

# Modified Unscented Kalman Filter for a Multirate INS/GPS Integrated Navigation System

Munkhzul Enkhtur, Seong Yun Cho, and Kyong-Ho Kim

*Instead of the extended Kalman filter, the unscented Kalman filter (UKF) has been used in nonlinear systems without initial accurate state estimates over the last decade because the UKF is robust against large initial estimation errors. However, in a multirate integrated system, such as an inertial navigation system (INS)/Global Positioning System (GPS) integrated navigation system, it is difficult to implement a UKF-based navigation algorithm in a low-grade or mid-grade microcontroller, owing to a large computational burden. To overcome this problem, this letter proposes a modified UKF that has a reduced computational burden based on the basic idea that the change of probability distribution for the state variables between measurement updates is small in a multirate INS/GPS integrated navigation filter. The performance of the modified UKF is verified through numerical simulations.*

*Keywords: UKF, modified UKF, multirate INS/GPS.*

## I. Introduction

Global Positioning System (GPS)-based navigation systems have been used in land vehicle navigation systems (LVNSs) due to their low price, easy installation, and other beneficial factors. The level of performance required of an LVNS recently increased with the successful implementation of LVNSs in

unmanned land vehicles, the development of augmented reality for land vehicles, and the availability of high-grade LVNSs [1]. To achieve strong performance, an inertial navigation system (INS)/GPS integrated navigation system can be used [2], [3]. Moreover, the performance enhancement of low-cost microelectromechanical systems (MEMS)-based inertial sensors has induced the commercialization of an INS/GPS integrated navigation system. The extended Kalman filter (EKF) has been used to integrate the INS and GPS receiver. One of the demerits of the EKF, however, is that the filter may diverge when an initial state estimation error is large [4]. To avoid this problem, the use of the unscented Kalman filter (UKF) was adopted during the last decade [5]-[9]. The UKF is robust against large initial state estimation errors because the UKF does not use the approximated Jacobian matrix used in the EKF for time propagation of a state error covariance matrix. On the other hand, the time propagation of the state error covariance matrix in the UKF is performed using the direct propagation of the sigma points through a nonlinear system function. An INS/GPS integrated navigation system generally has a multirate system structure. In other words, the updating frequency of the INS is high, such as 50 Hz, and the frequency of the measurement update is low, depending on the output frequency of the GPS receiver. In a UKF-based INS/GPS integrated navigation system, the sigma points must be propagated at a high frequency. During this process, the computational burden of the UKF becomes several times larger than that of the EKF. This problem may cause a restriction in the real-time implementation of an INS/GPS integrated navigation system. To overcome this problem, this letter proposes a modified UKF for a multirate INS/GPS integrated navigation system. The performance of the proposed filter is evaluated through a numerical simulation.

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## II. Modified UKF for Multirate INS/GPS Integrated Navigation System

Figure 1 shows the structures of the conventional UKF and the modified UKF proposed in this letter. The main difference between the filters can be confirmed from the time propagation process of the state variables.

A modified UKF in a multirate INS/GPS integrated navigation system can be constructed as follows:

0) The definition of a discrete-time nonlinear system is

$$\begin{aligned} x_{k+1} &= f(x_k, \omega_k^b, f_k^b, dt), \\ y_k &= h(x_k), \end{aligned} \quad (1)$$

where  $\omega^b$  and  $f^b$  are the gyro output and accelerometer output, respectively, and  $dt$  is the time propagation interval. State variables are set as a basic 15th-order model [10].

The number of sigma points is set to 17 (state dimension of  $15 + 2$ ) for a small computational load [6].

1) Initialization of the state, covariance matrix, and weights [9]:

$$\hat{x}_0 = E[x_0], \quad (2)$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T], \quad (3)$$

