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Modified Particle Swarm Optimization with Time Varying Acceleration Coefficients for Economic Load Dispatch with Generator Constraints

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Abstract – This paper proposes a Modified Particle Swarm Optimization with Time Varying Acceleration Coefficients (MPSO-TVAC) for solving economic load dispatch (ELD) problem. Due to prohibited operating zones (POZ) and ramp rate limits of the practical generators, the ELD problems become nonlinear and nonconvex optimization problem. Furthermore, the ELD problem may be more complicated if transmission losses are considered. Particle swarm optimization (PSO) is one of the famous heuristic methods for solving nonconvex problems. However, this method may suffer to trap at local minima especially for multimodal problem. To improve the solution quality and robustness of PSO algorithm, a new best neighbour particle called 'rbest' is proposed. The rbest provides extra information for each particle that is randomly selected from other best particles in order to diversify the movement of particle and avoid premature convergence. The effectiveness of MPSO-TVAC algorithm is tested on different power systems with POZ, ramp-rate limits and transmission loss constraints. To validate the performances of the proposed algorithm, comparative studies have been carried out in terms of convergence characteristic, solution quality, computation time and robustness. Simulation results found that the proposed MPSO-TVAC algorithm has good solution quality and more robust than other methods reported in previous work.

Keywords: Economic load dispatch, Particle Swarm Optimization (PSO), Prohibited operating zone (POZ), Ramp rate limits, Time varying acceleration coefficients (TVAC)

1. Introduction

Economic load dispatch (ELD) is one of the important tasks in power system operation and planning. The main purpose of ELD is to determine the real power output of scheduled generators to meet power demand at minimum cost whilst satisfying the equality and inequality constraints. Optimal combination of generator power output can reduce the cost of power plant operation significantly.

In general, the cost characteristic of generator is assumed to be convex and is represented by a single quadratic function for ELD problems. It was successfully solved by mathematical programming methods based on derivative information of cost function [1]. However, the cost function of a practical generator becomes highly non-linear and discontinuous due to prohibited operating zones (POZ) and ramp-rate limits of the generator [2, 3]. Therefore, ELD problems with equality and inequality constraints are nonconvex and very difficult to solve using a mathematical approach. Conventional methods such as gradient method, lambda iteration, base point participation and Newton methods are unable to solve nonconvex optimization problem [1]. On the other hand, dynamic programming can solve nonconvex ELD problem due to no restriction on the cost function, but suffers from 'curse of dimensionality' when involves with high number of variables [4].

Recently, modern heuristic methods such as genetic algorithm [5, 6], evolutionary programming [7], differential evolution [8], ant colony optimization [9], tabu search [10], simulated annealing [11], neural network [12], and particle swarm optimization (PSO) [13-18] have been successfully applied to nonconvex ELD problems. However, these approaches are not always promising a global optimum solution and sometimes are trapped at local point.

Among these techniques, PSO is widely used for solving nonconvex ELD problem due to its simple implementation, less complexity and most of the time able to find global solution. In classical PSO, premature convergence is always occurring due to the lack of diversity of PSO algorithm. This can lead the particles to converge at a local optimal solution especially for complex and nonconvex problems with multiple minima. To overcome

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this problem, many types of PSO variants were proposed in ELD application [16, 18-21]. However, most of these strategies do not produce consistent results in many different trials. In [22], a new index (iteration best) are introduced to improve the solution quality for unit commitment problem based on modification of velocity equation. Later, the method in [22] was applied to solve ELD problem, but the robustness of this algorithm is not discussed in [23]. In [16], the authors proposed a new PSO strategy based on the information of bad experience among particle. The updated particle will use this information to move away from the bad position that has been achieved from the population. The time varying acceleration coefficients (TVAC) approach is proposed by varying the value of the acceleration coefficient for the cognitive component (c_1) and social component (c_2) during the iterative process [24, 25]. By proper tuning of these coefficients (c_1 and c_2), the particles are guided towards optimum solution.

In this paper, a new PSO variant named MPSO-TVAC method is proposed for solving ELD problems with the objectives to improve the solution quality, robustness and to avoid premature convergence. A new best neighbour parameter (*rbest*) is introduced into velocity equation, which is randomly selected from the best position (*pbest*) that has been obtained by another particle. This can enhance the searching behaviour and exploration capability of particles through the entire solution space. Moreover, TVAC (for c_1, c_2 and c_3) can provide a balance exploration and exploitation for the particle to get a better optimum solution. To validate the proposed MPSO-TVAC method, it is tested on ELD problem with generator limits, POZ, ramp-rate limits and transmission loss constraints. A new constraint handling has been introduced to handle POZ and ramp-rate limits constraints instead of used penalty factor as in [16, 17] and discussed in Section 4 (step 5). The results obtained by MPSO-TVAC are compared with some PSO variants in term of convergence characteristic, solution quality and robustness. For the ELD problem, the results show that MPSO-TVAC approach provides lower cost and more robust than other PSO strategies and results of an existing method.

In this paper, Section 2 details mathematical formulations of ELD problem considering POZ, ramp rate limits and transmission loss. Section 3 describes the proposed MPSO-TVAC approach. Section 4 presents the detail procedure of implementing the MPSO-TVAC strategy for solving the nonconvex ELD problems. The simulation results and comparison study are presented and discussed in section 5. Finally, the conclusions are drawn in Section 6.

2. Formulation of ELD Problem

The primary objective of ELD problem is to minimize the total fuel cost (F_C) of thermal generator while satisfying the operational constraints of a power system. Therefore, ELD problem can be formulated based on single quadratic function as below:

$$F_C = \sum_{i=1}^{Ng} F_i(P_i) \tag{1}$$

where $F_i(P_i)$ is the fuel cost of the *i*th generator ($/h) which is defined by following equation:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{2}$$

where Ng is the number of generator, P_i is the active power of generator i (MW) and a_i, b_i and c_i are the fuel cost coefficients of *i*th generator.

In this paper, the POZ and ramp rate limits are considered as practical constraints of generator and transmission losses as network constraint. This result in the ELD becomes more complicated and nonconvex optimization problem that has multiple local minima which is difficult to find a global optimum solution. The ELD constraints are discussed as follows:

2.1 Power balance constraint

The total generated power must meet the total load demand and transmission losses as given in (3) and (4).

$$\sum_{i=1}^{Ng} P_i = P_D + P_L \tag{3}$$

where P_D is the total power demand and P_L is the transmission losses in the power network. Transmission losses in (3) can be calculated either using penalty factor or *B*-loss coefficient [1,26]. In ELD problem, *B*-loss coefficient is commonly used in the previous study and is adopted in this paper as following equation:

$$P_L = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_i B_{ij} P_j + \sum_{i=1}^{Ng} B_{i0} P_i + B_{00} \tag{4}$$

where B_{ij} is the *i,j* element of the loss coefficient matrix, B_{i0} is the *i*th element of the loss coefficient vector and B_{00} is the loss coefficient constant.

