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Strategy Equilibrium in Stackelberg Model with Transmission Congestion in Electricity Market

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Abstract – Nash Cournot Equilibrium (NCE) has been widely used in a competitive electricity market to analyze generation firms' strategic production quantities. Congestion on a transmission network may lead to a mixed strategy NCE. Mixed strategy is complicated to understand, difficult to compute, and hard to implement in practical market. However, Stackelberg model based equilibrium does not have any mixed strategy, even under congestion in a transmission line. A guide to understanding mixed strategy equilibrium is given by analyzing a cycling phenomenon in the players' best choices. This paper connects the concept of leader-follower in Stackelberg model with relations between generation firms on both sides of the congested line. From the viewpoint of social welfare, the surplus analysis is presented for comparison between the NCE and the Stackelberg equilibrium (SE).

Keywords : Stackelberg model, Nash equilibrium, Cournot model, Mixed strategy, Congestion, Social welfare, Leader-follower, Duopoly, Electricity market

1. Introduction

With the introduction of competition in the electricity market, competitive prices are of fundamental importance for designing and operating the electricity market. The prices are setbased on the market clearing rules and the market participants' behaviors for pursuing their profit maximization. In order to predict and estimate the participants' decisions under market rules, it is paramount to find equilibrium of generation firms' strategies under a competition model. Also computing the equilibrium is essential in predicting market prices and analyzing performance of a trade between supply and demand in electricity market.

Using game theory to model generation firm's strategic choices in production quantity has been attempted numerously in order to analyze competition in a market setting. The most extensively used among oligopoly market models has been the Cournot model in Nash Equilibrium (NE) analysis of generation wholesale markets [1-3]. In the Cournot model, generation firms choose a strategic quantity depending on the rivals' expected production quantity. This format of assuming rivals' behaviors is applicable equally to all the participants in the Cournot model. On the other hand in the Stackelberg model a certain player called "leader" assumes the others' responses differently. This paper states that the Stackelberg model is more suitable to analyze the electricity market with transmission network that may have congestion.

The transmission line congestion status varies with the value of the generation quantities. Accordingly, the decision space can be divided into subsets depending on whether the transmission lines are congested or uncongested [4]. This leads to discontinuities in the reaction curves and causes the profit functions to be nondifferentiable and nonconcave [5, 6]. In this situation, they may not have a pure strategy of NE. Instead, the NCE has a form of mixed strategies where the players find it optimal to randomly choose between strategies [7].

In a basic representation of game theory, the mixed strategy is comprehensive and acceptable within the sample game. However, in the electricity market, the production quantity of a mixed strategy is complicated to understand conceptually, difficult to compute, and hard to implement practically in the market. This mixed strategy Nash equilibrium (MSNE) makes it harder to analyze the market and to work out the strategy of the generation firms. It may be desirable that the situation causing the MSNE be avoided [8]. Since transmission congestion is unavoidable in practical power systems, the market clearing mechanism for the generation firms needs to be adjusted to avoid the MSNE. This is the main reason that this paper suggests the Stackelberg model for finding equilibrium.

The mathematical programming method [9-11] can solve for the NE of pure strategy in a game that has differentiable and concave profit functions. When a transmission line is congested, however, this method has difficulties in determining equilibrium because the profit function can be nondifferentiable and nonconcave. The Nikaido-Isoda function and a relaxation algorithm (NIRA) are combined in [12, 13] to calculate the NE for a noncooperative game.

The Stackelberg model focuses on a differential or inequitable position in an oligopolistic competition, and utilizes the differences into the concept of"leader-follower". This paper proposes a corresponding relation between the

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leader-follower and the generation firms in a transmission network. The generation quantity of each firm influences discriminately on the congested line. For example, the power injection in the receiving end of the congested line relieves congestion; on the other hand, the power in the supplying end intensifies congestion. This differential effect on congestion makes the firms behave differently in competition [14], where a certain firm plays the role of a leader, and the others play the role of followers. Based on this suggestion, the Stackelberg model is applied to analyze the competition in the electricity market with congestion.

