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# Assortment Optimization under Consumer Choice Behavior in Online Retailing

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**ABSTRACT**

This paper studies the assortment optimization problem in online retailing by using a multinomial logit model in order to take consumer choice behavior into account. We focus on two unique features of online purchase behavior: first, there exists increased amount of uncertainty (e.g., size and color of merchandize) in online shopping as customers cannot experience merchandize directly. This uncertainty is captured by the scale parameter of a Gumbel distribution; second, online shopping entails unique shopping-related disutility (e.g., waiting time for delivery and security concerns) compared to offline shopping. This disutility is controlled by the changes in the observed part of utility function in our model. The impact of changes in uncertainty and disutility on the expected profit does not exhibit obvious structure: the expected profit may increase or decrease depending on the assortment. However, by analyzing the structure of the optimal assortment based on convexity property of the profit function, we show that the cardinality of the optimal assortment decreases and the maximum expected profit increases as uncertainty or disutility decreases. Therefore, our study suggests that it is important for managers of online retailing to reduce uncertainty and disutility involved in online purchase process.

Keywords: Assortment Optimization, Online Retailing, Multinomial Logit, Consumer Choice Behavior

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## 1. INTRODUCTION

Assortment planning is defined as the specification of the set of products to be carried in a store in order to maximize profit margin (Kok *et al*., 2005). It has a substantial impact on sales and profit and it is one of the most important and challenging fields in academic study (Kok *et al*., 2005). Even though assortment planning is also a critical issue in online retailing (Jarvenpaa and Todd, 1997), there has been a lack of literature on this topic. Therefore, in this study we analyze how the optimal assortment changes under online retailing framework.

We focus on two unique features of online retailing in this paper. First, online shopping involves more uncertainty than offline shopping. When a customer buys merchandise offline, she can look, touch, and experience the product. On the other hand, in online shopping a customer's purchase decision usually resorts to virtual appearance such as pictures and video clips (Kolesar and Galbraith, 2000). Therefore, there could be more uncertainty involved in online shopping in terms of size, weight, color, and performance of merchandize, amongst others. In addition, other factors such as product misrepresentations and misleading advertisement may intensify the degree of uncertainty. Secondly, shopping-related disutility exists under online shopping environment including waiting time for delivery, order fulfillment glitches (e.g., wrong delivery), and security concerns (e.g., encryption, authentication, and information leaks), which may make customers uncomfortable (Rao *et al*., 2011; Monsuwé *et al*., 2004). Compared to offline shopping process, customers have to provide their personal information and have to wait for delivery when they purchase online, which creates a variety of disutility.

The difficulties of assortment planning come from the interdependency of products offered in an assortment. Therefore, consumer choice behavior, such as substitution, should be incorporated into assortment planning. Recently, there is a growing stream of research on assortment planning under consumer choice models (Mahajan and van Ryzin, 2001; Rusmevichientong and Topaloglu, 2012), but little literature studies this topic in regards to online retailing. Hence, we analyze the impact of uncertainty and shopping disutility on assortment planning in online retailing using a consumer choice model.

In the coming sections, we first show that the effect of reducing uncertainty or disutility is not straightforward: the expected profit may increase or decrease depending on the assortment. We then also show that the maximum expected profit increases and the size of the optimal assortment decreases as uncertainty or disutility reduces.

## 2. MODEL

Consider an assortment optimization problem for a retailer: she needs to choose an assortment *S* to offer out of a set of products indexed by $N = \{1, \cdots, n\}$. Let product 0 denote the no-purchasing option. Let $f \in R^n$ denote the associated vector of unit profits for *N*. Without loss of generality, we assume that profits are non-negative and the products are indexed in non-increasing profit order: $f_1 \ge f_2 \ge \cdots f_n \ge 0$.