2.2 Generator limit constraints

The active power output of each generator should satisfy the minimum and maximum limits as given:

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5}$$

where P_i^{min} and P_i^{max} are the minimum and maximum limits for ith generating unit.

2.3 Ramp rate limits

In practical, adjustments of power output are not instantaneous. Increasing or decreasing of power output is restricted by ramp rate limits of the generating unit by the following conditions:

If power generation increases:

$$P_i - P_i^0 \le UR_i \quad (6)$$

If power generation decreases:

$$P_i^0 - P_i \le DR_i \quad (7)$$

Therefore, the effective generator limits with the presence ramp rate limits are modified as follows:

$$\max(P_i^{\min}, P_i^0 - DR_i) \le P_i \le \min(P_i^{\max}, P_i^0 + UR_i) \quad (8)$$

where P_i^0 is the previous active power output of generator i (MW), DR_i and UR_i are the upper and lower ramp rate limits of generator i (MW/time period) respectively.

2.4 Prohibited Operating Zones (POZ)

The generating unit may have certain zones where the operation is not allowed due to vibration in shaft bearing or problem of machine components [2]. Thus, discontinuous and non smooth fuel cost characteristic is produced corresponding to the POZ as illustrated in Fig. 1.

In practical, adjustments of the output power of generator i must be avoided to operate within these zones. The allowable operating zones incorporated POZ constraints are formulated as follows:

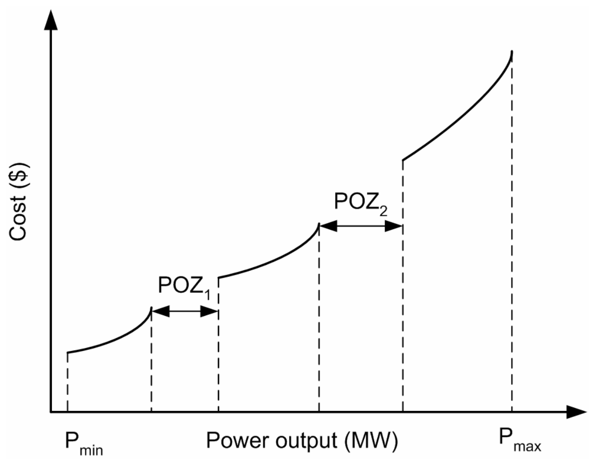

Fig. 1. Fuel cost characteristic with POZ

$$P_i \in \begin{cases} P_i^{\min} \le P_i \le P_{i,1}^{LB} \\ P_{i,z-1}^{UB} \le P_i \le P_{i,z}^{LB} & z=2,3\ldots,Nz \\ P_{i,Nz}^{UB} \le P_i \le P_i^{\max} \end{cases} \quad (9)$$

where $P_{i,z}^{LB}$ and $P_{i,z}^{UB}$ are the lower and upper bounds of zth POZ of ith generator in (MW) and Nz is the number of POZ of ith generator.

3. Proposed MPSO-TVAC Algorithm

3.1 Review PSO algorithm

The particle swarm optimization is a population based optimization technique that was introduced by Kennedy and Eberhart in 1995 [27]. This modern heuristic technique is inspired by social behaviour of the swarm of fishes and flocks of birds searching for the food. The main advantages of the PSO algorithm compared to other optimization methods are simple, easy to implement, less storage requirement and able to find a global optimum solution [28].

In PSO, each particle represents the possible solutions to the problem. Initially, a random population of particles (or solution) is generated in d-dimensional (or variable) search space. A particle i at iteration j is represented as position vector $x_i^j=[x_{i1}^j, x_{i2}^j, \ldots, x_{id}^j]$ and velocity vector $v_i^j = [v_{i1}^j, v_{i2}^j, \ldots, v_{id}^j]$. Based on the evaluation function value, each particle in current iteration has its own best position represented as $pbest_i^j = [pbest_{i1}^j, pbest_{i2}^j, \ldots, pbest_{id}^j]$. The best particle in a population is defined as global best $gbest_d^j = [gbest_1^j, gbest_2^j, \ldots, gbest_d^j]$. The velocity and position of each particle are updated using equations below:

$$v_{id}^{j+1} = w^j v_{id}^j + c_1 r_1 (pbest_{id}^j - x_{id}^j) + c_2 r_2 (gbest_d^j - x_{id}^j) \quad (10)$$

$$x_{id}^{j+1} = x_{id}^j + v_{id}^{j+1} \quad (11)$$

where r_1 and r_2 are random numbers between 0 and 1, c_1 is the cognitive acceleration coefficient which pushes the particles towards $pbest$, c_2 is the social acceleration coefficient which push the particles towards $gbest$ and w is the inertia weight factor.

The inertia weight controls the impact of the previous velocity on updating velocity of a particle. A proper selection of w can provide a good exploration and exploitation to find the optimum solution. A large initial value of w can provide a better global exploration while smaller values of w facilitates better exploitation in local search [29]. The linearly decreasing of w is computed as follows [15]:

$$w^j = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{j_{\max}} \right) \times j \qquad (12)$$

where w_{min} and w_{max} are the initial and final inertia weights respectively and j_{max} is the maximum iteration number. For effective balance between global and local searches, the inertia weight is decreased linearly from 0.9 to 0.4 during the optimization process [14, 30, 31].

3.2 Review IPSO algorithm

Iteration particle swarm optimization (IPSO) was introduced by Tsung-Ying Lee and Chung-Lung Chen in 2007 [22] in order to improve the solution quality of PSO. A new index named 'Iteration best' was introduced in (10) to enrich searching behaviour of PSO. A modified velocity equation of the particles is

$$\begin{aligned} v_{id}^{j+1} &= w^j v_{id}^j + c_1 r_1 (pbest_{id}^j - x_{id}^j) \\ &+ c_2 r_2 (gbest_d^j - x_{id}^j) + c_3 r_3 (I_{best}^j - x_{id}^j) \end{aligned} \qquad (13)$$

where I_{best}^j is the best value of fitness function that has been achieved by any particle in current iteration j and c_3 is the stochastic acceleration coefficient that pulls the particles towards I_{best}.

