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On Power Allocation Schemes for Bi-directional Communication in a Spectrum Sharing-based Cognitive Radio System

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** Regular Paper*

Abstract**:** This paper presents the results of an investigation into bi-directional communication in spectrum sharing-based cognitive radio (Bi-CR) systems. A Bi-CR system can increase the spectral efficiency significantly by sharing the spectrum and through the bi-directional use of spatial resources for two-way communication. On the other hand, the primary user experiences more interference from the secondary users in a Bi-CR system. Satisfying the interference constraint by simply reducing the transmission power results in performance degradation for secondary users. In addition, secondary users also experience self-interference from echo channels due to full duplexing. These imperfections may weaken the potential benefits of the Bi-CR system. Therefore, a new way to overcome these defects in the Bi-CR system is needed.

To address this need, this paper proposes some novel power allocation schemes for the Bi-CR system. This contribution is based on two major analytic environments, i.e., noise-limited and interference-limited environments, for providing useful analysis. This paper first proposes an optimal power allocation (OPA) scheme in a noise-limited environment and then analyzes the achievable sum rates. This OPA scheme has an effect in the noise-limited environment. In addition, a power allocation scheme for the Bi-CR system in an interference-limited environment was also investigated. The numerical results showed that the proposed schemes can achieve the full duplexing gain available from the bi-directional use of spatial resources.

Keywords**:** Cognitive radio, Spectrum sharing, Bi-directional communication

1. Introduction

In cognitive radio (CR) networks, unlicensed users, which are known as secondary users, can either access the primary user's spectrum opportunistically when they are not occupied by the primary users (a process known as ***spectrum overlay***), or the secondary users are allowed to transmit together with the primary users over the same spectra simultaneously (a process known as ***spectrum underlay*** or ***spectrum sharing***) [1]. This paper focuses on spectrum sharing-based CR networks. Theoretically, spectrum sharing offers a way of improving the spectrum utilization without affecting the existing legacy systems [2]

A number of active studies have been conducted on spectrum sharing-based CR systems. The fundamental capacity of a secondary user in fading environments was investigated in [3-5]. Ghasemi *et al.* and Musavian *et al.* examined the capacity of the secondary link in the Rayleigh and Nakagami fading models [3, 4]. Kang *et al.* analyzed the ergodic and outage capacities of the secondary link in the delay-limited environments [5].

On the other hand, these studies focused only on conventional one-way communication for the secondary users in spectrum sharing environments. Achieving inherent improvement in spectral efficiency means considering two-way communication for secondary users. The most common method used for two-way communication in conventional communication systems is to use time division duplexing (TDD) or frequency division duplexing (FDD). One of the fundamental problems with these conventional methods is that they significantly degrade the capacity performance by up to 50% [6, 7]. This is because the portion of time or bandwidth used in each link acts as a fractional pre-log factor of the achievable sum rate [6]. Any decrease in the pre-log factor degrades the system capacity significantly.

Attempts to resolve this problem have been made in the form of a scheme for the bi-directional use of spatial resources in multiple-input-multiple-output (MIMO) systems proposed to enhance the spectral efficiency in two-way communication [7-9]. Ju *et al.* proposed and analyzed a duplexing scheme that uses spatial resources bi-directionally [7]. In addition, they examined two-way communication in a correlated point-to-point MIMO channel with their proposed bi-directional beamforming scheme [8]. They also analyzed the capacity bounds in pairwise two-way communications using the bi-directional beamforming scheme [9]. These studies showed that the bi-directional use of a spatial resource is more beneficial in terms of the achievable sum rates when spatial multiplexing is used in MIMO channels.

Inspired by the potential benefits of spectrum sharing-based CR networks and the bi-directional use of spatial resources, this paper proposes a system of bi-directional communication in a spectrum sharing-based cognitive radio, or Bi-CR, as a way to increase the spectral efficiency. In the proposed Bi-CR system, the secondary users share the spectrum of the primary user simultaneously subject to an interference constraint. Furthermore, the secondary users transmit their signals by the bi-directional use of the spatial resource. Because this doubles the shared spectrum use compared to conventional CR systems, the Bi-CR system can theoretically enhance the spectral efficiency.

Unlike conventional spectrum sharing-based CR systems, the primary user receives more interference from the secondary users in a Bi-CR system. Consequently, to satisfy the interference constraint, the transmission powers of the secondary users need to be lower than that in the conventional systems. Reducing the transmission power arbitrarily, however, causes performance degradation of the secondary users in the Bi-CR system. In addition, the secondary users also experience the self-interference from the echo channel due to full duplexing. Basically, because self-interference is very strong without an interference cancellation (IC), it should ideally be cancelled for full duplexing systems. On the other hand, the residual interference left over after IC still causes performance degradation [10]. Consequently, these weaknesses may degrade the performance of the Bi-CR system. Therefore, a new power allocation scheme to overcome these imperfections is needed to utilize the benefits of the Bi-CR system.

To this end, this paper proposes a novel power allocation scheme for the Bi-CR system. This study deals with two significant environments in the CR system, noise-limited and interference-limited environments. In the noise-limited environment, it is assumed that the self-interference caused by echo channels can be eliminated sufficiently. On the other hand, in the interference-limited environment, it is assumed that the residual interference after echo cancellation is larger than the noise level. This paper first proposes an optimal power allocation (OPA) scheme in the noise-limited environment, and then analyzes the achievable sum rate of the secondary users. Next, a power allocation scheme in high signal-to-interference (SIR) and low SIR regions in interference limited-environments was evaluated.

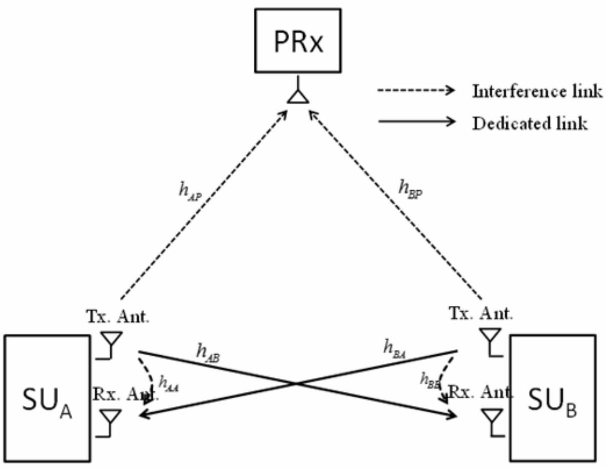


Fig. 1. Bi-directional communication in a spectrum sharing-based cognitive radio system.

2. Bi-Directional Communication in a Spectrum Sharing-based Cognitive Radio System Model

Fig. 1 shows the Bi-CR system model. In this figure, SU_A and SU_B represent the secondary users communicating using a bi-directional (BD) communication scheme, and PRx denotes the primary receiver. In the BD system, it is assumed that each node has two antennae, where one antenna is used for receiving and the other is used for transmission.