We use a multinomial logit (MNL) choice model for the choice probability of customers. The MNL model is based on a random utility framework and it is widely used in marketing and operations literature (Kok *et al*., 2005). Each customer associates utility $U_j = V_j + \varepsilon_j$ with product $j \in N \cup \{0\}$. We assume that $V_j$, the observed part of utility or the expected utility, consists of two components for $j \in N: V_j = Y_j - D$, where $Y_j$ denotes the utility from product *j*, and *D* represents the shopping-related disutility, e.g., waiting time for delivery and security concerns. Note that $U_0$ does not have component *D* because it is only associated with actual purchase transaction. We assume that $V_0 \le 0$ and $V_j \ge 0$ for $j \in N$, which implies that a customer incurs some visiting cost like searching cost and the observed part of utility is greater than the visiting cost for all products.

The unobserved part $\varepsilon_j$, $j \in N \cup \{0\}$, follows a Gumbel distribution with shift parameter 0 and scale parameter $\sigma$: the distribution is given as $\Pr\{X \le x\} = \exp(-\exp(-(x/\sigma + \gamma)))$, where γ is the Euler constant (≈ 0.577). Note that its mean is zero and variance is $\sigma^2\pi^2/6$ (Train, 2009).

The scale parameter governs the degree of uncertainty in utility across customers. Here, we assume that the scale parameter captures the uncertainty involved in online shopping. Note that the scale parameter itself does not affect the expected utility directly, as the mean of $\varepsilon_j$ is zero regardless of the value of the scale parameter. However, it is known that as the scale parameter increases, the relative probability of choosing a product with smaller expected utility increases (Train, 2009).

Let $w_j := exp(V_j/\sigma)$ for $j \in N \cup \{0\}$. The probability that a customer chooses product j from assortment S under the MNL model is (Train, 2009): $P_j(S) = w_j/(w_0 + \sum_{i \in S} w_i)$. The assortment optimization problem is given by

$$\pi^* := \max_{S \subseteq N} \pi(S) := \max_{S \subseteq N} \sum_{j \in S} f_j \frac{w_j}{w_0 + \sum_{i \in S} w_i}.$$

It is shown that profit-ordered assortments are optimal for this problem (Talluri and van Ryzin, 2004; Rusmevichientong and Topaloglu, 2012). This means that an optimal assortment is $\{1, 2, \cdots, k^*\}$ for some $k^*$. For ease of exposition, we use $\pi(k)$ to denote $\pi(\{1, 2, \cdots, k\})$. Rusmevichientong and Topaloglu (2012) present a simple algorithm to find the largest optimal assortment. They show that $\pi(k+1)$ is a convex combination of $\pi(k)$ and $f_{k+1}$:

$$\pi(k+1) = \frac{w_0 + \sum_{i=1}^k w_i}{w_0 + \sum_{i=1}^k w_i + w_{k+1}} \pi(k) + \frac{w_{k+1}}{w_0 + \sum_{i=1}^k w_i + w_{k+1}} f_{k+1}$$

Therefore, if $f_{k+1} \ge \pi(k)$, we need to add product $k+1$ into the assortment to be offered in order to increase the expected profit. Starting from product 1, we can find $k^*$ that satisfies $f_{k^*} \ge \pi(k^*-1)$ and $f_{k^*+1} < \pi(k^*)$. Then, $\{1, 2, \cdots, k^*\}$ is the largest optimal assortment. Note that, from the convexity argument,

$$\text{if } f_{k+1} \ge \pi(k), \text{ then } f_{k+1} \ge \pi(\tilde{k}), \text{ for any } \tilde{k} \ge k. \quad (1)$$

We focus on analyzing the impact of the changes in the uncertainty and the shopping disutility on the structure of optimal assortment and maximum profit. As we decrease uncertainty $\sigma$, $w_j = \exp(V_j/\sigma)$ increases for $j \in N$. On the other hand, $w_0$ increases because $V_0 < 0$. As we decrease shopping disutility $D$, $w_j = \exp(V_j/\sigma)$ for $j \in N$ increases and $w_0$ stays the same.