3.3 Proposed MPSO-TVAC algorithm

In order to improve the solution quality and robustness of PSO, a novel modified PSO with time varying acceleration coefficients (MPSO-TVAC) is proposed. This method introduces a new parameter named the best neighbour particles (*rbest*) in (10) based on randomizing the best position of neighbour particles. The idea is to provide the extra information to each particle, thus increasing the exploration capability and avoiding being trapped in a local optimum. In this strategy, each particle has its own $rbest_i^j=[rbest_{i1}^j, rbest_{i2}^j, \ldots, rbest_{id}^j]$, which is randomly selected from the best position (*pbest*) of other particles. Fig. 2 is an illustration on how to determine *rbest* value for particle 2, where the other *pbest* values (except its own *pbest*) are randomly preferred. A similar approach is applied to other particles in the swarm. The new updated velocity for proposed method is given in (14):

$$\begin{aligned} v_{id}^{j+1} &= w^j v_{id}^j + c_1 r_1 (pbest_{id}^j - x_{id}^j) \\ &+ c_2 r_2 (gbest_d^j - x_{id}^j) + c_3 r_3 (rbest_{id}^j - x_{id}^j) \end{aligned} \qquad (14)$$

where, c_3 is the acceleration coefficient that pulls each particle towards *rbest*.

The performance of PSO is dependent to the proper tuned parameters that results in the optimum solutions. Generally, the acceleration coefficients for cognitive (c_1) and social components (c_2) are set to constant values. The impact of acceleration coef ficients setting is reported in [13, 32]. A relatively high value of the social acceleration coefficient c_2 than cognitive acceleration coefficient c_1 is selected, the algorithm will converge to a local optimum solution (premature convergence). However, a relatively high value of cognitive acceleration coefficient c_1 compared to social acceleration coefficient c_2 results in wandering of particles around the search space [27].

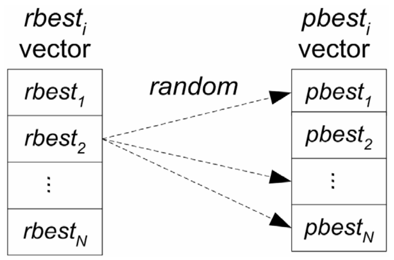

Fig. 2. Determination of *rbest* value of the *i*-th particle.

To enhance exploration and exploitation of particle towards optimum solution, both coefficients should be varies according to the iteration number [24]. A large value of cognitive component and small social component in initial iteration pushes the particles to move to the entire the solution space. As iteration increases, the value of cognitive will decrease and the value of the social components will increase, which pull the articles to the global solution. The acceleration coefficients are varied according the following formulas:

$$c_1 = c_{1i} + (c_{1f} - c_{1i}) \times \frac{j}{j_{\max}} \qquad (15)$$

$$c_2 = c_{2i} + (c_{2f} - c_{2i}) \times \frac{j}{j_{\max}} \qquad (16)$$

where c_{1i} and c_{1f} are the initial and final values of cognitive coefficient respectively and c_{2i} and c_{2f} are the initial and final values of social coefficient respectively.

Presenting a new parameter (*rbest*) in the velocity equation in (14), will encourage the particle movement to converge at optimum solution due to extra information provided by the *rbest* value in current iteration. The time varying acceleration coefficient for *rbest* component (c_3) is using the following Eq. [33]:

$$c_3 = c_1 \times (1 - \exp(-c_2 \times j)) \qquad (17)$$

The behaviour acceleration coefficients (c_1, c_2 and c_3) of the MPSO-TVAC algorithm are shown in Fig. 3. It assumed that, the c_1 value varies from 1 to 0.2 and c_2 varies from 0.2 to 1 during 100 iterations. At the initial iteration, the c_3 value is increased immediately which helps the particles to explore the entire possible solutions based on the best neighbour particle (*rbest*). This can avoid the

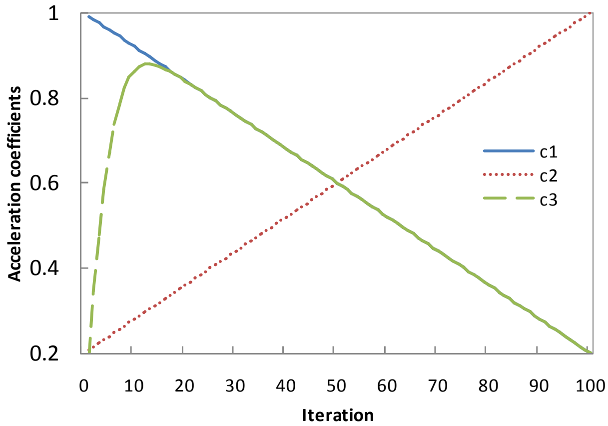

Fig. 3. Behaviour of acceleration coefficients (c_1,c_2 and c_3) during iteration.

particle to rapidly converge at the local *gbest*. As iteration proceeds, the c_2 value is linearly increased to encourage particles towards global *gbest* value. Therefore, the exploration and exploitation capability of MPSO-TVAC is improved, thus providing good solution quality and consistent results near to the global optimum.

4. Procedure of MPSO-TVAC Algorithm for ELD Problem

In this section, the proposed MPSO-TVAC method for solving non-smooth ELD problems with POZ and ramp rate limits constraints are explained. This paper also proposes a new strategy to handle POZ and ramp rate limit constraints during the optimization process without using penalty factor. The results obtained by this approach satisfy all the constraints at the minimum cost. The flowchart of the MPSO-TVAC algorithm is shown in Fig. 4. The detailed implementation of the proposed algorithm are described as follows:

Step 1: Initialization of the Swarm.

The active power output of the generator is defined as a variable (or dimension) for ELD problem. For a population size of N_{pop}, the particles are randomly generated between the generator limits in (5) and satisfy all constraints in (6) and (7). The *i*th particle for N_g generator number is represented by

$$P_i = (P_{i1}, P_{i2} ..., P_{id}) \tag{18}$$

$$P_{id} = P_{id}^{\min} + rand() \times (P_{id}^{\max} - P_{id}^{\min}) \tag{19}$$

Step 2: Evaluation Function.