In this system, the signals received at two nodes SU_A and SU_B can be expressed as

$$\begin{aligned} y_A &= h_{BA}x_B + h_{AA}x_A + n_A, \\ y_B &= h_{AB}x_A + h_{BB}x_B + n_B, \end{aligned} \tag{1}$$

where y_A and y_B are the received signals at node SU_A and SU_B, respectively, x_A and x_B denote the transmit signals at SU_A and SU_B, respectively, and n_A and n_B denote the additive white Gaussian noise (AWGN) at SU_A and SU_B which have zero means and σ_A^2 and σ_B^2 variances, respectively. In addition, h_{AB} and h_{BA} are channels for the SU_A - SU_B and SU_B - SU_A links, respectively, and h_{AA} and h_{BB} are the echo channels at

SU_A and SU_B, respectively.

In this BD system, each transmitter sends its information simultaneously. Therefore, the interference induced in the BD system is caused by the system's own transmit signal. This interference is included in the received signal. Each channel is assumed to have an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero means and variances λ_{AB} and λ_{BA}, λ_{AA} and λ_{BB}, respectively.

In spectrum sharing-based CR systems, the secondary users must satisfy an interference power constraint to protect the performance of the primary users. This applies well for the proposed system, where the secondary nodes, SU_A and SU_B, must also satisfy the interference power constraint. On the other hand, because the PRx receives interference from the secondary nodes, SU_A and SU_B, simultaneously, the transmission powers of SU_A and SU_B must satisfy the following constraint:

$$\left|h_{AP}\right|^2 P_A + \left|h_{BP}\right|^2 P_B \le I_{th}, \tag{2}$$

where h_{AP} and h_{BP} are the interference channels from SU_A and SU_B to the PRx, respectively, and P_A and P_B are the transmission powers of SU_A and SU_B, respectively. h_{AP} and h_{BP} are assumed to be distributed in complex Gaussian distribution with a zero mean and variance λ_{AP} and λ_{BP}, respectively.

In addition, SU_A and SU_B experience interference from the primary transmitter (PTx) as well in a spectrum sharing-based CR system. This interference is generally non-white [15]. By applying a noise-whitening filter at SU_A and SU_B, and incorporating the filter effects into the channels, the effective noise is assumed to be approximately white Gaussian [15-18]. The validity of this assumption is supported by considering the “low-interference regime” in [16]. Therefore, this interference was considered to be AWGN in this paper.

An achievable sum rate for the secondary users was used to evaluate the performance of the Bi-CR system. In two-way communication systems, an achievable sum rate for each pair is the sum of an achievable rate at each link normalized to the amount of resource used. An achievable sum rate in the Bi-CR system can be expressed as

$$\begin{aligned} C^{Bi} &= \max_{P_A, P_B \ge 0} C_{AB}^{Bi} + C_{BA}^{Bi}, \\ s.t \quad & \left|h_{AP}\right|^2 P_A + \left|h_{BP}\right|^2 P_B \le I_{th}, \end{aligned} \tag{3}$$

where C_{AB}^{Bi} and C_{BA}^{Bi} represent the achievable rates for the SU_A - SU_B and SU_B - SU_A links, respectively. As mentioned earlier, because the total power of SU_A and SU_B are constrained, the transmission powers of SU_A and SU_B, which maximize the achievable sum rate C^{Bi}, must be optimized.

3. Power Allocation for Secondary Users in Noise- and Interference-Limited Environments

This section focuses on the power allocation scheme for the Bi-CR system. For the power allocation of secondary users as a way to enhance the achievable sum rates, two significant environments in the CR system are considered, i.e., the noise-limited and interference-limited environments. We begin by proposing an optimal power allocation (OPA) scheme and then analyzing an achievable sum rate in the noise-limited environment. This OPA scheme has only the effect in the noise-limited environment. In addition, because the OPA does not exist in the interference-limited environment, the best power allocation in high and low SIR regions was also analyzed.

3.1 OPA and Achievable Sum Rate in Noise-Limited Environments

In BD systems, the induced interference is caused by the system's own transmit signal. When the antennae are separated into mutually exclusive transmit and receive antennae, it can be assumed that the echo interference can be suppressed by the insulation or subtraction of echo [11-14]. Up to 90dB of echo interference can be insulated by isolating the antennae [13, 14]. In addition, echo interference in the BD system can be subtracted from the desired signal because each node knows its transmitted signal and the amount of echo interference is sufficient to estimate the echo interference channel. If the echo interference can be eliminated sufficiently, it is reasonable to assume a noise-limited environment in the Bi-CR system.

In a noise-limited environment, the achievable rate for each link can be described as follows [20]:

$$\begin{aligned} C_{AB}^{Bi} &= E\left[\log\left(1+\frac{\left|h_{AB}\right|^2 P_A}{\sigma_B^2}\right)\right], \\ C_{BA}^{Bi} &= E\left[\log\left(1+\frac{\left|h_{BA}\right|^2 P_B}{\sigma_A^2}\right)\right], \end{aligned} \tag{4}$$

where $E\,[\cdot]$ denotes the expectation operator. Therefore, the optimization problem for maximizing the achievable sum rates of the secondary users can be expressed as

$$\begin{aligned} C_N^{Bi} = \max_{P_A, P_B \ge 0} \; & E\left[\log\left(1+\frac{\left|h_{AB}\right|^2 P_A}{\sigma_B^2}\right)\right] \\ & + E\left[\log\left(1+\frac{\left|h_{BA}\right|^2 P_B}{\sigma_A^2}\right)\right], \\ s.t \quad & \left|h_{AP}\right|^2 P_A + \left|h_{BP}\right|^2 P_B \le I_{th}. \end{aligned} \tag{5}$$

This achievable sum rate is a concave function with respect to P_A or P_B. Therefore, because an achievable sum rate of secondary users has a maximum value, the optimal transmission powers of P_A and P_B can be derived to maximize the achievable sum rate.

Therefore, the OPA is a solution where the first derivative of an achievable sum rate with respect to P_A is zero. The first derivative can be expressed as

$$\frac{\partial C_N^{Bi}}{\partial P_A} = \frac{|h_{AB}|^2}{\sigma_B^2 + |h_{AB}|^2 P_A} - \frac{|h_{AP}|^2 |h_{BA}|^2}{|h_{BP}|^2 \sigma_A^2 - |h_{AP}|^2 |h_{BA}|^2 P_A + |h_{BA}|^2 I_{th}}. \tag{6}$$

Therefore, the optimal transmission power P_A for maximizing an achievable sum rate can be derived as

$$P_A = \underbrace{\frac{|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 - |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{AP}|^2 |h_{AB}|^2 |h_{BA}|^2}}_{\text{joint allocation}} + \underbrace{\frac{I_{th}}{2|h_{AP}|^2}}_{\text{non-joint allocation}}. \tag{7}$$

The transmission power P_B can be expressed as a function of P_A as follows:

$$P_B = \frac{1}{|h_{BP}|^2}\left(I_{th} - |h_{AP}|^2 P_A\right). \tag{8}$$

Therefore, the optimal transmission power, P_B, is also derived as

$$P_B = \underbrace{\frac{-|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 + |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{BP}|^2 |h_{AB}|^2 |h_{BA}|^2}}_{\text{joint allocation}} + \underbrace{\frac{I_{th}}{2|h_{BP}|^2}}_{\text{non-joint allocation}}. \tag{9}$$

As shown in (7) and (9), the OPA for SU_A and SU_B consists of joint allocation and non-joint allocation parts. If SU_A and SU_B are not in a cooperative power allocation arrangement, then each node sets its own transmit power when the interference threshold is set to half the conventional value, i.e., $I_{th} \to I_{th}/2$. On the other hand, this does not entirely guarantee the full-duplexing performance gain because the non-joint power allocation cannot adapt to the channel conditions. Therefore, to use the OPA scheme, it is important to adopt a joint power allocation scheme between SU_A and SU_B.