The equation for solving the Stackelberg equilibrium (SE) is given, and the surplus analysis is derived comparatively in the NCE and the SE. In the numerical example, a "cycling" phenomenon is shown in a sample with congested network, and compared with the MSNE and the SE. The numerical result shows that the SE has pure strategy equilibrium and may result in better social welfare than in the NCE.

2. Cournot Nash Equilibrium

2.1 Optimization of cournot model

Nash equilibrium (NE) is the most widely used equilibrium concept in economics and in the electricity market. A set of strategies is called a NE if, holding the strategies of all other firms constant, no firm can obtain a higher profit by choosing a different strategy. Thus, in a NE, no firm wants to change its strategy.

Augustin Cournot presented a model of noncooperative oligopoly. The Cournot model assumes that each firm acts independently and attempts to maximize its profits by choosing its output [8]. A NE based on Cournot model is called Nash Cournot equilibrium (NCE) and widely used in studying an electricity market.

Assume there are *N* strategic firms having *N* generating units each. Firm $i \in \{1, \ldots, N_g\}$ has a unit *i* with a generating cost function $C_i(q_i) = b_i q_i + 0.5 m_i q_i^2$ and a marginal cost $c_i(q_i) = b_i + m_i q_i$, where q_i represents firm *i*'s generation power quantity, b_i and m_i are cost coefficients. The inverse demand function at bus $j \in \{1,\ldots, N_d\}$ is $p_j = a_j - r_j d_j$, where d_j is the power demand, a_j and r_j are its coefficients.

The market operator (MO) is responsible for running the electricity market while settling the accepted bids and offers, and keeping a balance between load demand and generation supply in real time. The objective of the MO is quite different from the other participants. The MO attempts to maximize the demand side benefit as defined by an integration of the inverse demand function instead of maximizing profits. Formulating the MO's objective into a quadratic program, we have

$$\underset{d_j}{Max} \;\; B(d) = \sum_{j \in D} (a_j d_j - 0.5 r_j d_j^2) \tag{1}$$

$$\text{s. t.} \quad \sum_{i \in G} q_i - \sum_{j \in D} d_j = 0 \tag{2}$$

$$0 \le T_l \le T_{l,\max} \qquad \forall l \in L \tag{3}$$

where *D* and *G* are the sets of all demand buses and generating units respectively. *L* is the set of all transmission lines and T_l acts as the power flow on the transmission line *l*. The constraint consists of a power balance equality (2), and transmission power inequalities (3).

Each rational strategic generation firm maximizes its profits (revenue minus generating costs) by selecting its own generation parameter accepting as given the strategic parameters of other firms [3]. Formulating firm *i*'s objective into a quadratic program with generation capacity limits, we have

$$\underset{q_i}{Max} \;\; \pi_i = p_i q_i - (b_i q_i + 0.5 m_i q_i^2) \tag{4}$$

$$\text{s. t.} \quad p_i = a_i - r_i d_i \tag{5}$$

$$q_i \le q_{Mi} \tag{6}$$

where π_i is the profit from unit *i*, and p_i is a nodal price determined by the local demand, d_i at the bus unit *i*, and q_{Mi} is the capacity limit of q_i.

When a transmission network has congestion, the nodal prices change differently at each node. In this paper, locational marginal pricing is adopted as a pricing method of the MO. As generation firms' profit maximization needs to be solved with the MO's problem simultaneously, these optimizations of decentralized decision makers require hierarchical coordination.

2.2 Congestion and cycling of choices

For understanding transmission congestion on generation firms' strategies, an electricity power market is given a simple network as shown in Fig. 1. The system consists of 2 firms, and 2 buses with a local market at each bus. The marginal cost functions of firm F_1, F_2, and the inverse demand functions are respectively, $MC_1 = 0.4q_1$, $MC_2 = 0.2q_2$, $p_1 = 100 - d_1$, $p_2 = 100 - d_2$. The transmission lines are assumed to be lossless and have the power flow limit T_{max}=5.