$$W_0^{(m)} = (w_0 - 1) / \alpha^2 + 1,$$

$$W_0^{(c)} = W_0^{(m)} + 1 + \beta - \alpha^2, \quad (4)$$

$$W_i^{(m)} = W_i^{(c)} = w_i / \alpha^2,$$

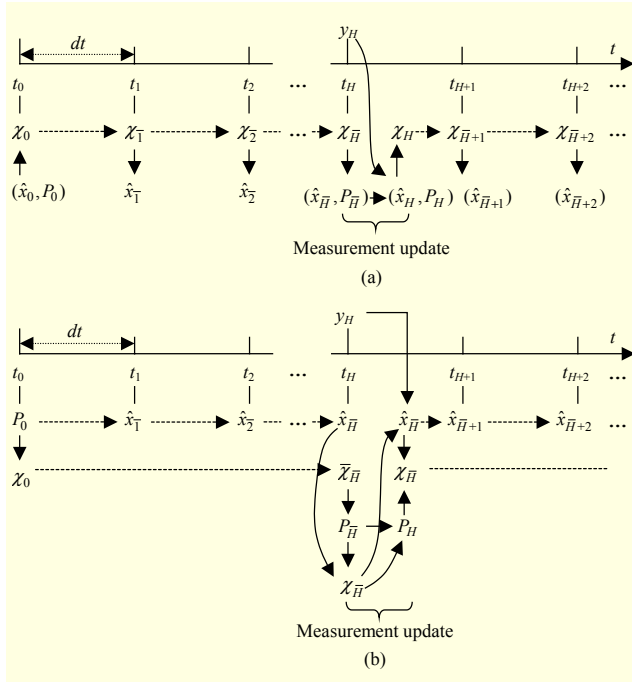


Fig. 1. Structures of (a) conventional UKF and (b) modified UKF.

where  $10^{-4} \leq \alpha \leq 1$  and  $0 \leq w_0 \leq 1$  are the scaling parameters and  $\beta$  is a parameter used to minimize the higher-order effects, which are minimized when  $\beta$  is 2 in a Gaussian distribution. In this letter,  $\alpha$  and  $w_0$  are set to 0.1. In addition,

$$w_i = (1 - w_0) / (L + 1), \quad i = 1, 2, \dots, 16. \quad (5)$$

2) Sigma point calculation [9]:

$$\begin{aligned} \chi_k &= \mathbf{0}_{L \times (L+2)}, \\ \chi_k(i, 2:i+1) &= \frac{-1}{\sqrt{i(i+1)w_1}}, \end{aligned} \quad (6)$$

$$\chi_k(i, i+2) = \frac{i}{\sqrt{i(i+1)w_1}},$$

$$\chi_k(1:15, j) = \hat{x}_k + \alpha S_k^T \chi_k(1:15, j),$$

where  $i = 1, 2, \dots, 15$  and  $j = 1, 2, \dots, 17$ . The matrix  $S$  can be obtained using the Cholesky decomposition as follows:

$$S_k^T S_k = P_k. \quad (7)$$

3) Time propagation: In this step, the difference between the conventional UKF and the modified UKF can be confirmed. In a conventional UKF, the time propagation of the state variables is performed based on the sigma points until the measurement is obtained. On the other hand, the state variables in the modified UKF are time propagated directly, using the system function similarly to the EKF for computational load reduction.

$$\hat{x}_k^- = f(\hat{x}_{k-1}^-, \omega_{k-1}^b, f_{k-1}^b, dt). \quad (8)$$

When the measurement is obtained, the sigma points are time propagated at intervals of  $dt \cdot H$  to calculate the time propagated covariance matrix:

$$\bar{\chi}_{tH}^- = f(\chi_{(t-1)H}^-, \bar{\omega}_{tH}^b, \bar{f}_{tH}^b, dt \cdot H), \quad (9)$$

where  $t = 1, 2, \dots$  and

$$\bar{\omega}_{tH}^b = \frac{1}{H} \sum_{i=0}^{H-1} \omega_{it}^b, \quad \bar{f}_{tH}^b = \frac{1}{H} \sum_{i=0}^{H-1} f_{it}^b. \quad (10)$$

The mean values of sensor data are used in (9). Therefore, it is assumed that changes of vehicular dynamics are small in the intervals of measurement updates.

The time propagated covariance matrix is calculated as

$$P_{tH}^- = \sum_{i=0}^{16} W_i^{(c)} [\bar{\chi}_{tH}^-(1:15, i) - \hat{x}_{tH}^-] [\bar{\chi}_{tH}^-(1:15, i) - \hat{x}_{tH}^-]^T, \quad (11)$$

where

$$\hat{x}_{tH}^- = \sum_{j=0}^{16} W_j^{(m)} \chi_{tH}^-(j). \quad (12)$$

4) Measurement update [9]:

$$\hat{x}_{tH} = \hat{x}_{tH}^- + K_{tH} (y_{tH} - \hat{y}_{tH}^-), \quad (13)$$

$$P_{tH} = P_{tH}^- - K_{tH} P_{y_{tH}y_{tH}} K_{tH}^T, \quad (14)$$

where

$$K_{tH} = P_{x_{tH}y_{tH}} P_{y_{tH}y_{tH}}^{-1}, \quad (15)$$

where

$$P_{x_{tH}y_{tH}} = \sum_{i=0}^{16} W_i^{(c)} \left[ \chi_{tH}^-(1:15, i) - \hat{x}_{tH}^- \right] \times \left[ h(\chi_{tH}^-(1:15, i)) - \hat{y}_{tH}^- \right]^T, \quad (16)$$

$$P_{y_{tH}y_{tH}} = \sum_{i=0}^{16} W_i^{(c)} \left[ h(\chi_{tH}^-(1:15, i)) - \hat{y}_{tH}^- \right] \left[ h(\chi_{tH}^-(1:15, i)) - \hat{y}_{tH}^- \right]^T, \quad (17)$$

where  $\chi_{tH}^-$  represents the calculated sigma points using the state estimate at time  $t_{tH}$  in (9) and

$$\hat{y}_{tH}^- = \sum_{i=0}^{16} W_i^{(m)} h(\chi_{tH}^-(1:15, i)). \quad (18)$$

### III. Simulation Results

Numerical simulations are performed to evaluate the performance of the proposed filter. The conditions of the simulations and specifications ( $1\sigma$ ) of the MEMS-type inertial sensors are listed below.

- Total trial number of Monte Carlo simulations: 100.
- Trajectory: S-turn, as shown in Fig. 2.
- Error distribution for initial attitude and heading:
  - leveling attitude:  $N(0, (1^\circ)^2)$ d.
  - heading:  $N(0, (45^\circ)^2)$ .
- Gyro random constant error: 0.3%/s.
- Accelerometer random constant error: 30 mg.

First, the processing time of each filter at intervals of measurement updates is calculated, as shown in Fig. 3. The mean processing times of the conventional UKF, modified UKF, and EKF in MATLAB are 0.11117 s, 0.01689 s, and 0.01545 s, respectively. The processing time of the modified UKF is 13.89% of that of the conventional UKF and is less than that of the EKF.

Basically, the performance of the filter for the LVNS depends significantly on the initial heading error. To confirm the performance of estimating the heading angle of each filter, the initial estimate of the heading angle is randomly generated with a normal distribution. The estimation errors of the heading angle are then denoted as in Fig. 4. The standard deviation ( $1\sigma$ ) of a heading error decreases from a large initial value. When the heading error approaches a steady state value, the mean of the heading error is almost 0 degrees. The performance of the modified UKF is approximately equal to that of the

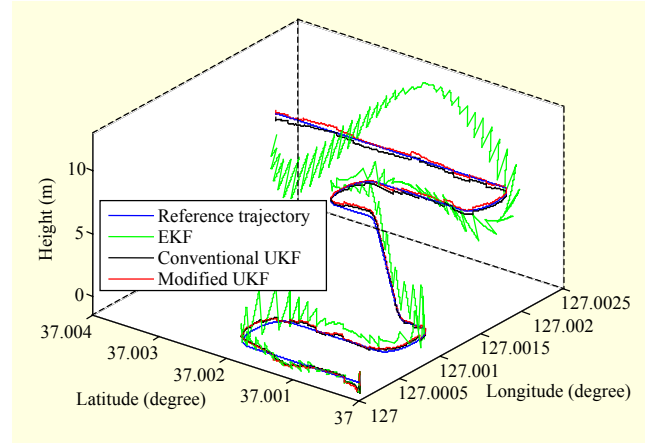


Fig. 2. Trajectory.

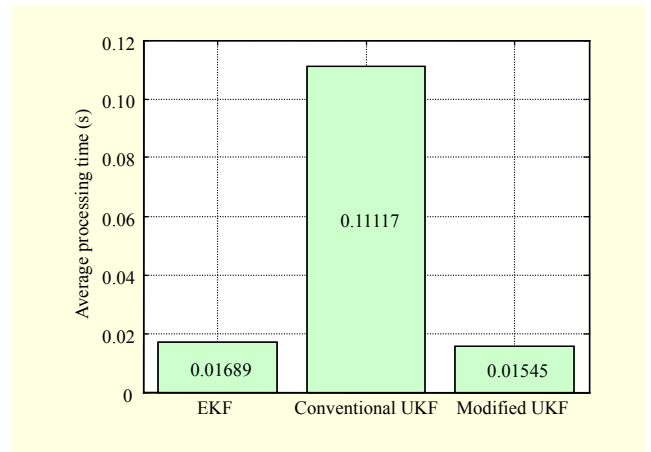


Fig. 3. Averaging processing time of each filter.

conventional UKF.