In (7) and (9), the transmission powers, P_A and P_B, are limited by the interference power constraint as follows:

$$0 \le P_A \le \frac{I_{th}}{|h_{AP}|^2}, \quad 0 \le P_B \le \frac{I_{th}}{|h_{BP}|^2} \tag{10}$$

Therefore, (10) can be rewritten as

$$\frac{\mu_A}{1+\mu_A} \le \mu_B \le \frac{\mu_A}{1-\mu_A}, \tag{11}$$

where μ_A and μ_B are the received signal-to-noise ratios (SNRs) at SU_A and SU_B, respectively, when each node transmits its own signal separately in a conventional spectrum sharing-based CR system as follows:

$$\mu_A = \frac{\|\mathbf{h}_{\mathbf{BA}}\|^2}{|h_{BP}|^2}\frac{I_{th}}{\sigma_A^2}, \quad \mu_B = \frac{\|\mathbf{h}_{\mathbf{AB}}\|^2}{|h_{AP}|^2}\frac{I_{th}}{\sigma_B^2}. \tag{12}$$

Consequently, μ_A and μ_B indicate the relative channel condition for power allocation for each node. For example, if $\mu_A > \mu_B$, the channel condition for SU_A is better than that for SU_B. In this case, more transmission power is allocated to SU_A to maximize the achievable sum rate. Therefore, the allocated power is determined by the ranges of μ_A and μ_B.

Accordingly, the OPA for maximizing the achievable sum rate of the secondary user in the Bi-CR system can be rewritten as

i. $\mu_A < 1$

$$P_A = \begin{cases} 0, & \mu_B < \dfrac{\mu_A}{1+\mu_A} \\ \dfrac{|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 - |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{AP}|^2 |h_{AB}|^2 |h_{BA}|^2} + \dfrac{I_{th}}{2|h_{AP}|^2}, & \dfrac{\mu_A}{1+\mu_A} \le \mu_B \le \dfrac{\mu_A}{1-\mu_A} \\ \dfrac{I_{th}}{|h_{AP}|^2}, & \mu_B > \dfrac{\mu_A}{1-\mu_A} \end{cases}$$

$$P_B = \begin{cases} \dfrac{I_{th}}{|h_{BP}|^2}, & \mu_B < \dfrac{\mu_A}{1+\mu_A} \\ \dfrac{-|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 + |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{BP}|^2 |h_{AB}|^2 |h_{BA}|^2} + \dfrac{I_{th}}{2|h_{BP}|^2}, & \dfrac{\mu_A}{1+\mu_A} \le \mu_B \le \dfrac{\mu_A}{1-\mu_A} \\ 0, & \mu_B > \dfrac{\mu_A}{1-\mu_A} \end{cases} \tag{13}$$

ii. $\mu_A \ge 1$

$$P_A = \begin{cases} 0, & \mu_B < \dfrac{\mu_A}{1+\mu_A} \\ \dfrac{|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 - |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{AP}|^2 |h_{AB}|^2 |h_{BA}|^2} + \dfrac{I_{th}}{2|h_{AP}|^2}, & \dfrac{\mu_A}{1+\mu_A} \le \mu_B \end{cases}$$

$$P_B = \begin{cases} \dfrac{I_{th}}{|h_{BP}|^2}, & \mu_B < \dfrac{\mu_A}{1+\mu_A} \\ \dfrac{-|h_{BP}|^2 |h_{AB}|^2 \sigma_A^2 + |h_{AP}|^2 |h_{BA}|^2 \sigma_B^2}{2|h_{BP}|^2 |h_{AB}|^2 |h_{BA}|^2} + \dfrac{I_{th}}{2|h_{BP}|^2}, & \dfrac{\mu_A}{1+\mu_A} \le \mu_B \end{cases} \tag{14}$$

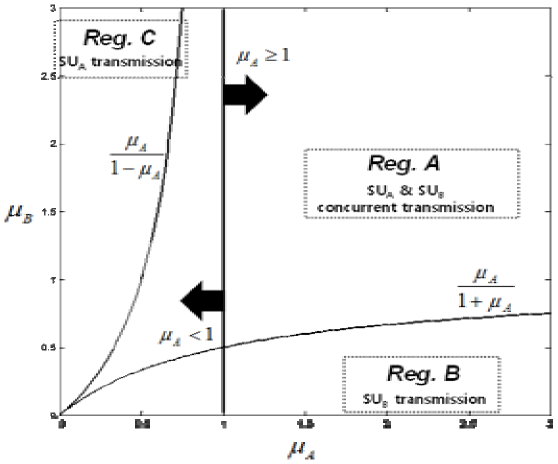


Fig. 2. Optimal power allocation region with respect to μ_A and μ_B in the Bi-CR system.

As shown in (13) and (14), the OPAs for SU_A and SU_B consist of all channel gains, noise variances and the interference threshold. Each OPA is limited by the range of μ_A and μ_B. This is because the maximum transmission power that satisfies the interference constraint at each node is limited in (10).

Therefore, the OPA region exists with respect to μ_A and μ_B, as shown in Fig. 2. Three regions exist in this figure: *Reg. A*, *B*, and *C*. *Reg. B* indicates a region where $\mu_B < \mu_A/(1+\mu_A)$. In this region, allocating more transmission power to P_B increases the achievable sum rate because μ_B is relatively lower than μ_A. Therefore, the transmission power is allocated only to P_B in *Reg. B*. On the other hand, the transmission power is only allocated to P_A in Reg. C because μ_A is relatively lower than μ_B. On the other hand, in *Reg. A*, because both channel conditions at SU_A and SU_B are relatively good, the transmission power is allocated to both SU_A and SU_B simultaneously using the proposed OPA scheme.

To obtain an achievable sum rate using the OPA scheme, the probability density functions (PDFs) of μ_A and μ_B are first derived in (12). Basically, when m and n follow an independent exponential distribution with mean values of λ_m and λ_n, respectively, the PDF of $z = k\frac{m}{n}$ ($k > 0$) is as follows [24]

$$f_Z(z) = \frac{k\lambda_m\lambda_n}{(k\lambda_m + \lambda_n z)^2} = \frac{\dfrac{\lambda_n}{k\lambda_m}}{\left(1+\dfrac{\lambda_n}{k\lambda_m}z\right)^2}. \tag{15}$$

Using (15), the PDFs of μ_A and μ_B can be derived as

$$f_{\mu_A}(\mu_A) = \frac{\alpha}{(1+\alpha\mu_A)^2}, \quad \alpha = \frac{\sigma_A^2\lambda_{BP}}{I_{th}\lambda_{BA}}, \tag{16}$$

$$f_{\mu_B}(\mu_B) = \frac{\beta}{(1+\beta\mu_B)^2}, \quad \beta = \frac{\sigma_B^2\lambda_{AP}}{I_{th}\lambda_{AB}}. \tag{17}$$

In (16) and (17), α and β are the reciprocals of the mean values of μ_A and μ_B, respectively. Therefore, the reciprocals of α and β are the average received SNRs at SU_A and SU_B when each node transmits its own signal separately in a conventional spectrum sharing-based uni-directional communication system.