The generation firm F_2 produces more power than F_1, since the two markets have identical demand function and

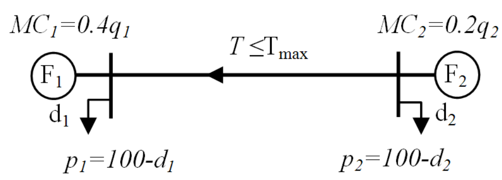


Fig.1. Duopoly system considering transmission congestion

F_1 has higher cost function. The power flow is q_2-d_2, or d_1-q_1, less than 5.0 with a direction from F_2 to F_1. Under no congestion, the price at both markets is identical, so $d_1=d_2=(q_1+q_2)/2$. Since $q_2-d_2 \le 5.0$, if the gap between q_1 and q_2 is greater than 10, the line is congested. Let's consider the firms' profit maximization alternatively.

A. Case ①: Assume a congestion on the line, then demand and generation power has a relation as $d_1=q_1+5$. And the price $p_1=95-q_1$, the profit of F_1 as in (4), $\pi_1=(95-q_1)\cdot q_1-0.2q_1^2$. Therefore F_1 will choose $q_1=39.58$ which is from an optimization of the π_1 under congestion. Let the quantity symbolized as 'q_{1c}' of F_1's critical point that means F_1 always choose this value when the line is congested.

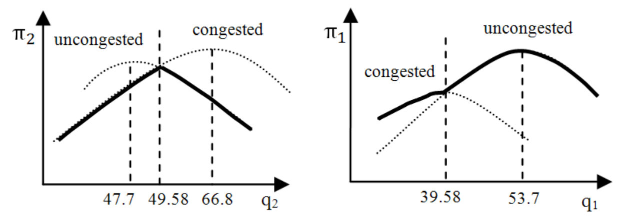
B. Case ②: Let's look into F_2's strategy under F_1's pick of q_{1c}. When F_2 picks $q_2$greater than 49.58, congestion arises on the line, since the gap between q_1 and q_2 is bigger than 10. Then $d_2=q_2-5$, and $p_2=105-q_2$, the profit of F_2, $\pi_2=(105-q_2)\cdot q_2-0.1q_2^2$. The crest point of π_2 is at $q_2=47.73$ that is out of the boundary $q_2 \ge 49.58$.

When F_2 picks q_2 smaller than 49.58, congestion will not arise. Under no congestion, $d_2=(q_1+q_2)/2$, the price of market 2, $p_2=100-d_2=80.2-0.5q_2$. Then the crest point of π_2 is at $q_2=66.84$ that is out of boundary $q_2 \le 49.58$. Fig. 2(a) shows the profit of F_2 with respect to q_2 when $q_1=q_{1c}$. Therefore the best choice of F_2 is $q_2=49.58$ which is the boundary point between congestion and uncongestion.

C. Case ③: Let's observe F_1's response to F_2's pick $q_2=49.58$. As checked before, when $q_1 \le q_{1c}$, the line is congested, and the optimal choice of F_1 is q_{1c}. However under no congestion, the demand is $d_1=(q_1+q_2)/2$, and the price is $p_1=75.1-0.5q_1$. Then the crest of theπ_1 is at $q_1=53.72$ that is in the boundary $q_1 \ge q_{1c}$. Fig. 2(b) shows the profit functions and the optimum point at $q_1=53.72$.

D. Case ④: The best response of F_2 with $q_1=53.72$ is as follows. When $p_1>p_2$, the line is congested, and $d_1<d_2$, $q_2 \ge 63.72$. Then the price is $p_2=105-q_2$, and the profit of F_2 has the crest point at $q_2=47.73$ as in the previous case that is out of the boundary $q_2 \ge 63.72$. When $q_2<63.72$, the demand is $d_2=(q_1+q_2)/2$ and the price is $p_2=73.14-0.5q_2$. The optimum of the function is at $q_2=60.95$ which is inside the region. Based on these profit functions, F_2 selects $q_2=60.95$ while discarding the choice before 49.58.