We use superscript 1 to denote the base parameters and superscript 2 to denote the changed parameters after we decrease uncertainty or shopping disutility. Then, $\sigma^1 > \sigma^2$ or $D^1 > D^2$ which leads to the following:

$$w_0^1 \ge w_0^2 \text{ and } w_j^1 < w_j^2 \text{ for } j \in N. \quad (2)$$

## 3. ANALYSIS

Let $k^{1*}$ and $k^{2*}$ denote the last product of the largest optimal assortment and $\pi^1$ and $\pi^2$ denote the expected profit before and after we decrease uncertainty or shopping disutility respectively. In addition, let $\pi^{1*}$ and $\pi^{2*}$ denote the corresponding maximum expected profit: $\pi^{1*} = \pi^1(k^{1*})$ and $\pi^{2*} = \pi^2(k^{2*})$.

We analyze how the changes in uncertainty or disutility affect the expected profit function and assortment planning. We first show that the structure of the changes in $\pi^1$ and $\pi^2$ is not trivial because neither $\pi^1(k)$ nor $\pi^2(k)$ dominates the other over all *k*.

**Lemma 1:** Decreasing uncertainty or disutility does not guarantee improvement in expected profit for all profit-ordered assortment: It is possible that $\pi^2(k) < \pi^1(k)$ for some *k*.

We prove this property by illustrating a counterexample. Suppose we have 7 products indexed in non-increasing profit order. Table 1 shows the profit and the expected utility. Suppose we decrease uncertainty of online shopping process by reducing the scale parameter from 2 to 1.2.

Table 1. Parameters of Each Product

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f_k$ | 0 | 100 | 80 | 79 | 60 | 48 | 45 | 44 |
| $V_k$ | -1 | 0.5 | 4 | 5 | 6 | 3 | 1 | 4 |

$\sigma^1 = 2,\ \sigma^2 = 1.2.$

As we can see in Figure 1, the structure of $\pi^1(k)$ and $\pi^2(k)$ is nontrivial: from $k=1$ to 3, $\pi^2(k)$ is higher than $\pi^1(k)$; but from $k=4$ to 6, $\pi^2(k)$ is lower than $\pi^1(k)$; for $k \geq 7$, $\pi^2(k)$ exceeds $\pi^1(k)$ again. Therefore, the effect of decreasing uncertainty or disutility on the structure of $\pi^1(k)$ and $\pi^2(k)$ is not obvious: it might increase or decrease the expected profit for given *k*.

Although the impact of decreasing uncertainty or disutility on the expected profit is not obvious for each profit-ordered assortment, we identify its effects on the structural properties of the optimal assortment and the maximum expected profit.

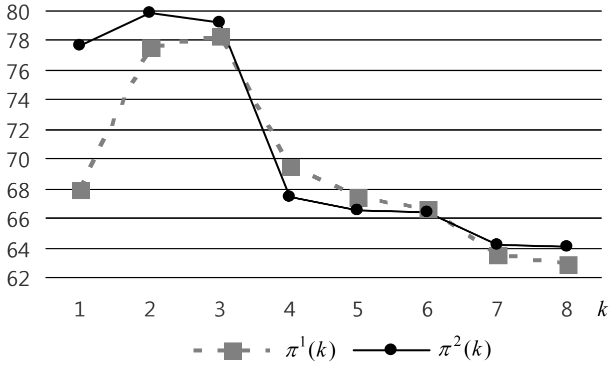


Figure 1. Changes in the Expected Profit

**Proposition 1:**

a) The maximum expected profit increases as the uncertainty or the shopping disutility decreases: $\pi^{1*} < \pi^{2*}$.

b) The cardinality of the largest optimal assortment decreases as the uncertainty or the shopping disutility decreases: $k^{1*} \geq k^{2*}$.

**Proof**: Since we know that the optimal solution is profit-ordered, we only need to investigate the value of objective function using the profit ordered assortment sequentially. We prove the proposition by induction on *k*.