To derive an achievable sum rate using the OPA scheme, the OPA is adopted based on the range of μ_A and μ_B. An achievable sum rate of secondary users can then be expressed as

$$\begin{aligned}
C_N^{Bi} &= E\left[\log\left(1+\frac{|h_{AB}|^2 P_A}{\sigma_B^2}\right)+\log\left(1+\frac{|h_{BA}|^2 P_B}{\sigma_A^2}\right)\right] \\
&= \int_0^\infty\int_0^{\frac{\mu_A}{1+\mu_A}}\log(1+\mu_A)\, f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A \\
&+\int_0^1\int_{\frac{\mu_A}{1-\mu_A}}^\infty \log(1+\mu_B)\, f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A \\
&+\int_0^1\int_{\frac{\mu_A}{1+\mu_A}}^{\frac{\mu_A}{1-\mu_A}}\left\{\log\left(\frac{1}{2}+\frac{\mu_B}{2\mu_A}+\frac{\mu_B}{2}\right)\right. \\
&\left.+\log\left(\frac{1}{2}+\frac{\mu_A}{2\mu_B}+\frac{\mu_A}{2}\right)\right\} f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A n \\
&+\int_1^\infty\int_{\frac{\mu_A}{1+\mu_A}}^\infty\left\{\log\left(\frac{1}{2}+\frac{\mu_B}{2\mu_A}+\frac{\mu_B}{2}\right)\right. \\
&\left.+\log\left(\frac{1}{2}+\frac{\mu_A}{2\mu_B}+\frac{\mu_A}{2}\right)\right\} f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A.
\end{aligned} \tag{18}$$

Because the optimal transmission powers of SU_A and SU_B are classified according to the range of μ_A and μ_B, as shown in Fig. 2, the transmission powers are employed based on the range of these values, as shown in (18).

Unfortunately, to the best of the authors' knowledge, the integrals in (18) cannot be evaluated in a closed form. Therefore, the bound of an achievable sum rate is derived using a high SNR approximation. Basically, the log term in the capacity is approximated as follows:

$$\log(1+x) \approx \log(x), \quad \text{for large } x. \tag{19}$$

Using this property, the lower bound of an achievable sum rate can be derived via the following Lemma.

Lemma 1. (**Lower Bound for an achievable Sum Rate in the Bi-CR System**)

The lower bound of an achievable sum rate in the Bi-CR system is derived as

$$C^{Bi} > C^{Bi}_{N_{LB}} = \frac{\alpha\beta\log\left(\frac{\beta}{1+\beta}\right)+\alpha\log\left(\frac{4\beta}{1+\beta}\right)+\log\left(\frac{1+\beta}{4\alpha\beta}\right)}{(\alpha-1)(\beta^2-1)} + \frac{\beta\left((\beta-1)\log(\alpha)+\log\left(\frac{\alpha(\beta+1)}{\beta}\right)\right)}{(\alpha-1)(\beta^2-1)} \tag{20}$$

Proof. The derivation process is provided in Appendix I

To investigate the effect of the interference link channel gain between the secondary and primary users, it is assumed that the channel gains and noise variances corresponding to SU_A and SU_B are symmetrical, i.e., $\alpha = \beta$ in (16) and (17) [8]. An achievable sum rate can then be

$$C^{Bi}_{N_{LB}} = \frac{\alpha\left(\log 4+2\alpha\log\alpha-\alpha\log(1+\alpha)\right)+\log\left(\frac{1+\alpha}{4\alpha^2}\right)}{(\alpha+1)(\alpha-1)^2}. \tag{21}$$

This sum rate is a decreasing function as α increases. α consists of the interference-threshold-to-noise-ratio (ITNR) and the mean values of the channel gains $|h_{AP}|^2$, $|h_{AB}|^2$. Therefore, an achievable sum rate increases with increasing ITNR or the channel gain of the secondary link. In addition, an achievable sum rate increases as the channel gain for the link between SU_A and PRx decreases. This is because the level of interference decreases with decreasing channel gain for the link between SU_A and PRx. Therefore, each node can increase its own transmission power within the interference power constraint in the Bi-CR system. This means that the secondary users in the Bi-CR system can achieve a performance gain by sharing the primary user's spectrum, provided the echo interference can be eliminated sufficiently.

3.2 Power Allocation for the Bi-CR System in the Interference-Limited Environment

If self-interference by the echo channel cannot be eliminated sufficiently, an interference-limited environment in the Bi-CR system can be assumed. Basically, because the self-interference power is greater than the noise and received signal powers, the residual interference can significantly affect the performance. This makes it important to analyze the power allocation for the Bi-CR system in an interference-limited environment.

In an interference limited environment, the achievable sum rate can be expressed as follows:

$$\begin{aligned} C_I^{Bi} = \max_{P_A,P_B\geq 0} & E\left[\log\left(1+\frac{|h_{AB}|^2 P_A}{|h_{BB}|^2 P_B}\right)\right] \\ & + E\left[\log\left(1+\frac{|h_{BA}|^2 P_B}{|h_{AA}|^2 P_A}\right)\right], \\ s.t \quad & |h_{AP}|^2 P_A + |h_{BP}|^2 P_B \leq I_{th}. \end{aligned} \tag{22}$$

Unfortunately, this achievable sum rate is non-convex with respect to P_A and P_B. Thus, it is difficult to find the optimal power in the interference-limited environment. Therefore, the local optimized solutions, i.e., the best power allocation schemes in the high and low SIR regions, need to be analyzed in this paper.

In the high SIR region, the residual interference is relatively low compared to the received signal but is still larger than the noise level. An achievable sum rate of the secondary users in this case can be approximated as

$$C_I^{Bi} \approx C^{Bi}_{I_{hi}} = \log\left(\frac{|h_{AB}|^2 P_A}{|h_{BB}|^2 P_B}\right)+\log\left(\frac{|h_{BA}|^2 P_B}{|h_{AA}|^2 P_A}\right), \tag{23}$$

where $C^{Bi}_{I_{hi}}$ is the approximated achievable sum rate in a high SIR region. The approximated achievable sum rate can then be rewritten as

$$\begin{aligned} C^{Bi}_{I_{hi}} &= \log\left(\frac{|h_{AB}|^2 P_A}{|h_{BB}|^2 P_B}\times\frac{|h_{BA}|^2 P_B}{|h_{AA}|^2 P_A}\right) \\ &= \log\left(\frac{|h_{AB}|^2 |h_{BA}|^2}{|h_{BB}|^2 |h_{AA}|^2}\right) \\ &= \log\left(\frac{|h_{AB}|^2}{|h_{BB}|^2}\right)+\log\left(\frac{|h_{BA}|^2}{|h_{AA}|^2}\right). \end{aligned} \tag{24}$$

