it makes 3rd term relatively large. Thus, it will lead big transient response.

As zeros is given as $s=-b/a$ from Eq. (4), it can be formulated in terms of K_p and K_i as shown in Eq. (13).

$$Z_o=-\frac{K_i}{K_p} \tag{13}$$

In case of $K_i \gg K_p$, the zero gives small influence to the system, when it is distant from imaginary axis in complex left half plane(LHP). On the contrary, in case of $Z_0 \leq \omega_n$, namely $K_i \leq \omega_n K_p$, there is large effect to percent overshoot.

3. Controller design and simulation for refrigeration system

3.1 Oil cooler control system

Oil cooler control system is based on vapor compression refrigeration cycle which keeps oil outlet temperature constant by changing mass flow rate of refrigerant. Fig. 3 represents conceptual diagram of oil cooler control system composed of refrigeration cycle.

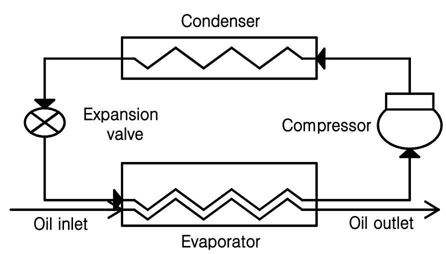

Fig. 3 Conceptual diagram of an oil cooler control system

The oil outlet temperature is regulated by change of compressor frequency. However, superheat is also controlled to avoid the decrease of COP and to prevent system damage due to abrupt change of compressor frequency. It is accomplished by controlling of opening angle in EEV(Electronic Expansion Valve).

This paper only addresses the design of oil outlet temperature controller, because EEV controllers can be designed in the same way as compressor.

Table 1 shows main components of experimental equipment, and Table 2 represents experimental conditions and design specifications.

Table 1 Specifications of the test unit

Component	Note
Compressor	Rotary type, 2.2[㎾]
Condenser	Air-cooled fin and tube type
Evaporator	Bare tube type
Refrigerant	R-22

Table 2 Experimental conditions and design specifications

Item	Note
Oil flow rate	22.5[ℓ/min]
Ambient air temperature	27[℃]
Sampling time	1[sec]
Target temperature	25[℃]
Rising time	100[sec]
Percent maximum overshoot	10[%]
Settling time	300[sec]

Eq. (14) was obtained by the experiment for the response of oil outlet temperature by changing of compressor frequency with stepwise reference.[6)]

$$G(s)=\frac{-0.0307}{55s+1} \tag{14}$$

This plant model was applied to the controller design to verify the validity of the proposed design method.

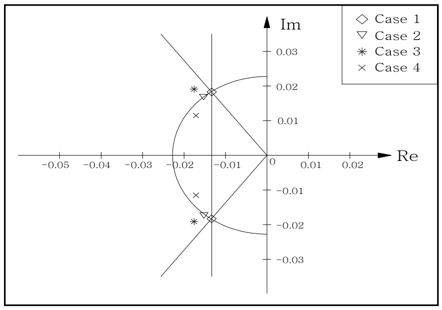

Fig. 4 The allocation of poles by designed PI gains

3.2 PI controller design for oil cooler system

Characteristic parameters, ω_n, ζ, and $\zeta\omega_n$ were obtained from Eq. (5)~(7) considering design specification prior to the PI controller design. Fig. 4 depicts pole allocation satisfying Eq. (5)~(7).

PI gains were decided by Eq. (8) and (9) considering the design specifications in Table 2. Table 3 represents designed four cases of K_p and K_i, and characteristic parameters, ζ and ω_n.

Table 3 Designed PI gains and characteristic parameters

Method	Compressor PI gain		Characteristic parameter	
	K_p	K_i	ζ	ω_n
Case 1	15.2	0.92	0.59	0.023
Case 2	21.06	0.95	0.65	0.023
Case 3	30	1.2	0.67	0.026
Case 4	28.6	0.17	0.83	0.020

Case 1 was obtained from the given design specification. Case 2 was obtained from a little adjustment on ζ in Case 1 to reduce percent overshoot. Case 3 assumed bigger $\zeta\omega_n$ than Case 2 to reduce settling time. Case 4 assumed avoidance of saturation in real manipulated variable due to excessive high gain.

The poles were located as shown in Fig. 4 from the Eq. (4) using these four PI gains. From this figure, dynamic behaviour including design specifications can be analyzed intuitively. Therefore, retuning will be readily performed by adjusting ω_n, ζ, and $\zeta\omega_n$ to accomplish good control performance.

3.3 Simulation results and analysis

Fig. 5 indicates indicial responses of closed loop system by applying gains of Table 3 to Eq. (4).