E. Case ⑤: The F_1's best reaction under $q_2=60.95$ is also based on the profit functions with respect to q_1. While the line is not congested, $d_1-q_1 \le 5$, $d_1=(q_1+q_2)/2$, and $q_1 \ge 50.95$. The profit of F_1 under no congestion has a crest at $q_1=49.6$, which is not feasible. The attainable optimum point is $q_1=50.95$ and $\pi_1=1725.1$ on the boundary case. As checked before, F_1's choice under congestion is q_{1c} and $\pi_1=1880.2$ larger than 1725.1. Therefore F_1's strategic response under $q_2=60.95$ is to pick the $q_{1c}=39.58$ discarding the pick before 53.72. This means to return to the case ①, and it will change the situation to cases ②, ③, ④, and ⑤ successively. These choices are not called equilibrium strategy, since they keep changing their choices when opposite player changes. This phenomenon is called "cycling". Table 1 shows this cycling by summarizing their strategies.



(a) (b)

Fig. 2. Profit functions of (a) F_2 at $q_1=39.58$ and (b) F_1 at $q_2=49.58$

Table1. Summarized best response and cycling of F_1 and F_2's reactions

q_1 \ q_2	49. 58	60.95
39.58	Case 2	Case 5
53.72	Case 3	Case 4

2.3 Mixed strategy equilibrium

The cycling phenomenon results from the discontinuities in their reaction curves. The transmission line congestion makes the decision space divide into subsets depending on whether they are congested or uncongested. This leads to discontinuities and nondifferentiable profit functions.

The hierarchical optimization of (1)~(6) can be solved easily by the mathematical programming method, once no inequality binds underlying decision space of differentiable functions in the Cournot model. Let the solved quantity parameters be q^*. At a pure strategy NE, the strategies of all participants satisfy

$$\pi_i(q_i^*, q_{-i}^*) \ge \pi_i(q_i, q_{-i}^*) \qquad \forall i \in G \tag{7}$$

where q_i^* is the solved quantity parameter of firm *i*, q_i is the possible quantity parameter firm *i* can choose, and q_{-i}^* is the solved quantity parameter set of all participants excluding firm *i*. By unilaterally altering their choices, none of the generating firms can improve their profits.

It may be that none such pure strategies satisfy the definition of Nash equilibrium (7). Instead, the firms may discover that they must play a combination of pure strategies, choosing amongst them randomly. This is a "mixed strategy," which is specified by the probability distribution of the choice of pure strategies [7, 15].

Solving the two-level optimization for the sample case as summarized in Table 1 gives a mixed strategy Nash equilibrium (MSNE); $q_2=54.89$, $q_1=[39.58, 51.83]$ with probability [0.657, 0.343]. At the NE, the firm, F_1 chooses a mixed strategy consisting of two pure strategies with the probabilities, while F_2 chooses a pure strategy. One of the two choices of F_1 gives rise to congestion on a line, for example q_1=51.83. The other, q_1=39.58, on the contrary, makes no congestion. The former can be called a congestion strategy, the latter an uncongestion strategy. F_1 uses more strategic choices, sometimes congestion strategy; other times uncongestion strategy, whereas F_2 takes only one choice.

This paper asserts that the player choosing a mixed strategy plays the role of a leader, and the other plays the role of a follower from the viewpoint of the Stackelberg model. The leading player locates in a receiving area over a congested line, whereas the follower locates in a sending area. The leader, F_1, is at bus 1on the receiving end in Fig. 1. This assumption is very useful in calculating a MSNE, and its usefulness has been verified over a variety of case studies.

3. Stackelberg Model

3.1 Stackelberg equilibrium at duopoly

It is assumed in Cournot model that all the competitors in a market have equal opportunity. However, in some industries, historical, institutional, or legal factors put the competitors into a differential or inequitable position in the market. For example, the firm that discovers and develops a new product has a natural first-mover advantage.

Heinrich von Stackelberg presented an important oligopoly model in 1934. In the Stackelberg model, one firm acts before the others. The leader firm picks its output level and then the other firms are free to choose their optimal quantities given their knowledge of the leader's output. The follower's best response is determined as in the Cournot model that competitors' output is assumed to be fixed. However, the leader's action is different from the followers' actions in Cournot model [8].

The leader picks the output to maximize its profit subject to the constraint that the follower firm chooses its corresponding output using its Cournot best response function [16, 17]. In Stackelberg equilibrium (SE), the leader is better off and the follower is worse off than in a Cournot equilibrium (CE).