As shown in (24), the effect of the power allocation vanishes in the high SIR region in the Bi-CR system. Because the signal of each node creates the self-interference caused by echo channel in the Bi-CR system, the SIR at each node can be expressed as the received signal power over the interference power at the other node when a high SIR approximation is used. In other words, the boosted transmission power causes severe interference due to full duplexing. Therefore, the power allocation in the high SIR region in the Bi-CR system cannot enhance the achievable sum rate. This means that a simple power allocation scheme for the Bi-CR system within the interference constraint represents the most effective approach in the high SIR region. Therefore, an effective non-joint power allocation scheme in the high SIR region is as follows:

$$P_A = \frac{I_{th}}{2|h_{AP}|^2}, \quad P_B = \frac{I_{th}}{2|h_{BP}|^2}. \tag{25}$$

The approximated achievable sum rate in the high SIR region can be derived as

$$\begin{aligned}
C_{I_{hi}}^{Bi} &= E\left[\log\left(\underbrace{\frac{|h_{AB}|^2}{|h_{BB}|^2}}_{=a}\right)\right] + E\left[\log\left(\underbrace{\frac{|h_{BA}|^2}{|h_{AA}|^2}}_{=b}\right)\right] \\
&= \int_0^\infty \log(a) f_A(a)da + \int_0^\infty \log(b) f_B(b)db \\
&= \int_0^\infty \log(a)\frac{\lambda_{AB}\lambda_{BB}}{(\lambda_{AB}+\lambda_{BB}a)^2}da \qquad (26)\\
&+ \int_0^\infty \log(b)\frac{\lambda_{BA}\lambda_{AA}}{(\lambda_{BA}+\lambda_{AA}b)^2}db = \\
&\log\left(\frac{\lambda_{AB}}{\lambda_{BB}}\right) + \log\left(\frac{\lambda_{BA}}{\lambda_{AA}}\right) = \log\left(\frac{\lambda_{AB}\lambda_{BA}}{\lambda_{BB}\lambda_{AA}}\right),
\end{aligned}$$

where $a = |h_{AB}|^2 / |h_{BB}|^2$ and $b = |h_{BA}|^2 / |h_{AA}|^2$, $f_A(a)$ and $f_B(b)$ are the PDFs of a and b, respectively. As shown in (26), the approximated achievable sum rate is expressed simply as a log function. Therefore, an achievable sum rate increases with increasing dedicated link gain or decreasing interference link gain. In addition, the link gains between the primary and secondary users do not affect the achievable sum rate despite the CR system because the power allocation cannot enhance the secondary user's performance in the high SIR region.

Next, the power allocation in the low SIR region was analyzed. In this region, it is the residual interference that is the primary cause of performance degradation. An achievable sum rate in this case can be approximated as follows:

$$C_{I_{lo}}^{Bi} = \frac{|h_{AB}|^2 P_A}{|h_{BB}|^2 P_B} + \frac{|h_{BA}|^2 P_B}{|h_{AA}|^2 P_A}, \qquad (27)$$

where $C_{I_{lo}}^{Bi}$ is an approximated achievable sum rate in the low SIR region. Therefore, the optimization problem can be expressed as

$$\begin{aligned}
C_{I_{lo}}^{Bi} &= \max_{P_A,P_B\geq 0} \frac{|h_{AB}|^2 P_A}{|h_{BB}|^2 P_B} + \frac{|h_{BA}|^2 P_B}{|h_{AA}|^2 P_A}, \qquad (28)\\
&s.t \quad |h_{AP}|^2 P_A + |h_{BP}|^2 P_B \leq I_{th}.
\end{aligned}$$

Substituting (8) into (28), the approximated achievable sum rate can be rewritten as

$$\begin{aligned}
C_{lo}^{Bi} &= \frac{|h_{BP}|^2 |h_{AB}|^2 P_A}{|h_{BB}|^2\left(I_{th} - |h_{AP}|^2 P_A\right)} \\
&+ \frac{|h_{BA}|^2\left(I_{th} - |h_{AP}|^2 P_A\right)}{|h_{BP}|^2 |h_{BB}|^2 P_A}.
\end{aligned} \qquad (29)$$

C_{lo}^{Bi} in (29) is a convex function with respect to P_A or P_B. Therefore, it has a minimum value within the range (10). Consequently, an achievable sum rate has a maximum value when the power allocation at each node is an end value within the range. The end value of the range is 0 or $I_{th} / |h_{AP}|^2$. This means that an achievable sum rate is maximized when only that node that can achieve a better rate than the other transmits in the Bi-CR system. This is because the self-interference caused by echo channel is a critical factor that causes performance degradation in the low SIR region. Concurrent transmission in the Bi-CR system cannot enhance an achievable sum rate, making uni-directional communication more efficient than BD communication in the low SIR region. Therefore, obtaining the performance gain offered by full duplexing in CR systems will require sufficient elimination of the self-interference.

4. Simulation Results

This section first examines the conventional schemes that will be compared with the proposed scheme. Next, the performance of the proposed schemes in the noise- and interference-limited environments is evaluated.

4.1 Conventional Schemes

To evaluate the performance of the Bi-CR system, a non-joint power allocation scheme in the Bi-CR system, called the equal power allocation on average (EPA) scheme, is first introduced. If two nodes in the Bi-CR system are not cooperating with each other, they use only their own CSI for power allocation. In this case, a simple way to allocate power is to have each node cut its own interference threshold in half as follows:

$$P_A^E = \frac{I_{th}}{2|h_{AP}|^2}, \quad P_B^E = \frac{I_{th}}{2|h_{BP}|^2}. \qquad (30)$$

This EPA scheme can easily satisfy the interference constraint without the need for joint power allocation.

Next, a system for conventional uni-directional communication in spectrum sharing-based cognitive radio (Uni-CR) is introduced. To keep the comparison fair, two schemes, i.e., uni-directional beamforming communication in spectrum sharing-based cognitive radio (UBF-CR) and MIMO transmission (UM-CR), are assessed. In the UBF-CR scheme, a receive beamforming scheme, i.e., a maximum ratio combining (MRC) scheme, is considered [21]. Because each secondary node shares the spectrum of the primary user on different resources, the transmission powers at each node can be expressed as [22]

$$P_A = \frac{I_{th}}{|h_{AP}|^2}, \quad P_B = \frac{I_{th}}{|h_{BP}|^2}, \qquad (31)$$

where $\|h_{AB}\|^2$ and $\|h_{BA}\|^2$ are the channel vectors for the links of SU_A - SU_B and SU_B - SU_A, respectively, and ω is a pre-log factor, and it is assumed that $\omega = 1/2$.