The fitness of each particle is evaluated based on the defined evaluation function. The evaluation function should minimize the total cost function and satisfy the constraints. Commonly, the penalty factor method is

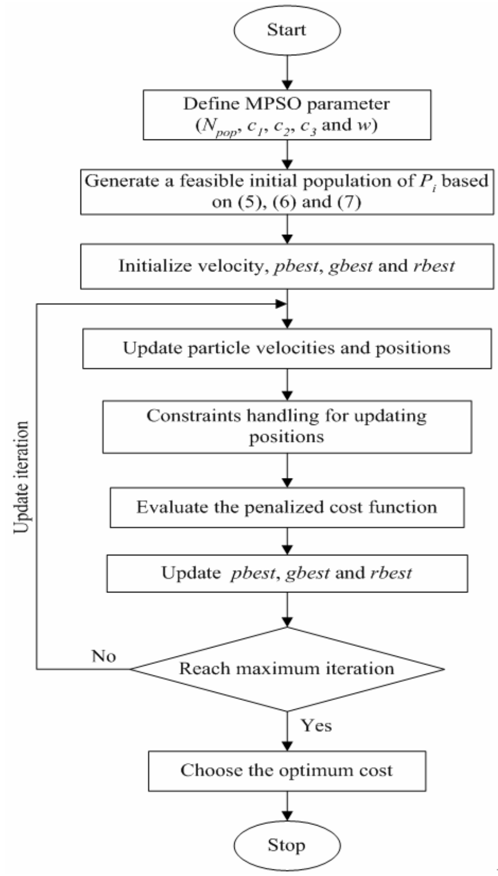

Fig. 4. Flowchart of proposed MPSO-TVAC algorithm.

widely implemented in solving ELD problem, which is adopted here. In this method, the penalty function is integrated with the objective function in order to satisfy the power balance constraint in (3). The penalty parameter must be chosen carefully to distinguish between feasible and infeasible solutions. The evaluation function $f(P_i)$ is defined as

$$f(P_i) = \sum_{i=1}^{Ng} F_i(P_i) + k \times abs\left(\sum_{i=1}^{Ng} P_i - (P_D + P_L)\right) \tag{20}$$

where, k is the penalty factor for the total active power which does not satisfy the power balance constraints.

Step 3: Initialization of *pbest*, *gbest* and *rbest*

The fitness value of each particle is calculated using (20). Initial particles in Step 1 are set as initial *pbest* values. The best fitness function among the *pbest* value is defined as *gbest*. Then, *rbest* of each particle is randomly selected from other best particles.

Step 4: Update Velocities and Particles Position.

The velocities are updated using (14) within the range of [-v_d^{min}, v_d^{max}]. The maximum velocity of *d*th dimension is computed by

Table 1. Parameter setting for the selected algorithm

Algorithm	w_{min}	w_{max}	c_1	c_2	N_{pop}	J_{max}	R
MPSO-TVAC	0.4	0.9	c_{1i} = 1.0 c_{1f} = 0.2	c_{2i} = 0.2 c_{2f} = 1.0	Test system 1 = 30 Test system 2 = 150 Test system 3 = 200	Test system 1 = 500 Test system 2 = 500 Test system 3 = 700	5
IPSO	0.4	0.9	1.5	1.5			
PSO	0.4	0.9	2.0	2.0			

$$v_d^{\max} = \frac{P_d^{\max} - P_d^{\min}}{R} \tag{21}$$

where, R is the chosen number of interval in d-th dimension. The maximum velocity (v_d^{max}) is set to 20% of the dynamic range of each variable (P^{max}-P^{min}). Then, every particle in the swarm is moved to a new position using (11).

Step 5: Constraints Handling.

The updated position in Step 4 may have violated from inequality constraints in (8) and (9) due to the over or under velocity. If the updated position of ith particle in dth dimension (or generator) is larger/lower than the effective maximum/minimum, the updated position is set to the effective maximum/minimum. This approach ensures that the particles in a swarm are moved around feasible solution only. The adjustments of the updated position to satisfy both constraints are

$$P_{id}^{j+1} = \begin{cases} P_{id}^{j} + v_{id}^{j+1} & if\ P_d^{\min\_new} \le P_{id}^{j+1} \le P_d^{\max\_new} \\ P_d^{\min\_new} & if\ P_{id}^{j+1} \le P_d^{\min\_new} \\ P_d^{\max\_new} & if\ P_{id}^{j+1} \ge P_d^{\max\_new} \end{cases} \tag{22}$$

where,

$$P_d^{\min\_new} = \max(P_d^{\min}, P_d^0 - DR_d) \tag{23}$$

$$P_d^{\max\_new} = \min(P_d^{\max}, P_d^0 + UR_d) \tag{24}$$

where, $P_d^{min\_new}$ and $P_d^{max\_new}$ are the effective minimum and maximum of d-generator.

If the generator output i is violated the POZ constraints in (9), it will be pushed to the nearest boundary of z-th POZ as follows :

$$P_i = \begin{cases} P_{i,z}^{LB} & if\ P_{i,z}^{LB} < P_i \le P_{i,z}^{mean} \\ P_{i,z}^{UB} & if\ P_{i,z}^{mean} < P_i \le P_{i,z}^{UB} \end{cases} \tag{25}$$

where, $P_{i,z}^{mean}$ is the average value of the z-th POZ which calculated as follows:

$$P_{i,Z}^{mean} = \left(\frac{P_{i,z}^{LB} + P_{i,z}^{UB}}{2} \right) \tag{26}$$

Step 6: Update the Swarm.

The updated particles are evaluated using (20). If the current value is better than the previous *pbest*, the current value is stored as *pbest*. Otherwise, it is remained as the previous *pbest*. The *gbest* value is updated as similar to the *pbest*. Then, the *rbest* value for each particle is defined.

Step 7: Termination Condition.

A maximum iteration is applied as the stopping criteria for the algorithm. If the maximum iteration is reached, then MPSO-TVAC algorithm is stopped and the best solution is selected. Otherwise, the algorithm returns to 4.

5. Simulation Results and Performance Analysis

The proposed MPSO-TVAC method is tested on 6, 15 and 38-generator ELD problems with different sizes and complexity. To validate the effectiveness of the proposed algorithm, the test results are compared with PSO and IPSO after 50 different runs.

The obtained results are compared with the results reported in previous work. In this study, the parameter setting used for every case study is listed in Table 1. The simulation was performed using MATLAB 7.6 on Core 2 Quad processor, 2.66 GHz and 4 GB RAM.

5.1 Test system

The first test system consists of six generators with POZ, ramp rate limits and load demand of 1263MW. The cost data and B-loss coefficients are given in [14]. All the generators have ramp rate limits and POZ. The best result reported to date is $15450 [16].

