

# Statistical notes for clinical researchers: Nonparametric statistical methods: 2. Nonparametric methods for comparing three or more groups and repeated measures

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For comparison of three or more independent groups, Kruskal-Wallis test which is comparable to one-way ANOVA is used. Also for analysis of correlated data with three or more occasions or conditions, Friedman test, comparable to repeated measures one-way ANOVA, is used as a nonparametric method.

### 1. Kruskal-Wallis test

The Kruskal-Wallis test, an extension of the Wilcoxon rank-sum test, may be applied to data of independent three or more groups whose outcome measurements are at least ordinal. The null hypothesis is that three sets of samples came from the same population and they do not differ systematically.

1<sup>st</sup> step: Transform observed data into ranks. Tied values get average ranks.

Observed score				Rank			
Groups	A	B	C	Groups	A	B	C
Data	20	27	65	1	3	18	
	25	39	59	2	7	16	
	50	29	55	11	4.5	13	
	51	44	61	12	8	17	
	29	47	56	4.5	9	14.5	
	36	49	56	6	10	14.5	
				Sum of ranks	36.5	41.5	93

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2<sup>nd</sup> step: Calculation of rank sum for each group ( $T_g$ ).

$$T_1 = 36.5, T_2 = 41.5, T_3 = 93.$$

3<sup>rd</sup> step: Calculate H value

$$H = \left[ \frac{12}{N(N+1)} * \sum \frac{T_g^2}{n_g} \right] - 3(N+1) = \frac{12}{18(18+1)} * 1950.6 - 3(18+1) = 11.4$$

$$N = 18, n_g = 6, \sum \frac{T_g^2}{n_g} = \frac{36.5^2}{6} + \frac{41.5^2}{6} + \frac{93^2}{6} = 1950.6$$

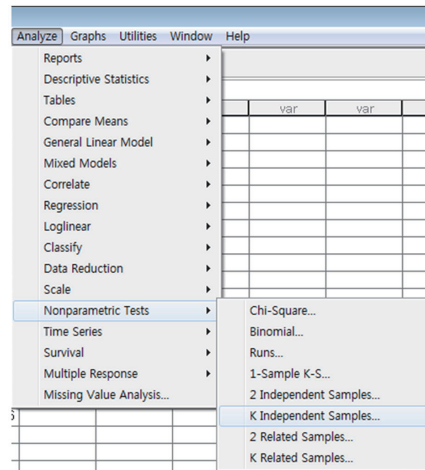
- 4<sup>th</sup> step: Compare with the chi-square distribution with degree of freedom (number of groups-1).  
 Reject null hypothesis if the calculated H value is greater than the critical chi-square value. The calculated H value 11.4 is larger than the critical value,  $\chi^2(d.f. = 2, p = 0.05) = 5.99$ . Therefore the null hypothesis is rejected, and it is concluded that at least distribution of one group is different from other groups.
- 5<sup>th</sup> step: If the group difference is statistically significant, a *post-hoc* paired comparison is implemented using the nonparametric two group comparison method with adjusting Bonferroni-corrected alpha level.
- 1) *Post-hoc* paired comparison may be done using the Wilcoxon rank-sum test
  - 2) Bonferroni-corrected alpha level is an alpha level divided by the number of comparisons. For example, if three groups were compared by three different pair-wise comparisons, an alpha level divided by 3 should be applied, e.g.,  $0.05/3 = 0.0167$ .

The Krusal-Wallis test using the SPSS statistical package is according to the following procedures:

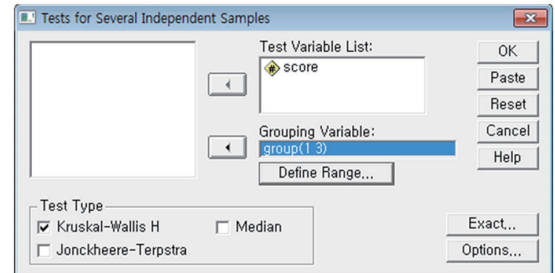
(a) data

	group	score
1	1	20
2	1	25
3	1	50
4	1	51
5	1	29
6	1	36
7	2	27
8	2	39
9	2	29
10	2	44
11	2	47
12	2	49
13	3	65
14	3	59
15	3	55
16	3	61
17	3	56
18	3	56

(b) Analyze-Nonparametric tests  
 - k independent samples



(c) Define test variable and grouping variable



(d) Mean ranks

	group	N	Mean Rank
score	1	6	6,08
	2	6	6,92
	3	6	15,50
Total		18	

(e) Chi-square value and p value

	score
Chi-Square	11,465
df	2
Asymp. Sig.	,003

a. Kruskal Wallis Test  
 b. Grouping Variable: group

## 2. Friedman test

The Friedman test, an extension of the Wilcoxon signed rank test, is used for within-subject design. It is used for data with three or more correlated or repeated outcomes whose distribution is not normal. The null hypothesis is that the distribution is the same across repeated measures.