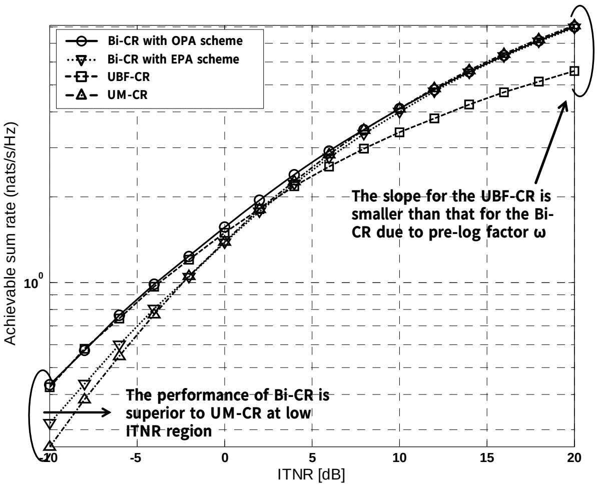
In addition, in the UM-CR system, equal power allocation to each transmit antenna is assumed because sophisticated encoding and decoding methods need to be used to achieve the optimal capacity for the secondary users [23]. An achievable sum rate for the UM-CR system can then be expressed as

$$\begin{aligned} C^{UM} &= \omega E\left[\log\det\left|\mathbf{I} + \frac{I_{th}}{2\sigma_B^2}\mathbf{H_{AB}}\mathbf{H_{AB}^H}\right|\right] \\ &\quad + (1-\omega)E\left[\log\det\left|\mathbf{I} + \frac{I_{th}}{2\sigma_A^2}\mathbf{H_{BA}}\mathbf{H_{BA}^H}\right|\right], \end{aligned} \tag{32}$$

where $\mathbf{H_{AB}}$ and $\mathbf{H_{BA}}$ are the channel matrices for the links of SU_A - SU_B and SU_B - SU_A, respectively. The equal power allocation for each transmit antenna can enable the secondary users to achieve optimal capacity in the high SNR regime [20].

4.2 Achievable Sum Rate in the Noise-Limited Environments

To evaluate the performance of the Bi-CR system, the achievable sum rates of the proposed scheme are compared with the conventional schemes with respect to the ITNR. The ITNR is the basic parameter used to evaluate the performance of a spectrum sharing-based CR system [3]. Because the ITNR is the ratio of the allowable interference threshold to noise variance, it depends on the primary user's sensitivity to interference. If the allowable interference at the PRx increases on account of a decrease in power attenuation due to small path-loss or an increase in channel gain for the primary link, the secondary users can increase their own transmission power to satisfy their own quality of service (QoS). On the other hand, a decrease in ITNR means that the amount of interference allowed by the primary user has decreased due to an increase in the QoS for the primary user or an increase in power attenuation. In this case, the secondary users share the spectrum of the primary user with their transmission at low power, which results in reduced secondary user throughput. In particular, in the Bi-CR system, as mentioned above, the transmission powers of the secondary users are more constrained than in a conventional Uni-CR system.



The slope for the UBF-CR is smaller than that for the Bi-CR due to pre-log factor ω

The performance of Bi-CR is superior to UM-CR at low ITNR region

Fig. 3. Achievable sum rates for the secondary users in the Bi-C R and conventional systems with respect to the ITNR.

Fig. 3 presents the achievable sum rates of the secondary users with respect to the ITNR in the noise-limited environment. First of all, compare the performance of the OPA and EPA schemes in the Bi-CR system. the performance of the EPA scheme is close to that of the OPA scheme in the high ITNR region. When the allowable interference to the primary user is large, the transmission powers of the secondary users for both the EPA and OPA schemes can increase so that the performance of the secondary users is less vulnerable to the channel conditions and noise powers. Because the transmission powers are dominant in the performance of the secondary users, the performance of the EPA scheme approaches that of the OPA scheme in the high ITNR region. On the other hand, the OPA scheme is superior to the EPA scheme in the low ITNR region. When the EPA scheme is used, SU_A and SU_B do not cooperate with each other as stated above.

Hence, the EPA scheme cannot achieve any cooperation gain. On the other hand, because the OPA scheme uses the information for all links and noise variances at each node, it obtains cooperation gain in the low ITNR region. Therefore, Fig. 3 shows that as the ITNR decreases, the OPA scheme has better performance than the EPA scheme.

Next, the performance of the Bi-CR and Uni-CR systems are compared. The results show that the achievable sum rates for the Bi-CR system with OPA scheme and UBF-CR system increase with increasing ITNR, as shown in Fig. 3. On the other hand, the slope of an achievable sum rate for the Bi-CR system is much greater than that for the UBF-CR system. This is because each link of the Bi-CR system uses its own spatial resources and shares frequency and time resources. Conversely, each link of the UBF-CR system shares spatial resources but uses its own frequency or time resource. This divided resource is expressed as a pre-log factor ω in an achievable rate, as shown in (32). Basically, this pre-log factor causes a decrease in the achievable sum rate, which is expressed in Fig. 3 as a decrease in slope. Therefore, the slope of an achievable sum rate of the UBF-CR system is smaller than that of the Bi-CR system. Accordingly, the Bi-CR system outperforms the UBF-CR system as the ITNR increases, i.e., in the high ITNR region.

In addition, the performance of the Bi-CR system with the OPA scheme is also superior to that of the UM-CR system in the low ITNR region. Basically, the transmission power of the Bi-CR system is lower than that of the UM-CR system because the interference from SU_A and SU_B to the PRx is more severe than that with the Bi-CR system

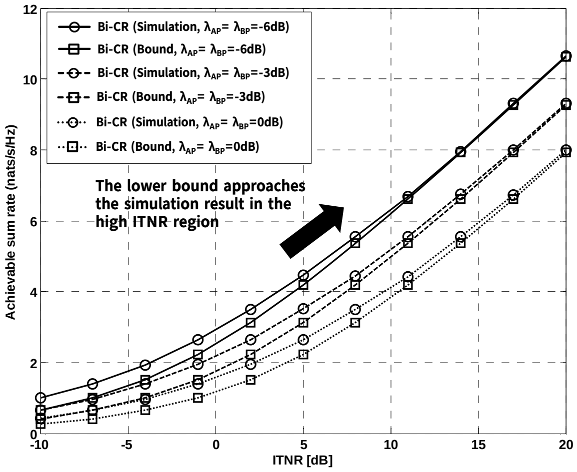


Fig. 4. Lower bound and simulated results of the achievable sum rates for secondary users in the Bi-CR system with respect to the ITNR.

because the transmission occurs from multiple transmit antennae. Moreover, the UM-CR system cannot achieve diversity or spatial multiplexing gains in the low ITNR region, whereas the performance of the UM-CR is the same as that of the Bi-CR in the high ITNR region. Therefore, the Bi-CR system with the OPA scheme performs better than the UM-CR system in the high ITNR region in a noise-limited environment.

Fig. 4 shows the achievable sum rates of the secondary users in the Bi-CR system as well as the lower bound of an achievable sum rates with respect to the mean value of the interference link channel gain. The results show that the derived lower bound for an achievable sum rate is very close to the simulated result in the high ITNR region. Of course, in the low ITNR region, the difference between the lower bound and the simulation results is small. Therefore, the derived lower bound is appropriate for analyzing the performance of a Bi-CR system. In addition, an achievable sum rate increases with decreasing mean value of the interference link channel gain (λ_{AP} and λ_{BP}). A small mean value of interference channel gain means that the interference from the secondary users to the primary user is weak. Therefore, the secondary users can increase their transmission powers within the interference constraint. Moreover, the performance of the Bi-CR system has a high performance gain with a weak interference link channel gain.