The second test system consists of 15 generators with ramp rate limit and POZ. A load demand of 2630 MW is considered in this case. The input data are taken from [14]. This system has many local minima and the optimum cost reported to date is $32704.50 [34]. All the generators have ramp rate limits and four generators with POZ. The transmission losses are considered both test systems and are calculated using (4).

The third test system consists 38 generators and 6000 MW of load demand. The input data are given in [35].

5.2 Parameter tuning for MPSO-TVAC

The performance of PSO algorithm is influenced by the setting of cognitive and social coefficients (c_1 and c_2). The

Table 2. Influence of acceleration coefficient on MPSO-TVAC performance

Test No.	Acceleration coefficients				6-generator system			15-generator system		
	c_{1i}	c_{1f}	c_{2i}	c_{2f}	Best	Worst	Mean	Best	Worst	Mean
1	1.0	0.4	0.4	1.0	15449.95	15463.27	15451.35	32704.63	32757.88	32707.39
2	1.5	0.4	0.4	1.5	15450.12	15475.6	15452.08	32704.57	32736.45	32706.13
3	2.0	0.4	0.4	2.0	15450.32	15455.82	15451.69	32704.48	32710.15	32705.2
4	2.5	0.4	0.4	2.5	15450.38	15454.8	15451.79	32704.49	32707.69	32704.95
5	**1.0**	**0.2**	**0.2**	**1.0**	**15449.92**	15451.57	15450.17	**32704.47**	32728.99	32705.8
6	1.5	0.2	0.2	1.5	15450.00	15462.56	15451.31	32704.57	32738.54	32706.39
7	2.0	0.2	0.2	2.0	15450.23	15455.04	15451.45	32704.48	32707.73	32705.2
8	2.5	0.2	0.2	2.5	15450.11	15454.1	15451.45	32704.49	32707.69	32704.95

Table 3. Statistical Results of various PSO algorithms (6-generator system)

N_{pop}	Method	Best	Worst	Mean	SD
15	PSO	15459.07	15583.63	15504.23	28.83
	IPSO	15455.84	15585.12	15491.99	26.90
	MPSO-TVAC	**15449.96**	**15477.98**	**15455.45**	**7.38**
20	PSO	15454.03	15592.72	15496.31	28.02
	IPSO	15453.98	15568.81	15489.94	27.08
	MPSO-TVAC	**15449.95**	**15474.66**	**15452.68**	**4.88**
25	PSO	15454.44	15564.56	15485.59	26.74
	IPSO	15453.48	15552.84	15491.86	26.07
	MPSO-TVAC	**15449.92**	**15474.77**	**15452.07**	**5.90**
30	PSO	15454.89	15562.99	15489.48	24.64
	IPSO	15453.49	15534.41	15483.08	18.67
	MPSO-TVAC	**15449.92**	**15451.57**	**15450.17**	**0.37**

Table 4. Statistical Results of various PSO algorithms (15-generator system)

N_{pop}	Method	Best	Worst	Mean	SD
30	PSO	32887.04	33385.72	33148.28	112.80
	IPSO	32751.45	35409.77	33130.82	364.55
	MPSO-TVAC	**32704.89**	**32869.94**	**32751.41**	**44.00**
50	PSO	32785.94	33379.55	33072.68	140.30
	IPSO	32778.81	33303.40	33066.08	136.72
	MPSO-TVAC	**32704.64**	**32780.31**	**32721.59**	**20.54**
100	PSO	32782.78	33504.75	33011.49	142.63
	IPSO	32747.04	33439.69	33041.69	142.42
	MPSO-TVAC	**32704.53**	**32803.98**	**32712.5**	**18.72**
150	PSO	32731.96	33353.83	33013.85	149.77
	IPSO	32721.49	33252.66	32994.95	132.51
	MPSO-TVAC	**32704.47**	**32728.99**	**32705.8**	**3.51**

best combination of c_1 and c_2 depends on the problem. To determine the best combination of c_1 and c_2, the different range of c_1 and c_2 is tested. The value of c_3 varies according to c_1 and c_2 as in (17).

Table 2 shows the results of the best, worst and mean costs after 50 independent runs for test system 1 and 2. Most of the combinations of c_1 and c_2 produce results near to the global optimum solution. However, the combination of $c_{1i}=c_{2f}=1.0$ and $c_{1f}=c_{2i}=0.2$ is found to be the optimum results than others. The same combination of c_1 and c_2 also provides the best results for the 38 generators system.

Tables 3 and 4 show the performances of MPSO-TVAC for different population size according to the number of dimension and complexicty of the problem.It clearly shows that the proposed MPSO-TVAC can obtain the global or near to a global solution for all population size while other PSO methods reach near to the global solution with a large number of population. The population sizes of 30 and 150 were found the best results for both systems respectively. Meanwhile, population size of 200 was found to be the best results for 38 generators system.

5.3 Convergence characteristic

The convergence characteristics of three different PSO strategies are shown in Figs. 5 and 6. It shows that the PSO and IPSO have loss diversify and converge at local minimal after certain iterations. However, the MPSO_TVAC method can reach near to the global value due the extra information provided by other best neighbour particle (*rbest*) and proper tuning of TVAC values. In the early iteration, *rbest* value in (14) helps every particle to explore the entire search space and high value of c_2 exploits the best solution in the latter iteration. It will lead the algorithm to find near to the global optimum effectively.

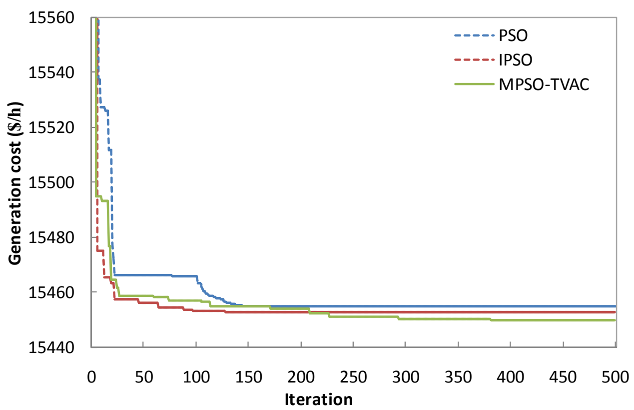

Fig. 5. Convergence characteristic of three PSO strategies for 6-generator system.