Another simulation is carried out. The simulation conditions are the same as those of the previous simulation with the exception that the initial heading error is fixed at 90 degrees, a very large value. Figure 5 shows that the convergence speed of the EKF is slow owing to the erroneous Jacobian matrix. On the other hand, the convergence speed of each UKF is fast because the UKFs have less of an effect on the large initial heading error than does the EKF. Moreover, it is clear that the heading estimation performance of the modified UKF is similar to that of the conventional UKF with a large heading estimation error. Therefore, each UKF can successfully estimate the heading error, unlike the EKF. As shown in Fig. 2, the position estimates are denoted when the initial heading error is set to 45 degrees. As shown in this figure, the two UKFs perform well. Notably, the modified UKF can accurately estimate the position even in the case of a large initial heading error. Analyzing the results, it is clear that the proposed modified UKF has not only an innovative computational advantage but also a strong estimation

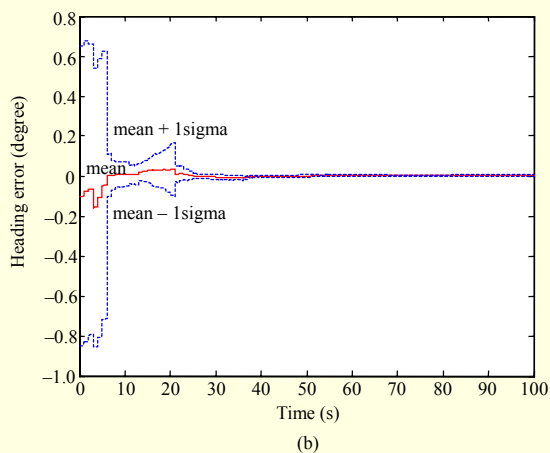
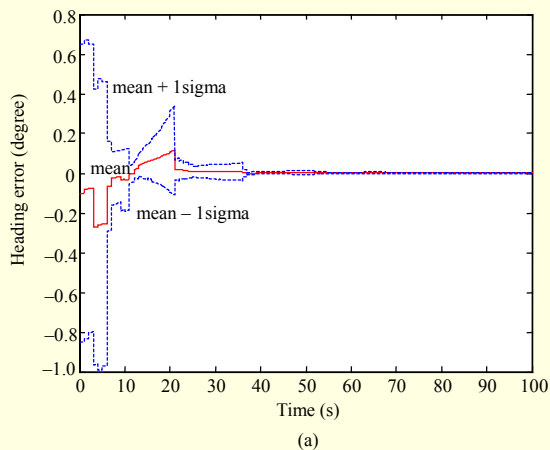


Fig. 4. Heading errors for (a) conventional and (b) modified UKF.

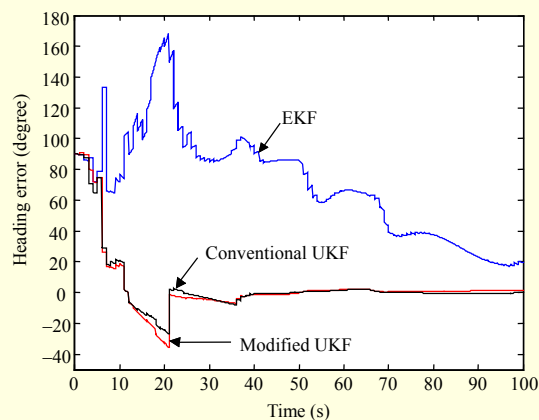


Fig. 5. Heading error comparison.

performance compared to the other filters.

#### IV. Conclusion

A modified UKF was proposed for a multirate INS/GPS

integrated system. To evaluate the efficiency of the proposed filter, a Monte Carlo simulation was performed. In the simulation, the heading estimation performance of the modified UKF was compared with that of a conventional UKF and EKF for the case in which the initial heading estimation error is large. It was shown that the proposed modified UKF has a computational advantage over the conventional UKF. In addition, the performance of the modified UKF is similar to that of the conventional UKF.

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