Consequently, the proposed OPA scheme in the Bi-CR system performs better than the conventional systems in terms of the achievable sum rates. Therefore, if the self-interference caused by the echo channel can be eliminated sufficiently, the advantage of BD communication will also be available to CR systems.

4.3 Achievable Sum Rate in the Interference-Limited Environments

This section discusses the achievable sum rate of the Bi-CR system in the interference-limited environment.

Fig. 5 shows an achievable sum rate of the secondary users versus the mean value of echo channel gain. Because an interference-limited environment was assumed, the ITNR was set to 50 dB in this simulation. As mentioned above, the Bi-CR system cannot achieve full duplexing performance gain in the low SIR region, as confirmed in Fig. 5. The performance of the uni-directional communication is better than the Bi-CR system in the low SIR region because the performance of Bi-CR system is limited by the self-interference caused by the echo channel in a full duplex configuration. Therefore, uni-directional communication is a better choice in this region than BD communication. In the high SIR region, however, the Bi-CR system with an EPA scheme performs better than uni-directional communication. This is because the full duplexing performance gain can be achieved in the high SIR region.

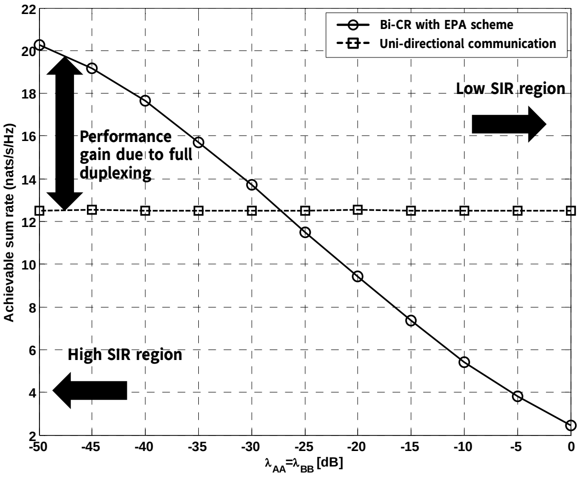


Fig. 5. Achievable sum rates for secondary users in the Bi-CR system with an EPA scheme and uni-directional communication system with respect to the mean value of echo channel gain ($\lambda_{AA}=\lambda_{BB}$).

In addition, as mentioned above, the impact of power allocation was proven to disappear in the high SIR region in Section III.B. As the Bi-CR system performs the same in the high SIR region regardless of the power allocation scheme, the EPA scheme with a simple power allocation scheme was chosen. Although simple power allocation was used, performance gain could be achieved in the high SIR region.

To investigate the relationship between the power allocation and self-interference caused by the echo channel, the achievable sum rate of the secondary users is depicted as a function of the interference channel gain in Fig. 6. As shown in Fig. 6, the achievable sum rate of the secondary users increases as the mean of the echo channel gain ($\lambda_{AA}, \lambda_{BB}$) decreases. The reason for this is that each node receives a small amount of interference with respect to the weak echo channel gain.

On the other hand, the achievable sum rates become saturated as the mean of the interference link channel gain decreases. Basically, an achievable sum rate in the CR system increases with decreasing interference link channel gain because of the boost in transmission power, as shown for uni-directional communication in Fig. 6. Because the

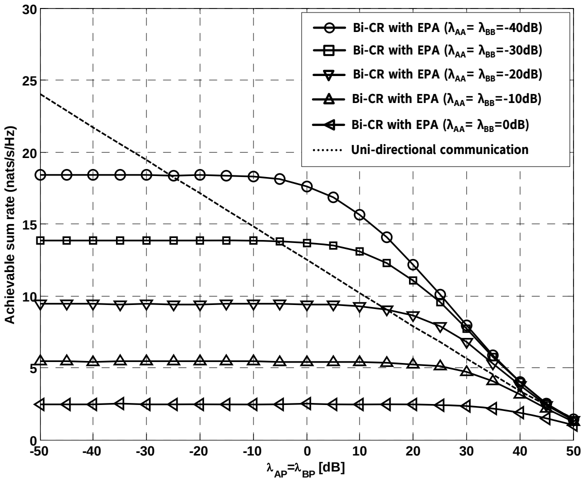


Fig. 6. Achievable sum rates for secondary users with respect to the interference link mean channel gain ($\lambda_{AP} = \lambda_{BP}$).

boosted transmission power causes severe interference at each node in the Bi-CR system, however, an achievable sum rate of the Bi-CR system becomes saturated as the mean of the interference link channel gain decreases. This result shows that the power allocation does not affect the achievable sum rate of the secondary users.

Moreover, in the low SIR region, e.g., when $\lambda_{AA} = \lambda_{BB} = 0, -10dB$, an achievable sum rate for unidirectional communication is superior to that of the Bi-CR system (Fig. 6) because of the strong self-interference caused by the echo channel. On the other hand, there is a region where the achievable sum rate for the secondary users in the Bi-CR system is superior to that with the uni-directional communication in the high SIR region. Therefore, in the interference-limited environment, there are regions where the Bi-CR system has an advantage over uni-directional communication.

Consequently, in an interference-limited environment, self-interference cancellation is essential for obtaining the performance gain from full duplexing in the Bi-CR system. If echo cancellation cannot be guaranteed adequately, then uni-directional communication is the better choice in terms of the achievable sum rate. Therefore, sufficient echo interference cancellation is indispensable if the promised performance gain is to be achieved by full duplexing in CR systems.

5. Conclusions

This study examined a Bi-CR system in noise- and interference-limited environments. An OPA scheme was first proposed as a way of maximizing the achievable sum rate of the secondary users in a noise-limited environment and the lower bound of an achievable sum rate was derived using the proposed scheme. In addition, power allocation schemes in the interference-limited environment were also analyzed.

The analytical and simulation results showed that echo cancellation in the Bi-CR system is essential for achieving the performance gain offered by full duplexing. Moreover, if the self-interference is eliminated sufficiently, the results show that significant performance gain can be achieved from BD communication, even though the overall secondary user transmission powers are constrained within the Bi-CR system.