Stackelberg pointed out history, institution, law, discovery, and development as factors affect the determination of the leader and the followers in general industries. But in an electricity market, the transmission network is an important factor. Because the physical limits of transmission lines can restrict the economic dispatch of the generation power, the generation firms change their strategies by depending on the site with respect to the congested line. This paper postulates that the firm at the receiving area of a congested line has a leading opportunity, and the firm at the sending area has a follower position. When it is not clear if a firm locates in a sending area or a receiving area, PTDF(Power Transfer Distribution Factor) is a useful index to clear the vagueness [14,18,19].

Let's look into the duopoly system and derive the Stackelberg equilibrium (SE). Firms have generating marginal cost; $MC_i=m_iq_i$, and the inverse demand functions of the market is $p_i=a-r(q_1+q_2)$. The firm F_1 is assumed a leader, F_2 follower. The follower's best response condition is as follows;

$$\frac{\partial \pi_2}{\partial q_2}=\frac{\partial}{\partial q_2}\{p\cdot q_2-C(q_2)\}$$
$$=\frac{\partial p}{\partial q_2}\cdot q_2+p-m_2q_2=-r\cdot q_2+p-m_2q_2=0 \qquad (8)$$

The leader's best response condition is derived by considering the follower reaction.

$$\frac{d\pi_1}{dq_1}=\frac{\partial \pi_1}{\partial q_1}+\frac{\partial \pi_1}{\partial q_2}\cdot\frac{\partial q_2}{\partial q_1}$$
$$=-r\cdot q_1+p-m_1q_1+(-r\cdot q_1)\frac{\partial q_2}{\partial q_1}=0 \qquad (9)$$

where, $\partial q_2/\partial q_1$ is obtained from (8).

The SE is derived from (7), (8) and the demand function is as follows;

$$q_{s1}=\frac{a(m_2+r)}{\Delta_A},\ q_{s2}=\frac{\Delta_B}{\Delta_A}\cdot\frac{a}{m_2+2r},\ p_s=\frac{\Delta_B}{\Delta_A}\cdot\frac{a(m_2+r)}{m_2+2r}$$

where, $\Delta_A= m_1m_2+2r(m_1+m_2)+2r^2$, $\Delta_B= m_1m_2+r(2m_1+m_2)+r^2$. q_{s1} is the leader's output, q_{s2} is the follower's output, and p_s is the market price. The subscript's' denotes for the SE and the subscript 'c' is used for the NCE.

$$q_{c1}=\frac{a(m_2+r)}{\Delta_A+r^2},\ q_{c2}=\frac{a(m_1+r)}{\Delta_A+r^2},\ p_c=\frac{a(m_1+r)(m_2+r)}{\Delta_A+r^2}$$

where, Δ_A and Δ_B are the same as before.

3.2 Social welfare comparison between NCE and SE

The NCE and the SE are compared from the viewpoint of social welfare which is used universally as an index for the trading value in a market in microeconomics. This section shows that social welfare in the SE is greater than in the NCE in no congestion situation by comparing price and total quantity supplied.

The ratio of the prices between the NCE and the SE is as follows;

$$\frac{p_s}{p_c}=\frac{\Delta_A+r^2}{\Delta_A}\cdot\frac{\Delta_B}{(m_1+r)(m_2+2r)},$$
$$=\frac{\Delta_A+r^2}{\Delta_A}\cdot\frac{\Delta_B}{\Delta_B+r^2}=\frac{\Delta_A\Delta_B+\Delta_Br^2}{\Delta_A\Delta_B+\Delta_Ar^2}<1$$

where, $\Delta_A>\Delta_B=\Delta_A-r\cdot m_2-r^2$. Therefore, the price in SE is lower than that in the NCE.

The generation quantity of F_1 in the SE and the NCE is compared by the following ratio;

$$\frac{q_{s1}}{q_{c1}} = \frac{\Delta_A + r^2}{\Delta_A} > 1.$$

The total quantity of generation, q_1+q_2, is derived from (8) in the SE and the NCE in common as follows;

$$q_1 + q_2 = \frac{a+(m_2+r)q_1}{m_2+2r}.$$

The total quantity is proportional to the leader's output, q_1. Since the quantities, $q_{s1}>q_{c1}$ from the previous equation, the total quantity in the SE is greater than in the NCE. From the viewpoint of "Benefit" in demand function in microeconomics, the SE is better than the NCE, because the price in the SE is lower and the total demand in the SE surpasses that in the NCE. But the generation cost in the SE is larger than in the NCE because of greater demand in the SE.