Initialization: $k=1$.

$$\pi^1(1) = \frac{w_1^1 f_1}{w_0^1 + w_1^1} > 0 \text{ and } \pi^2(1) = \frac{w_1^2 f_1}{w_0^2 + w_1^2} > 0.$$

Therefore, product 1 should be included in the optimal assortment.

$$\pi^2(1) - \pi^1(1) = \frac{f_1(w_1^2 w_0^1 - w_1^1 w_0^2)}{(w_0^1 + w_1^1)(w_0^2 + w_1^2)} > 0.$$

Thus, $\pi^2(1) > \pi^1(1)$.

Induction on k: Suppose $\pi^2(\mathrm{k}) > \pi^1(\mathrm{k})$. We proceed from $k=1$ only until we find $k^{2*}$.

Case 1: $\pi^2(k) > \pi^1(k) > f_{k+1}$.
From the convexity result, $k^{1*} = k^{2*} = k$. Therefore, by the induction assumption, $\pi^2(k^{2*}) > \pi^1(k^{1*})$. We do not need to proceed to product $k+1$.

Case 2: $\pi^2(k) > f_{k+1} \geq \pi^1(k)$.
From the convexity result, $k^{2*} = k$ and $k < k^{1*}$. In addition, $\pi^2(k^{2*}) > f_{k+1} \geq \pi^1(k^{1*})$, from (1). We do not need to proceed product to $k+1$.

Case 3: $f_{k+1} \geq \pi^2(k) > \pi^1(k)$.
From the assumption,

$$f_{k+1} - \pi^1(k) = \frac{f_{k+1}\left(w_0^1 + \sum_{i=1}^k w_i^1\right) - \sum_{i=1}^k w_i^1 f_i}{w_0^1 + \sum_{i=1}^k w_i^1} > 0.$$

Thus, $f_{k+1} w_0^1 + \sum_{i=1}^k w_i^1 (f_{k+1} - f_i) > 0.$ (3)

The relationship between $\pi^1(k)$ and $\pi^2(k)$ is not straightforward in this case. We introduce an intermediate profit function to facilitate our analysis. Let

$$\bar{\pi}(k+1) := \frac{\sum_{i=1}^k w_i^1 f_i + w_{k+1}^2 f_{k+1}}{w_0^1 + \sum_{i=1}^k w_i^1 + w_{k+1}^2}.$$

Then, we first compare this intermediate profit function with $\pi^1(k+1)$:

$$\begin{aligned}
&\bar{\pi}(k+1)-\pi^1(k+1)\\
&=\frac{\sum_{i=1}^{k} w_i^1 f_i + w_{k+1}^2 f_{k+1}}{w_0^1+\sum_{i=1}^{k} w_i^1 + w_{k+1}^2}-\frac{\sum_{i=1}^{k} w_i^1 f_i + w_{k+1}^1 f_{k+1}}{w_0^1+\sum_{i=1}^{k} w_i^1 + w_{k+1}^1}\\
&=\frac{\left(w_{k+1}^2-w_{k+1}^1\right)\left(f_{k+1}w_0^1+\sum_{i=1}^{k} w_i^1\left(f_{k+1}-f_i\right)\right)}{\left(w_0^1+\sum_{i=1}^{k} w_i^1+w_{k+1}^2\right)\left(w_0^1+\sum_{i=1}^{k} w_i^1+w_{k+1}^1\right)}\\
&>0, \text{ from (2) and (3).}
\end{aligned}$$