Appendix I

This appendix outlines the process for deriving the lower bound of an achievable sum rate in a Bi-CR system with two antennae at each node. Using the proposed OPA scheme, an achievable sum rate of the secondary user can be expressed as follows:

$$\begin{aligned}
C_N^{Bi} &= E\left[\log\left(1+\frac{|h_{AB}|^2 P_A}{\sigma_B^2}\right)+\log\left(1+\frac{|h_{BA}|^2 P_B}{\sigma_A^2}\right)\right] \\
&= \int_0^{\infty}\int_0^{\frac{\mu_A}{1+\mu_A}} \log(1+\mu_A)\, f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A \\
&+ \int_0^1 \int_{\frac{\mu_A}{1-\mu_A}}^{\infty} \log(1+\mu_B)\, f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A \\
&+ \int_0^1 \int_{\frac{\mu_A}{1+\mu_A}}^{\frac{\mu_A}{1-\mu_A}} \left\{\log\left(\frac{1}{2}+\frac{\mu_B}{2\mu_A}+\frac{\mu_B}{2}\right)\right. \\
&\left. + \log\left(\frac{1}{2}+\frac{\mu_A}{2\mu_B}+\frac{\mu_A}{2}\right)\right\} f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A \\
&+ \int_1^{\infty} \int_{\frac{\mu_A}{1+\mu_A}}^{\infty} \left\{\log\left(\frac{1}{2}+\frac{\mu_B}{2\mu_A}+\frac{\mu_B}{2}\right)\right. \\
&\left. + \log\left(\frac{1}{2}+\frac{\mu_A}{2\mu_B}+\frac{\mu_A}{2}\right)\right\} f_{\mu_B}(\mu_B) f_{\mu_A}(\mu_A) d\mu_B d\mu_A .
\end{aligned} \tag{33}$$

In (34), a high SNR approximation on the log term is used to obtain a bound for the achievable sum rate. Basically, a log function can be approximated as follows:

$$\log(1+x) \approx \log(x) \quad \text{for large } x. \tag{34}$$

Therefore, under the assumption of a large μ_A and μ_B in (33), the log terms of the third and fourth terms in (33) can be approximated as

$$\begin{aligned}
\log\left(\frac{1}{2}\left(1+\frac{\mu_B}{\mu_A}+\mu_B\right)\right) &\approx \log\left(\frac{1}{2}\left(\frac{\mu_B}{\mu_A}+\mu_B\right)\right), \\
\log\left(\frac{1}{2}\left(1+\frac{\mu_A}{\mu_B}+\mu_A\right)\right) &\approx \log\left(\frac{1}{2}\left(\frac{\mu_A}{\mu_B}+\mu_B\right)\right).
\end{aligned} \tag{35}$$

In addition, $\mu_A/(1+\mu_A)$ of the first and fourth term in (33) can be approximated as 1 for large μ_A. Therefore, an achievable sum rate can be rewritten as follows:

$$\begin{aligned} C^{Bi}_{N_{LB}} &= \int_0^\infty \int_0^1 \log(1+\mu_A)\, f_{\mu_B}(\mu_B)\, f_{\mu_A}(\mu_A)\, d\mu_B\, d\mu_A \\ &+ \int_0^\infty \int_1^\infty \left\{ \log\left(\frac{1}{2}\left(\mu_B + \frac{\mu_B}{\mu_A}\right)\right)\right. \\ &\left. + \log\left(\frac{1}{2}\left(\mu_A + \frac{\mu_A}{\mu_B}\right)\right)\right\} f_{\mu_B}(\mu_B)\, f_{\mu_A}(\mu_A) d\mu_B\, d\mu_A. \end{aligned} \tag{36}$$

The second and third terms in (33) can be omitted because of the high SNR approximation. Instead, the range of μ_A is expanded from 0 to ∞ in the fourth term. In addition, the log terms of the fourth term can be formulated as

$$\begin{aligned} &\log\left(\frac{1}{2}\left(\mu_B + \frac{\mu_B}{\mu_A}\right)\right) + \log\left(\frac{1}{2}\left(\mu_A + \frac{\mu_A}{\mu_B}\right)\right) \\ &= \log\left(\frac{1}{4}\left(\mu_B + \frac{\mu_B}{\mu_A}\right)\left(\mu_A + \frac{\mu_A}{\mu_B}\right)\right) \\ &= \log\left(\frac{1}{4}(1+\mu_B)(1+\mu_A)\right) \\ &= \log\left(\frac{1+\mu_B}{2}\right) + \log\left(\frac{1+\mu_A}{2}\right). \end{aligned} \tag{37}$$

Therefore, the lower bound of an achievable sum rate can be reformulated as

$$\begin{aligned} C^{Bi}_{N_{LB}} &= \underbrace{\int_0^\infty \int_0^1 \log(1+\mu_A)\, f_{\mu_B}(\mu_B)\, f_{\mu_A}(\mu_A)\, d\mu_B\, d\mu_A}_{U} \\ &+ \underbrace{\int_0^\infty \int_1^\infty \left\{ \log\left(\frac{\mu_B+1}{2}\right) + \log\left(\frac{\mu_A+1}{2}\right)\right\}}_{W} \\ &\underbrace{\times f_{\mu_B}(\mu_B)\, f_{\mu_A}(\mu_A) d\mu_B\, d\mu_A}_{W} \end{aligned} \tag{38}$$

First, the U term in (38) can be derived as

$$\begin{aligned} U &= \int_0^\infty \frac{\alpha}{(1+\alpha\mu_A)^2} \int_0^1 \log(1+\mu_A) \frac{\beta}{(1+\beta\mu_B)^2} d\mu_B\, d\mu_A \\ &= \int_0^\infty \frac{\beta \log(1+\mu_A)}{1+\beta} \times \frac{\alpha}{(1+\alpha\mu_A)^2} d\mu_A \\ &= \frac{\beta \log(\alpha)}{(\alpha-1)(1+\beta)}. \end{aligned} \tag{39}$$

(39) is derived using [26].

Next, the W term in (38) can also be derived as follows:

$$\begin{aligned} W &= \int_0^\infty \frac{\alpha}{(1+\alpha\mu_A)^2} \int_1^\infty \left\{ \log\left(\frac{\mu_B+1}{2}\right) + \log\left(\frac{\mu_A+1}{2}\right)\right\} \\ &\times \frac{\beta}{(1+\beta\mu_B)^2} d\mu_B\, d\mu_A \\ &= \int_0^\infty \frac{\beta \log\left(\frac{\beta(1+\mu_A)}{1+\beta}\right) + \log\left(\frac{4\beta}{(1+\beta)(1+\mu_A)}\right)}{\beta^2-1} \\ &\times \frac{\alpha}{(1+\alpha\mu_A)^2} d\mu_A \\ &= \frac{\alpha\beta \log\left(\frac{\beta}{1+\beta}\right) + \alpha \log\left(\frac{4\beta}{1+\beta}\right) + \log\left(\frac{1+\beta}{4\alpha\beta}\right)}{\beta^2-1} \\ &+ \frac{+\beta \log\left(\frac{(\beta+1)\alpha}{\beta}\right)}{\beta^2-1}. \end{aligned} \tag{40}$$

(40) is derived with the help of [26].

Therefore, the lower bound of the achievable sum rate in the Bi-CR system with two antennae at each node can be derived as

$$\begin{aligned} C^{Bi}_{N_{LB}} &= \frac{\alpha\beta \log\left(\frac{\beta}{1+\beta}\right) + \alpha \log\left(\frac{4\beta}{1+\beta}\right) + \log\left(\frac{1+\beta}{4\alpha\beta}\right)}{(\alpha-1)(\beta^2-1)} \\ &+ \frac{\beta\left((\beta-1)\log(\alpha) + \log\left(\frac{\alpha(\beta+1)}{\beta}\right)\right)}{(\alpha-1)(\beta^2-1)}. \end{aligned} \tag{41}$$

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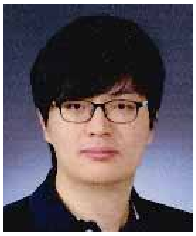

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