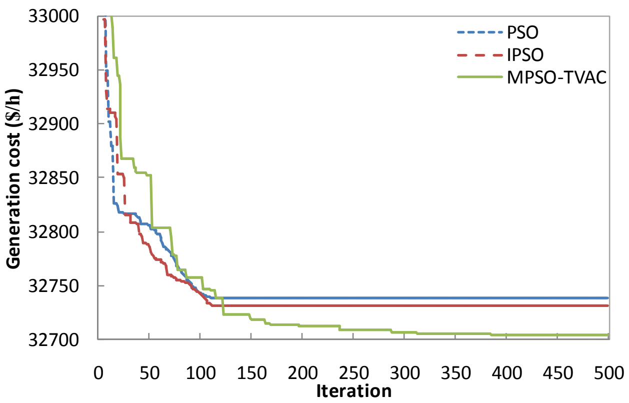

Fig. 6. Convergence characteristic of three PSO strategies for 15-generator system.

5.4 Solution quality

Tables 3 and 4 show the best, worst, mean and standard deviation (SD) cost obtained from 50 runs for three PSO approaches. The best, mean and SD cost obtained by the MPSO-TVAC is lower than other methods, which demonstrates the high solution quality of the proposed method. Tables 5-6 present the best generator output obtained by three PSO algorithms. Due to limited space, only the comparison of the best generation cost for 38 generator system shown in Table 7. It shows that the generation cost obtained by MPSO-TVAC is better than other PSO while satisfying all the operational constraints.

Table 5. Best simulation result for 6-generator system

Power output (MW)	PSO	IPSO	MPSO-TVAC
P_1	446.986	449.802	448.170
P_2	170.196	171.042	173.291
P_3	252.902	250.865	263.145
P_4	150.000	150.000	138.714
P_5	178.780	159.347	165.960
P_6	77.085	94.633	86.691
Generated power	1275.95	1275.69	1275.97
Power loss (P_L)	12.95	12.69	12.97
Total cost ($/h)	15454.90	15453.50	15449.92

Table 6. Best simulation result for 15-generator system

Power output (MW)	PSO	IPSO	MPSO-TVAC
P_1	455.00	455.00	455.00
P_2	380.00	380.00	380.00
P_3	130.00	129.83	130.00
P_4	130.00	130.00	130.00
P_5	154.42	168.25	170.00
P_6	460.00	459.93	459.99
P_7	430.00	430.00	430.00
P_8	60.00	98.26	72.60
P_9	74.27	37.59	58.32
P_{10}	160.00	160.00	159.73
P_{11}	80.00	73.39	80.00
P_{12}	79.60	80.00	80.00
P_{13}	25.00	25.00	25.01
P_{14}	27.59	18.49	15.00
P_{15}	15.00	15.66	15.00
Generated Power	2660.88	2661.40	2660.66
Power loss (P_L)	30.88	31.40	30.66
Total cost ($/h)	32731.96	32721.49	32704.47

Table 7. Best simulation result for 38-generator system

Unit power output	PSO	IPSO	PSO_TVAC [25]	MPSO-TVAC
P_1	220.000	493.570	443.659	425.622
P_2	274.181	445.343	342.956	423.379
P_3	423.862	306.866	433.117	430.236
P_4	500.000	332.036	500.000	426.658
P_5	473.981	305.999	410.539	431.205
P_6	500.000	285.042	482.864	436.116
P_7	317.095	496.616	409.483	430.120
P_8	221.174	498.425	446.079	428.938
P_9	114.000	116.229	119.566	114.152
P_{10}	114.002	114.002	137.274	114.000
P_{11}	143.402	131.982	138.933	117.736
P_{12}	327.418	153.678	155.401	125.966
P_{13}	361.093	148.301	121.719	110.001
P_{14}	145.341	291.907	90.924	90.012
P_{15}	82.000	90.583	97.941	82.000
P_{16}	120.000	122.474	128.106	120.000
P_{17}	110.688	163.420	189.108	159.380
P_{18}	65.000	67.308	65.000	65.031
P_{19}	65.000	103.185	65.000	65.091
P_{20}	245.592	142.167	267.422	271.994
P_{21}	253.769	271.985	221.383	272.000
P_{22}	202.735	254.788	130.804	259.944
P_{23}	80.901	80.008	124.269	128.546
P_{24}	23.239	10.021	11.535	10.030
P_{25}	109.093	70.536	77.103	115.083
P_{26}	91.491	107.021	55.018	88.746
P_{27}	62.193	40.282	75.000	36.511
P_{28}	33.633	20.000	21.682	20.017
P_{29}	21.012	20.461	29.829	20.000
P_{30}	20.000	38.654	20.326	20.000
P_{31}	34.351	44.083	20.000	20.000
P_{32}	25.675	57.134	21.840	20.000
P_{33}	43.475	25.044	25.620	25.001
P_{34}	32.860	34.952	24.261	18.001
P_{35}	21.566	8.000	9.667	8.000
P_{36}	52.511	37.187	25.000	25.000
P_{37}	38.000	37.887	31.642	23.063
P_{38}	29.666	32.824	29.935	22.419
Generated power	6000.000	6000.000	6000.005	6000.000
Total cost ($/h)	9,854,846.460	9,817,444.504	9,500,448.307	9417430.000

5.5 Robustness test

The performance of heuristic method such as PSO algorithm cannot be evaluated by a single run due to the inherent randomness involved in the optimization process. Therefore, the robustness of each PSO algorithm are evaluated based on 50 different runs. The algorithm is robust when it capable to produce consistence results. The best results obtained by three PSO variants after 50 runs are plotted in Figs. 7 and 8. It can be seen that the MPSO-TVAC method achieves consistent result at the lowest cost in every run compared to other PSO methods. Moreover, the smallest SD obtained by the MPSO-TVAC in Tables 3 and 4 also shows that the MPSO-TVAC is more robust than other PSO methods.

5.6 Comparison of best solution

The best result archived by the MPSO-TVAC for 6-generators system is compared with the previous publications of GA [14], PSO [14], PSO_LRS [16], NPSO [16], NPSO_LRS [16] and PSO-TVAC [36] in Table 8. The results show that the MPSO-TVAC provides the minimum cost with less computational time compared to other methods . For 15-generators system, the results obtained by the MPSO-TVAC are compared with GA [14], PSO [14], BF [37], SOH_PSO [17],GA-API [6], PSO-MSAF [38], PSO-TVAC [36] and FA [34] in Table 9. It shows that MPSO-TVAC can produce a better cost and less computational time compared to other methods. Similarly, the results obtained by MPSO-TVAC for 38-generators system are compared with PSO, IPSO and PSO_TVAC [25] in Table 7. From these results, it clearly shows that the proposed MPSO-TVAC has been found to be successful in solving ELD with generators constraints.