Next, we compare this intermediate profit function with $\pi^2(k+1)$:

$$\begin{aligned}
&\pi^2(k+1)-\bar{\pi}(k+1)\\
&=\frac{\sum_{i=1}^{k} w_i^2 f_i + w_{k+1}^2 f_{k+1}}{w_0^2+\sum_{i=1}^{k} w_i^2 + w_{k+1}^2}-\frac{\sum_{i=1}^{k} w_i^1 f_i + w_{k+1}^2 f_{k+1}}{w_0^1+\sum_{i=1}^{k} w_i^1 + w_{k+1}^2}\\
&=\frac{\sum_{i=1}^{k} f_i\left(w_i^2 w_0^1 - w_i^1 w_0^2\right)}{\left(w_0^2+\sum_{i=1}^{k} w_i^2+w_{k+1}^2\right)\left(w_0^1+\sum_{i=1}^{k} w_i^1+w_{k+1}^2\right)}\\
&+\frac{\sum_{i=1}^{k}\left(w_i^2-w_i^1\right)\left(f_i-f_{k+1}\right)+w_{k+1}^2 f_{k+1}\left(w_0^1-w_0^2\right)}{\left(w_0^2+\sum_{i=1}^{k} w_i^2+w_{k+1}^2\right)\left(w_0^1+\sum_{i=1}^{k} w_i^1+w_{k+1}^2\right)}\\
&>0, \text{ from (2).}
\end{aligned}$$

Thus, $\pi^2(k+1)>\bar{\pi}(k+1)>\pi^1(k+1)$. From the assumption $f_{k+1}\geq\pi^2(k)>\pi^1(k)$, we need to include $k+1$ to both of the optimal assortments and proceed to product $k+1$ with the property of $\pi^2(k+1)>\pi^1(k+1)$. We will stop at some $k$ when we reach case (1) or (2). Therefore, we conclude that $\pi^{1*}<\pi^{2*}$ and $k^{1*}\geq k^{2*}$. ■

From the lemma and the proposition, we show that the impact of decreasing uncertainty or disutility on the structure of the optimal assortment and the maximum expected profit: Proposition 1-a and Lemma 1 show that the maximum expected profit increases as we decrease uncertainty or disutility although the expected profit of each profit-ordered assortment may not; Proposition 1-b states that the cardinality of the optimal assortment decreases as we decrease uncertainty or disutility, which might not be very intuitive.

The previous example demonstrates the proposition: the maximum expected profit increases $\left(\pi^{1*}<\pi^{2*}\right)$ and the optimal assortment shrinks from $\{1, 2, 3\}$ to $\{1, 2\}$. Note that we do not include product 3 into the optimal assortment because $f_3$ is lower than $\pi^2(2)$.

## 4. DISCUSSION

In this study, we analyze the impact of the changes in uncertainty and shopping disutility on assortment planning in online retailing. We use the scale parameter of an MNL model to capture the change in uncertainty and use the fixed component of expected utility to capture the changes in disutility. It is shown that the impact of the changes on the expected profit is not conclusive: the expected profit may increase or decrease depending on the assortment. However, we prove that the largest optimal assortment shrinks and the maximum expected profit increases as uncertainty or disutility decreases.

Uncertainty and shopping-related disutility involved in online markets are major concerns for customers. By reducing the magnitude of them, managers can increase profit and decrease the size of optimal assortment. This reduced size of assortment can result in additional advantages by decreasing the complexity and the cost of inventory management through improved product availability, and reduced handling cost (Kok *et al.*, 2005). There could be various ways to decrease uncertainty: managers can provide more high quality images, sound and video applications, detailed information, reliable reviews, and online live support (Lohse and Spiller, 1999). Shopping-related disutility can be reduced by improving security, lead-time, and delivery reliability.

Even though we mainly focus on online shopping because uncertainty and disutility issues are more prominent under online shopping environment, these issues also exist in offline shopping. The main results of this paper can be carried over to offline shopping environment as well.

A limitation of this study is that we used an MNL model, which exhibits independence from irrelevant alternatives (IIA) property (Train, 2009). Introducing generalized extreme value models can resolve this problem. This paper can be extended by incorporating pricing decisions into the optimization model as simultaneous optimization on pricing and assortment planning is one of the most important issues in online retailing. In addition, incorporating competition into the basic model is another direction of future research.

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