Instead of the benefit in demand, the social welfare is suitable to evaluate a trade in a market. Since the social welfare is computed by benefit minus generation cost, the greater quantity may result in less social welfare.

Let's compare quantity A and quantity B in Fig. 4. Even though the higher quantity B gives larger benefit, the social welfare in B is less than in A. The social welfare increases with increasing quantity until point C. On the other hand it decreases with increasing quantity after point C. So it is the point at issue whether the SE with greater quantity locates in region less or greater than C.

In the region A less than C, the market price(p_a) is higher than the marginal cost(MC_a). On the contrary, p_b is lower than MC_b in region B. Therefore the prices in the SE need to compare with the marginal cost of the two firms for observing the SW.

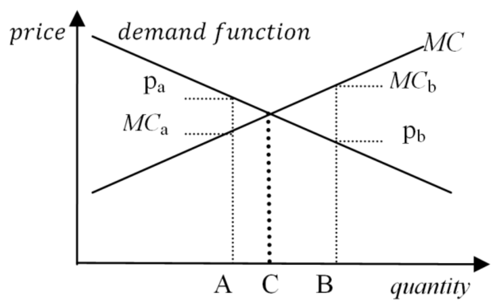


Fig. 3. Relation between market price and marginal cost

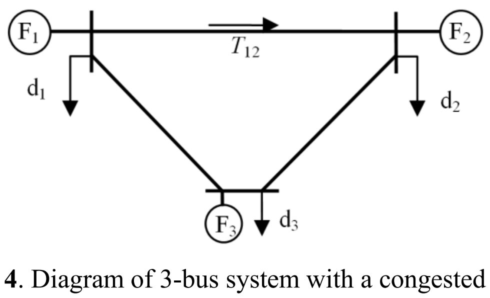


Fig. 4. Diagram of 3-bus system with a congested line

$$MC_1 = m_1 \cdot q_1 = \frac{a(m_2+r)m_1}{\Delta_A}, \quad MC_2 = m_2 \cdot q_2 = \frac{a(m_2+r)(m_1+r)m_2}{\Delta_A(2r+m_2)}$$

For an easy comparison, the market price(p_s) derived in 3.1 is rearranged here;

$$p_s = \frac{a(m_2+r)}{\Delta_A}\{m_1 + \frac{r(m_2+r)}{m_2+2r}\} > MC_1,$$

$$p_s = \frac{a(m_2+r)}{\Delta_A(m_2+2r)}\{m_2(m_1+r) + r(2m_1+r)\} > MC_2$$

Both the leader and the follower have lower marginal cost than the market price in the SE. So it is shown that the SE with a lower price and a larger total quantity has greater social welfare than in the NCE.

4. Results and Comparison

4.1 Equilibrium in duopoly competition

A. Uncongestion Case

In the duopoly system as Fig. 1, letting the transmission capacity be limitless leads to the NCE and the SE as in Table 2. In the SE, more demand quantity is supplied at a lower price than in the NCE. Therefore the SE gives more social welfare 7271.6 rather than 7188.6 of the CNE.

B. Congestion Case

When the limit value, T, affect the strategies of the generation companies, the equilibria of Cournot and Stackelberg in Table 2 will not be retained. The physical limit of the power flow on a transmission line might be advantageous to a potential supplier. The change of the NCE and the SE with a consideration of T=5.0 is in the Table 3.