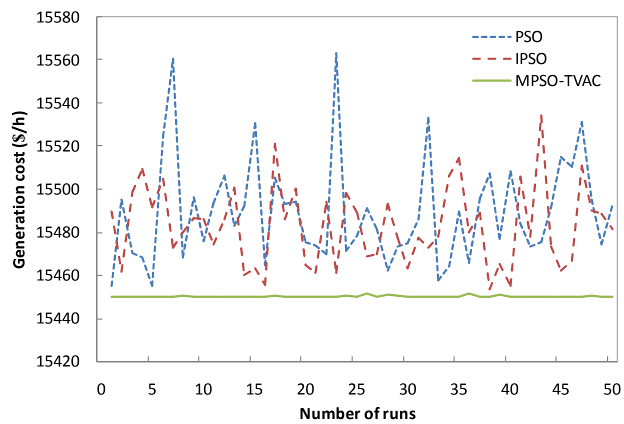

Fig. 7. Best result of variant PSO algorithms for 50 runs (6-generator system).

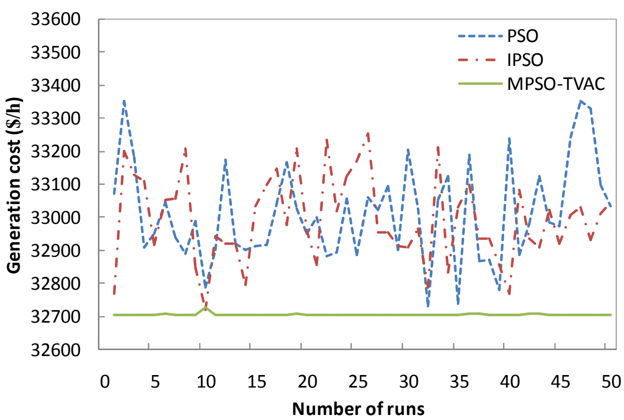

Fig. 8. Best result of variant PSO algorithms for 50 runs (15-generator system).

Table 8. Comparison among different methods after 50 trials (6-generator system)

Method	Generation cost ($/h)				Time (s)
	Best	Worst	Mean	SD	
GA [14]	15459.00	15524.00	15469.00	-	41.58
PSO [14]	15450.00	15455.00	15454.00	-	14.89
PSO_LRS [16]	15450.00	15455.00	15454.00	-	-
NPSO [16]	15450.00	15454.00	15452.00	-	-
NPSO_LRS [16]	15450.00	15452.00	15450.50	-	-
PSO-TVAC [36]	1542.6[a]	1544	1543	-	-
MPSO-TVAC	15449.91	15451.57	15450.17	0.37	1.68

[a] The solution provided in [36] is violated the equality constraints ($\sum P_i \neq P_D + P_L$).
'-': Not reported in the refereed literature.

Table 9. Comparison among different methods after 50 trials (15-generator system)

Method	Generation cost ($/h)				Time (S)
	Best	Worst	Mean	SD	
GA [14]	33113.00	33337.00	33228.00	-	49.31
PSO [14]	32858.00	33331.00	33039.00	-	26.59
BF [37]	32784.50	-	32796.80	85.77	-
SOH_PSO [17]	32751.00	32945.00	32878.00	-	0.0936
GA-API [6]	32732.95	32756.01	32736.06	-	-
PSO-MSAF [38]	32713.09	32798.25	32759.64	-	19.15
PSO-TVAC [36]	32705.75[a]	33197.01	32954.4	-	-
FA [34]	32704.50	33175.00	32856.10	147.17	16.05
MPSO-TVAC	32704.47	32728.99	32705.00	3.51	12.78

[a] The solution provided in [36] is violated the equality constraints ($\sum P_i \neq P_D + P_L$).
'-': not available in the refereed literature

6. Conclusion

This paper has proposed a MPSO-TVAC algorithm for solving nonconvex ELD problem considering generator limit, POZ, ramp rate limits and transmission loss. The proposed algorithm introduced a new best neighbour particle (*rbest*) in velocity equation which helps the particle to explore the entire solution space thus avoiding a premature convergence. Moreover, the used of time varying acceleration coefficients (TVAC) for c_1, c_2 and c_3 enhanced exploration and exploitation of the proposed algorithm. The MPSO-TVAC performances have been compared with some PSO variants for three benchmark power systems after 50 different runs. The simulation results have shown that the MPSO-TVAC has the ability to obtain lower generation cost and is more robust compared to PSO and other method reported in literature. These studies validate the effectiveness and applicability of the proposed algorithm for solving ELD problems.

Acknowledgements

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References

[1] A.J. Wood and B.F. Wollenberg. Power Generation, Operation and Control. 2nd ed. John Wiley and Sons, New York, 1996.

[2] S.O. Orero and M.R. Irving. Economic dispatch of generators with prohibited operating zones: a genetic algorithm approach. *IEE Proceedings Generation, Transmission and Distribution*. 143 (1996) 529-34.

[3] C. Wang and S.M. Shahidehpour. Effects of ramp-rate limits on unit commitment and economic dispatch. *IEEE Transactions on Power Systems*. 8 (1993) 1341-50.

[4] R.R. Shoults, S.V. Venkatesh, S.D. Helmick, G.L. Ward and M.J. Lollar. A Dynamic Programming Based Method for Developing Dispatch Curves When Incremental Heat Rate Curves Are Non-Monotonically Increasing. *IEEE Transactions on Power Systems*. 1 (1986) 10-6.

[5] N. Amjady and H. Nasiri-Rad. Economic dispatch using an efficient real-coded genetic algorithm. *IET Generation, Transmission & Distribution*. 3 (2009) 266-78.

[6] I. Ciornei and E. Kyriakides. A GA-API Solution for the Economic Dispatch of Generation in Power System Operation. *IEEE Transactions on Power Systems*. 27 (2012) 233-42.

[7] N. Sinha, R. Chakrabarti and P.K. Chattopadhyay. Evolutionary programming techniques for economic load dispatch. *IEEE Transactions on Evolutionary Computation*. 7 (2003) 83-94.

[8] M. Sharma, M. Pandit and L. Srivastava. Reserve constrained multi-area economic dispatch employing differential evolution with time-varying mutation. *International Journal of Electrical Power & Energy Systems*. 33 (2011) 753-66.

[9] S. Pothiya, I. Ngamroo and W. Kongprawechnon. Ant colony optimisation for economic dispatch problem with non-smooth cost functions. *International Journal of Electrical Power & Energy Systems*. 32 (2010) 478-87.