1<sup>st</sup> step: Transform observed data into ranks within the subjects

Observed score					Rank			
Subjects	Time 1	Time 2	Time 3		Subjects	Time 1	Time 2	Time 3
1	4	6	13	→	1	1	2	3
2	3	7	12		2	1	2	3
3	2	8	9		3	1	2	3
4	3	8	7		4	1	3	2
5	2	6	9		5	1	2	3
6	3	8	9		6	1	2	3
7	2	6	4		7	1	3	2
8	6	10	15		8	1	2	3
					Sum of ranks	8	18	22

2<sup>nd</sup> step: Calculation of rank sum for each time point ( $R_i$ ).

$R_1 = 8, R_2 = 18, R_3 = 22$ .

3<sup>rd</sup> step: Calculate Fr value

$$Fr = \left[ \frac{12}{Nk(k+1)} \sum R_i^2 \right] - 3N(k+1) = \left[ \frac{12}{8 * 3(3+1)} * 872 \right] - 3 * 8(3+1) = 13$$

N: number of subjects, 8; k: number of repetition, 3;  $\sum R_i^2 = 8^2 + 18^2 + 22^2 = 872$

4<sup>th</sup> step: Compare with the chi-square distribution with degree of freedom (number of repetitions-1).

Reject null hypothesis if the calculated Fr value is greater than the critical chi-square value. The calculated Fr value 13 is larger than the critical value,  $\chi^2(d.f. = 2, p = 0.05) = 5.99$ . Therefore, the null hypothesis was rejected and it is concluded that at least distribution of one occasion is different from other time points.

5<sup>th</sup> step: If the difference is statistically significant, a *post-hoc* paired comparison is implemented using the nonparametric two related sample comparison method with adjusting Bonferroni-corrected alpha level.

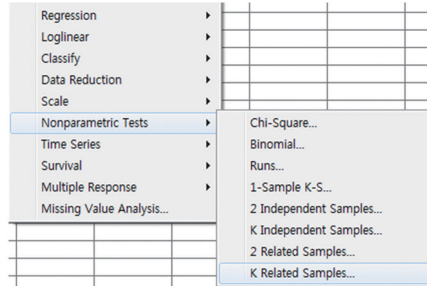
- 1) *Post-hoc* paired comparison may be done using the Wilcoxon signed rank test
- 2) Bonferroni-corrected alpha level should be applied for each comparison, e.g.,  $0.05/3 = 0.0167$  for three pairwise comparisons.

The Friedman test using the SPSS statistical package is according to the following procedures:

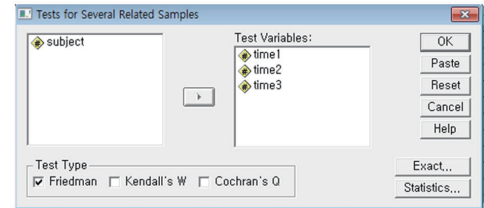
(a) data

	subject	time1	time2	time3
1	1	4	6	13
2	2	3	7	12
3	3	2	8	8
4	4	3	8	7
5	5	2	6	9
6	6	3	8	9
7	7	2	6	4
8	8	6	10	15

(b) Analyze-Nonparametric tests  
- k Related samples



(c) Define test variables (repeated)



(d) Mean ranks

Ranks	
	Mean Rank
time1	1,00
time2	2,31
time3	2,69

(e) Chi-square and *p* value

Test Statistics <sup>a</sup>	
N	8
Chi-Square	12,968
df	2
Asymp. Sig.	,002

a. Friedman Test

**Reference**

1. McDonald JH. Handbook of biological statistics. Baltimore, Maryland: Sparky House Publishing; 2008. p153-160.