As mentioned in 2.2 in this paper, taking a consideration of the limit T=5.0 results in "cycling" of firms' choices instead of a pure equilibrium. To avoid the cycling phenomenon, a mixed strategy of the NCE is to be solved,

Table 2. Results of NCE and SE in duopoly system without congestion

	q_1	q_2	d_1	d_2	T	Price	SW
NCE	48.95	62.94	55.94	55.94	6.99	44.06	7188.6
SE	59.32	58.62	58.97	58.97	0.35	41.03	7271.6

Table 3. Results of NCE and SE in duopoly system considering congestion

		q_1	q_2	d_1	d_2	T	Price	SW
NCE	Uncon.	51.83	54.89	53.36	53.36	1.53	46.65	6851.6
	Con.	39.58		44.58	49.89	5.0	55.42/50.11	
SE		39.58	49.58	44.58	44.58	5.0	55.42	6369.5

and the mixed strategy equilibrium of the duopolyis given with probability α=0.657, β=0.343 for uncongestion and congestion case respectively. In the uncongestion case, the power flow is 1.53 less than T, and the price is identical at both buses. In the congestion case, the flow is equal to T, and the price at bus 1 is 55.42, the price at bus 2 is 50.11. The social welfare is calculated as an expectation value with the probability.

In the SE, the state corresponds to the case 2 in Table 1 about the cycling reactions. From the viewpoint of the NCE, the firm F_1 assumes F_2 keeps the quantity q_2=49.58, and changes q_1 from 39.58 to 53.72 as in Table 1. However, in Stackelberg, F_1 believes that F_2 keeps following F_1's choice on the basis of F_2's profit maximization. No cycling happens in the SE as in Table 3, and the power flow is equal to the limit T with identical prices. So this state is a borderbetween uncongestion and congestion. In the case of Table 2, the profit of F_1 is computed to 1730 in the SE, however, in the case of Table 3, it is 1880.2. The leader, F_1, can increase its profit by utilizing the equipment scarcity of transmission capacity.

In the case of Table 3, the social welfare in the SE is 6369.5 lower than 6851.6 in the NCE. This paper shows that social welfare is higher in the SE than in the NCE as in Table 2, but when transmission congestion is considered it may not be true as in Table 3. The next section shows another result that the SE gives higher social welfare than the NCE even under consideration of congestion. The verification of the equilibria is provided in appendix A.

4.2 Equilibrium in 3-Players' competition

An electricity power market is given with a simple network as shown in Fig. 4. The system consists of 3 generation firms, and 3 buses with a local market at each bus. The marginal cost functions of firm F_1, F_2, F_3, and the inverse demand functions are respectively, $MC_1 = 10+0.3q_1$, $MC_2 = 20+0.4q_2$, $MC_3 = 15+0.45q_3$, $p_1 = 70-0.7d_1$, $p_2 = 80-0.5d_2$, $p_3 = 90-0.4d_3$. The transmission lines are assumed to be lossless and have the reactance satisfying $x_{12} = x_{13} = 2x_{23}$.

The transmission limit is assumed T_{max}=15.0, and the flow direction from 1 to 2 is determined by the inverse demand function at each bus and the marginal cost functions. The results of a mixed strategy of the NCE and a pure strategy of the SE are given in Table 4 and verified in appendix B.

Due to the transmission limit, the NCE does not have a pure strategy, rather a mixed strategy. The uncongestion strategy in the MSNE has a probability 0.49. On the other hand, the congestion strategy has a probability 0.51, and leads to the nodal prices 52.3, 54.3 and 53.7 in bus 1, 2, and 3 respectively. The node of the highest price is bus 2, and it is a receiving end on the congestion line. Therefore the F_2 that locates in bus 2 plays a role of a leader and the F_3 and $F_1$are the followers.

This market system also gives a pure strategy of the SE. The SE state corresponds to a border between congestion and uncongestion states. The identical price at each node denotes an uncongestion state, and the power flow equal to the limit means a congestion state. The total generation is 173.93 in the SE, which is higher than 172.3 in the NCE computed using an expected value with the probabilities. The price in SE is 52.47, which is lower than 52.8 in the NCE calculated using weighted average over the buses and utilizing expected value with the probabilities. The more quantity and lower price leads to a higher social welfare in the SE than in the NCE.

In this 3-player market, whether congestion arises or not,the social welfare in the SE is 7374.6 higher 7329.2 than in the NCE. This is one of the reasons that this paper recommends Stackelberg model for analyzing an electricity market with congestion. As mentioned before, the main reason is that the SE does not have a mixed strategy, while the NCE may have a mixed strategy. In order to implement this idea to the electricity market, amarked effort is needed to implement the concept of leader- follower in the electricity market and to adjust the market rule to acknowledge the first-mover advantage for the firms in the receiving area of the congested line.