[10] W. Sa-ngiamvibool, S. Pothiya and I. Ngamroo. Multiple tabu search algorithm for economic dispatch problem considering valve-point effects. International *Journal of Electrical Power & Energy Systems*. 33 (2011) 846-54.

[11] K.P. Wong and C.C. Fung. Simulated annealing based economic dispatch algorithm. *IEE Proceedings C Generation, Transmission and Distribution*. 140 (1993) 509-15.

[12] V.B.A. Kasangaki, H.M. Sendaula and S.K. Biswas. Stochastic Hopfield artificial neural network for unit commitment and economic power dispatch. *Electric Power Systems Research*. 42 (1997) 215-23.

[13] Y. Wang, J. Zhou, Y. Lu, H. Qin and Y. Wang. Chaotic self-adaptive particle swarm optimization algorithm for dynamic economic dispatch problem with valve-point effects. *Expert Systems with Applications*. 38 (2011) 14231-7.

[14] G. Zwe-Lee. Particle swarm optimization to solving the economic dispatch considering the generator constraints. *IEEE Transactions on Power Systems*. 18 (2003) 1187-95.

[15] J.-B. Park, K.-S. Lee, J.-R. Shin and K.Y. Lee. A particle swarm optimization for economic dispatch with nonsmooth cost functions. *IEEE Transactions on Power Systems*. 20 (2005) 34-42.

[16] A.I. Selvakumar and K. Thanushkodi. A New Particle Swarm Optimization Solution to Nonconvex Economic Dispatch Problems. *IEEE Transactions on Power Systems*. 22 (2007) 42-51.

[17] K.T. Chaturvedi, M. Pandit and L. Srivastava. Self-Organizing Hierarchical Particle Swarm Optimization for Nonconvex Economic Dispatch. *IEEE Transactions on Power Systems*. 23 (2008) 1079-87.

[18] J. Cai, X. Ma, L. Li and P. Haipeng. Chaotic particle swarm optimization for economic dispatch considering the generator constraints. *Energy Conversion and Management*. 48 (2007) 645-53.

[19] D. Vo Ngoc, P. Schegner and W. Ongsakul. A newly improved particle swarm optimization for economic dispatch with valve point loading effects. *2011 IEEE Power and Energy Society General Meeting*. 2011. pp. 1-8.

[20] J.-B. Park, Y.-W. Jeong, J.-R. Shin and K.Y. Lee. An Improved Particle Swarm Optimization for Nonconvex Economic Dispatch Problems. *IEEE Transactions on Power Systems*. 25 (2010) 156-66.

[21] H. Lu, P. Sriyanyong, Y.H. Song and T. Dillon. Experimental study of a new hybrid PSO with mutation for economic dispatch with non-smooth cost function. *International Journal of Electrical Power & Energy Systems*. 32 (2010) 921-35.

[22] T.-Y. Lee and C.-L. Chen. Unit commitment with probabilistic reserve: An IPSO approach. *Energy Conversion and Management*. 48 (2007) 486-93.

[23] A. Safari and H. Shayeghi. Iteration particle swarm optimization procedure for economic load dispatch with generator constraints. *Expert Systems with Applications*. 38 (2011) 6043-8.

[24] A. Ratnaweera, S.K. Halgamuge and H.C. Watson. Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. *IEEE Transactions on Evolutionary Computation*. 8 (2004)

240-55.

[25] K.T. Chaturvedi, M. Pandit and L. Srivastava. Particle swarm optimization with time varying acceleration coefficients for non-convex economic power dispatch. *International Journal of Electrical Power & Energy Systems*. 31 (2009) 249-57.

[26] H. Saadat. Power System Analysis. 2 ed. *McGraw Hill* 2002.

[27] J. Kennedy and R. Eberhart. Particle swarm optimization. *IEEE International Conference on Neural Networks*. 1995. pp. 1942-8, Vol. 4.

[28] T. Niknam, H.D. Mojarrad and M. Nayeripour. A new fuzzy adaptive particle swarm optimization for non-smooth economic dispatch. *Energy*. 35 (2010) 1764-78.

[29] Y. Shi and R.C. Eberhart. Empirical study of particle swarm optimization. *1999 CEC 99 Proceedings of the 1999 Congress on Evolutionary Computation*. 1999. p. 1950, Vol. 3.

[30] D.N. Jeyakumar, T. Jayabarathi and T. Raghunathan. Particle swarm optimization for various types of economic dispatch problems. *International Journal of Electrical Power & Energy Systems*. 28 (2006) 36-42.

[31] R.C. Eberhart and Y. Shi. Comparing inertia weights and constriction factors in particle swarm optimization. *Proceedings of the Congress on Evolutionary Computation*. 2000. pp. 84-8, Vol. 1.

[32] X. Yuan, A. Su, Y. Yuan, H. Nie and L. Wang. An improved PSO for dynamic load dispatch of generators with valve-point effects. *Energy*. 34 (2009) 67-74.

[33] B. Mohammadi-ivatloo, A. Rabiee and M. Ehsan. Time-varying acceleration coefficients IPSO for solving dynamic economic dispatch with non-smooth cost function. *Energy Conversion and Management*. 56 (2012) 175-83.

[34] X.-S. Yang, S.S. Sadat Hosseini and A.H. Gandomi. Firefly Algorithm for solving non-convex economic dispatch problems with valve loading effect. *Applied Soft Computing*. 12 (2012) 1180-6.

[35] M. Sydulu. A very fast and effective non-iterative “lambda logic based” algorithm for economic dispatch of thermal units. *TENCON 99 Proceedings of the IEEE Region 10 Conference1999*. pp. 1434-7, Vol. 2.

[36] O. Abedinia, N. Amjady and K. Kiani. Optimal Complex Economic Load Dispatch Solution Using Particle Swarm Optimization with Time Varying Acceleration Coefficient. *International Review of Electrical Engineering*. 7 (2012) 8.

[37] B.K. Panigrahi and V.R. Pandi. Bacterial foraging optimisation: Nelder-Mead hybrid algorithm for economic load dispatch. *IET Generation, Transmission & Distribution*. 2 (2008) 556-65.

[38] P. Subbaraj, R. Rengaraj, S. Salivahanan and T.R. Senthilkumar. Parallel particle swarm optimization with modified stochastic acceleration factors for solving large scale economic dispatch problem. *International Journal of Electrical Power & Energy Systems*. 32 (2010) 1014-23.

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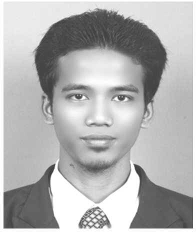

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