Table 4 represents rising time, percent overshot, and settling time by analyzing the indicial responses shown in Fig. 5.

Table 4 System characteristics in the 4 cases

Characteristic	Case 1	Case 2	Case 3	Case 4
Rising time[sec]	73.67	72.84	61.15	90.62
Percent overshoot[%]	11.02	8.24	7.94	1.98
Settling time[sec]	243.9	236.9	205.9	128.95

Case 1 turned out as dissatisfaction of the control specifications in percent overshoot. This is because the K_p and K_i were designed to be located poles at the boundary point of ζ and ω_n. It means that the desirable control response can not be obtained due to various errors during gain design process. In reality, there exist several errors such as decimal point approximation and approximate expression in t_r, M_p, t_s. Therefore, retuning is inevitable for K_p and K_i satisfying control specifications completely within the range of poles as shown in Fig. 4(d). As Case 1 did not satisfy design specification M_p, ζ is increased in Case 2 without change of ω_n. As the result, percent overshoot was reduced remarkably with almost the same control performance in rising time and settling time. Case 3 acquired acceptable response by increasing both ζ and ω_n to reduce all of t_r, M_p, t_s. However, this case may not applicable because of high gain problem. To avoid the excessive high gain, ζ is increased, and ω_n is decreased in case 4. Thus, Case 4 got the best control performance in t_s and M_p among four cases. Moreover, it can be noticed that settling time was more shortened than another cases in spite of smaller percent overshoot. This is due to the effect of zero point. As seen in Eq. (13), the zero point($|Z_o|=0.02658$) in Case 4 is smaller than one($|Z_o|=0.04$) in Case 3. Therefore, it can be found that settling time was shortened due to the bigger impact by zeros in Case 4.

Table 5 represents gain margin and phase margin to show the robustness of control system in four cases.

Table 5 Gain margins and phase margins in the 4 cases

Margin	Case 1	Case 2	Case 3	Case 4
GM	∞	∞	∞	∞
PM	60.69	65.84	68.01	79.24

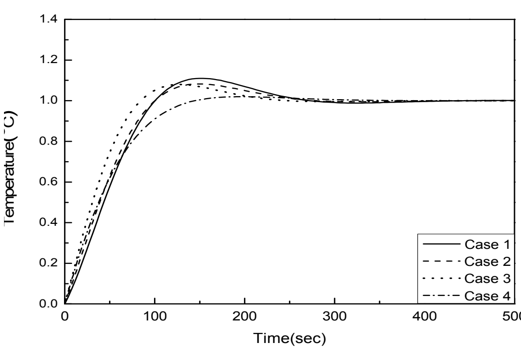

Fig. 5 Indicial responses applied PI gains using 4 cases

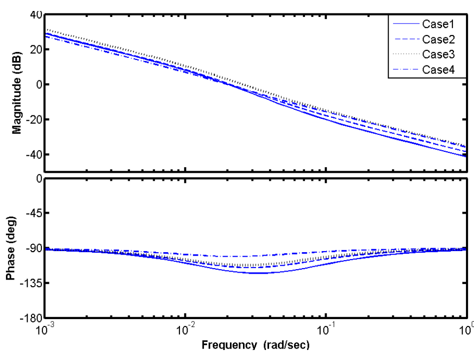

Fig. 6 Bode diagram of an oil cooler control system applied PI gains using 4 cases

Fig. 6 describes Bode diagram for 4 cases to analyze relative stability.

It can be found from the result that phase margin is an important factor to secure the system stability because gain margin is infinite. Case 4 has higher phase margin than other 3 cases, and it has better control performances in t_s and M_p. From this result, it can be concluded that Case 4 is more desirable.

Finally, the most desirable PI gains can be obtained by retuning for ζ, ω_n with focus on the most strict design specification. The retuning is easily performed by the suggested design method.

4. Conclusions

This paper suggested a novel design method of PI controller based on dynamic characteristic parameters for the oil cooler system in a systematic and deterministic way. It is possible for designers to estimate dynamic behaviour of the system intuitively because it provides the desirable range of poles represented by characteristic parameters which are decided design specifications. Therefore, this method makes reasonable and quick design of PI controller for the oil cooler system. Moreover, retuning of PI gains to obtain the best control performance in real system application is applicable.

The proposed method was successfully validated by applying it to the oil cooler system using the verified PI gains through experiments. The effect of zeros and phase margin to promote control performance and to guarantee system robustness were also analyzed.

This PI control design method is expected to be the most reasonable and powerful approach for the oil cooler system which need precise control specifications.

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