5. Conclusion

Transmission congestion may cause a mixed strategy equilibrium based on the Cournot model in an electricity market. A mixed strategy equilibrium is complicated to understand, difficult to compute, and hard to implement in a practical market. In order to avoid the mixed strategy Nash equilibrium, the Stackelberg model with a concept of leader-follower is suggested and analyzed in this paper. The leader's advantage corresponds to a major influence to congestion of a generation firm which is at the receiving end. The Stackelberg equilibrium is identified not to have mixed strategies, and have superiority to the Cournot Nash equilibrium from the viewpoint of social welfare in an uncongestion situation. When there is congestion in a transmission network, the Stackelberg equilibrium may have less social welfare. However, congestion is less frequent than uncongestion, so the SE will give better

Table 4. Results of NCE and SE in 3 firm competition systems under congestion

		q1	q2	q3	d1	d2	d3	T	Price	SW
NCE	Uncon. Con.	60.23	56.1 46.3	60.95	25.85 25.26	56.19 51.36	95.24 90.87	13.77 15.0	51.9 52.3/54.3/53.7	7329.2
SE		61.55	53.07	59.31	25.04	55.06	93.83	15.0	52.47	7374.6

social welfare. This analysis will help to suppress bothersome arguments about mixed strategy equilibria in an electricity market.

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Appendix. Verification of Equilibria

A. Duopoly Market Equilibria under Congestion

In the NCE of Fig. 6, the expected profit is calculated with the other player's quantity fixed. The profit curve, π_1, has two peak points with equivalent values, since F_1 chooses a mixed strategy. This means that each player in Fig. 6 has no incentive to deviate from its choice, given the choices of the other players [20]. In the SE of Fig. 7, the profit of the follower, F_2, is calculated in the same way as in the Cournot model. However, the profit of the

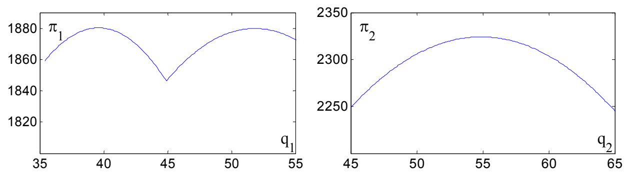

Fig. 5. Expected profits at nash equilibrium in a duopoly Market

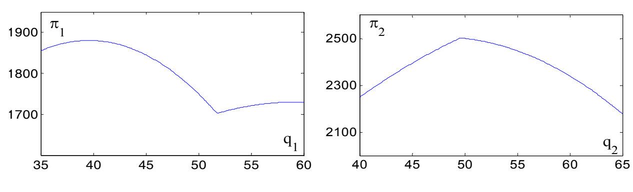

Fig. 6. Expected profits at stackelberg equilibrium in a duopoly market

leader, F_1, is calculated by considering F_2's reaction instead of assuming F_2's choice as being fixed. The highest points in the curves correspond to the equilibrium strategy in Table 3.

B. 3-player market equilibria under congestion

As in the NCE of the case in Table 4, F_2 has a mixed strategy, and it is shown that the peaks correspond to the strategies in the NCE. In the SE, the leader, F_2, and the followers, F_3 and F_1 have one peak point in each profit curve, which corresponds to its pure strategy.

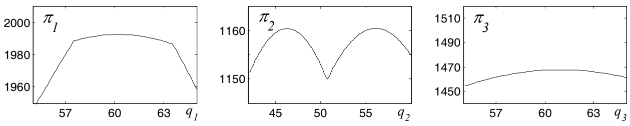

Fig. 7. Expected profits at NE in 3 firm competition

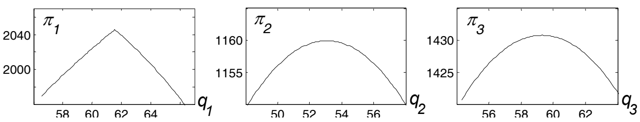

Fig. 8. Expected profits at stackelberg equilibrium in 3 firm competition

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